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Games as a means of motivating more students to participate in argumentation

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Mathematical argumentation and proof require students to engage in complex thought processes in which they explore logical connections, solve problems, and learn to trust their own assertions. While in most mathematics classrooms there are some students who show an aptitude for trusting their own reasoning and making logical inferences, this does not generally hold true for the majority of students. In the context of playing games, however, many students overcome such obstacles and become motivated to use logical reasoning and argumentation. In this paper I examine the potential of exploiting logical structures in games as a means of fostering the motivation of students to engage in inner-mathematical argumentation. A three-step-approach leading from a board game into argumentation in calculus is introduced and indications towards the usefulness of the approach for fostering students’ motivation are presented.

Argumentation, logical games, calculus, motivation.

Argumentation – a mathematical activity for the few?

Mathematics is a discipline based on logical thinking and inferences, with argumentation and proof at its heart. Proving activities in mathematics provide opportunities to autonomously discover mathematical knowledge and reach a deeper understanding (de Villiers, 1990) and can therefore promote an experience of empowerment to students. They may also teach us how to reason logically outside of mathematics (Grabiner, 2012). The mathematics classroom should thus aim at engaging all students in “suitable mathematical activities of argumentation and proof” (Boero, 2011, p. 1). This demand is even more pressing in the light of existing gaps in the accessibility of high level mathematical activities for students from different backgrounds (Boaler, 2016, p. 173).

Argumentation and proof threaten to act as social filters that may increase performance gaps between students of different backgrounds if they are only accessible to few students (Knipping, 2012).

Being important mathematical activities, argumentation and proof have been receiving increasing attention in mathematics education and in school curricula for decades. However, in spite of efforts made to promote argumentation in class, only one in three students reported in the PISA study of 2012 that they like to engage in complex problem solving activities (OECD, 2013, p. 67f), and about 30% of students reported a feeling of helplessness when doing mathematics problems (OECD, 2013, p. 88f). While problem solving is not synonymous to arguing and proving, such tasks almost always rely heavily on logical reasoning, and the results may be taken as an indication for students’ attitudes towards argumentation activities: few students appear to enjoy such activities, and a significant number of students is reluctant to engage in them at all. This is surprising, as more than half of the same students questioned for PISA 2012 stated that they sought explanations for
things and could easily link facts together (OECD, 2013, p. 67f), and both of these skills are crucial for argumentation.

Mathematical argumentation is a discursive practice, into which students need to be introduced in order to be able to participate (Boaler, 2000). Logical games and puzzles may provide a suitable way of overcoming obstacles to participation, as they require forms of logical, deductive reasoning similar to the reasoning used in mathematical argumentation and proof. Clear rules, balanced starting positions for all players, and the limited scope of knowledge required for argumentation in a game context may help to facilitate students’ participation in discourse as they provide circumstances which may support fulfilling discourse ethical requirements (Habermas, 1990). These potential positive aspects of games have been discussed before (Cramer, 2014).

Another positive aspect of games lies in their potential to appeal to the intrinsic motivation of students. High motivation in games can be explained by the needs for competency, autonomy and relatedness postulated by Deci & Ryan’s (1993) self-determination theory. The motivational potential of games has been well documented for video games (Rigby & Ryan, 2011), and similar effects may be expected for board games. The need for competency describes an inner desire to master challenges and new situations. Games provide such challenges to their players. A fulfillment of the need for autonomy is given when we perceive our actions as self-guided. While this is generally difficult to achieve in a school setting, a playful approach may help to fulfill this need. Lastly, the need for relatedness describes a need for meaningful social interaction. Board games are characterized by interaction and could prove even more helpful in this area than video games.

**From game to graph: A three step approach**

In this paper, a three-step approach to fostering the motivation of students to engage in argumentation is presented. It includes several rounds of the logical game Uluru, puzzles in the logical game environment, and puzzles in the area of calculus that aim at finding the graph of a function based on certain requirements. The research was conducted in a regular mathematics class with 15-16 year-old students in their tenth year of education (E-Phase) at a German high school (Gymnasium) over the course of five double lessons on four school days. Results are based on students’ responses to three questionnaires at different points of the study. Figure 1 shows a timeline of the intervention.

![Timeline of the intervention](image)

**Fig 1: Temporal overview of the study**

**Step 1: Playing Uluru**

In the first step of the intervention, the game *Uluru* (by Lauge Luchau, published by Kosmos) was presented to the students. In Uluru’s scenario, animals in Australia transform into dream birds of different colors (white, yellow, orange, pink, red, green, blue and black) at night and fly towards
Uluru (Ayers Rock). There are exactly eight spots around the Uluru on which birds may be placed. Each player receives bird tokens in the eight different colors and a game board. At the beginning of each round, wishes are randomly generated from a set of cards to guide the placement of the dream birds. The wishes correspond either to positions around the Uluru (e.g. “on the short edge”) or to positions relative to birds of other colors (e.g. “next to the green bird”). Situations can arise in which not all wishes can be fulfilled. All players simultaneously try to place their birds according to the wishes within a set time limit. When the time has run out, points are given to the players according to the number of birds correctly placed. The game is played in several rounds.

The types of arguments generated in Uluru are very similar to typical argumentation patterns in mathematics. When evaluating the positions of the birds that each player has found, it is easy to tell whether the requirements given by the wish for a certain bird have been fulfilled or not, and the only possible results of this evaluation are true or false. Furthermore, the game naturally leads to questions characteristically mathematical, such as: “(Why) is this the only possible constellation?”, “Is it possible to fulfill all wishes?”, or “Why can’t all conditions be met at the same time?”. Thus, the game naturally provokes argumentation between players at the end of each round, especially in situations in which no player found a solution yielding eight points or when different solutions occur.

**Step 2: Solving Uluru puzzles**

For the second step of the intervention, I created eight different puzzles in which different wish constellations were depicted. Students were given the task to figure out the maximum of achievable points for each situation. Two of the puzzles could be solved in exactly one way, two could be solved in different ways, three puzzles had a maximum of seven possible points, and in one puzzle six points were the maximum. The students were asked to record and justify their solutions on a protocol sheet. They were allowed to use the game board and the bird tokens to find a solution.

![Fig 2: An example for an Uluru-puzzle (colors annotated)](image)

In the example puzzle shown in Figure 2, three wishes (white, pink and green) refer to positions around the Uluru and five wishes (yellow, orange, red, blue and black) refer to birds of other colors. This puzzle has a unique solution. Due to the wishes of the red snake and the black emu, red and yellow need to be placed on adjacent spots and the black bird must be placed opposite the red bird.
The blue bat’s wish means that black and blue need to be on adjacent spots. The orange kangaroo’s bird must be placed opposite the blue bird and thus on the same side as yellow and red. There is only one side of the Uluru that has three spots (cf. Figure 3). The wishes of the yellow dingo (shared corner with pink) and the pink lizard determine the corner in which these two dream birds must be placed.

Combined with the already discussed wishes of the other animals, there remains only one spot of the places defined by the wish of the white echidna. The green bird token takes the remaining spot.

The students were allowed to work on these puzzles with a partner. They handed in their protocols at the end of the lesson. The examination of students’ answers showed that while they were almost always capable of finding the best possible solution, many students had problems with justifying their answers. Before resuming work on Uluru puzzles in the next lesson, a plenary phase was initiated in which proof by exhaustion of cases and proof by contradiction were discussed in the context of Uluru puzzles.

**Step 3: f(u)-luru puzzles**

In a third step, the concept of $f(u)$-luru puzzles was introduced. In these puzzles, the animals of Australia wished for properties of the graph of a function. The puzzles followed the game design. To develop the graph, the students received a laminated coordinate system in a design inspired by the game board, as well as removable foil pens. Table 1 gives an example for conditions of a puzzle.

<table>
<thead>
<tr>
<th>white</th>
<th>pink</th>
<th>yellow</th>
<th>orange</th>
<th>red</th>
<th>green</th>
<th>blue</th>
<th>black</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(-1) = -0.5$</td>
<td>exactly 1 maximum</td>
<td>$f''(0) &gt; 0$</td>
<td>$f(-2) = 0$</td>
<td>$f'(0) = 0$</td>
<td>symmetrical to y-axis</td>
<td>$f(0) = 1$</td>
<td>minimum for $x = -1.5$</td>
</tr>
</tbody>
</table>

**Table 1 Conditions in the f(u)-luru-"lightning puzzle" (simplified design)**

For this puzzle, a maximum of 7 points can be achieved. The wish of green combined with the wishes of orange, white and black provide additional points $f(2) = 0$, $f(1) = -0.5$ and another minimum for $x = 1.5$. The blue and red wishes define the y-intercept as a local maximum or minimum. As the black and green wishes define two minima, pink’s wish determines that the y-intercept must be a maximum. This contradicts yellow’s wish $f''(0) > 0$. Thus, not all conditions can be fulfilled simultaneously. As seven is the second largest number of achievable points, this is an ideal solution to this puzzle. Figure 4 shows the solution.
To help students determine possible solutions and formulate arguments, they received an overview which served to clarify and simplify the possible wishes in the f(u)-luru puzzles and to make the mathematical task more playful. Figure 5 shows an excerpt of the overview. The students also received protocol sheets in a design analogous to the protocol sheets for the Uluru puzzles to document their solutions. Like in the Uluru puzzles, the students were again allowed to work in pairs. The puzzles did not ask the students to find an algebraic solution (no functional equations were expected from the students).

**From game to graph: A three step approach**

To assess motivation, the Intrinsic Motivation Inventory (IMI, McAuley et al., 1989) was used. It is a suitable tool to measure motivation based on the needs postulated by self-determination theory. The questionnaire consists of 18 items and covers the dimensions interest-enjoyment (int-enj), perceived competence (perc-comp), effort-importance (eff-imp) and tension-pressure (tens-pres). For each item, agreement or disagreement was measured on a 7-point Likert-scale. Assessments took place at three points during the intervention. All examined tasks target solving a problem involving a variety of conditions that require reasoning and justifying. The students answered to the questionnaires using a pseudonym, allowing them to freely utter criticism (the researcher being their teacher).

Questionnaire 1 (Q1) was given to the students before the intervention to act as a point of comparison for later responses (cf. Field & Hole, 2003, p. 68). It was coupled to a task from the math textbook in which the graph of a function was to be created from certain requirements. This task had briefly been discussed in class in a lesson before the intervention. Q1’s purpose was to facilitate capturing students’ attitudes towards tasks in the regular mathematics classroom involving reasoning.

The second questionnaire (Q2) was given after the students had played the game Uluru and after they had been working on Uluru-puzzles for approximately one hour. I decided to forego an assessment of motivation after playing so as not to overstrain students with too many questionnaires.

The students responded to the third questionnaire (Q3) after approximately one hour of f(u)-luru-puzzles. This last questionnaire also covered some free text questions and items focusing on an evaluation of the intervention by the students, which were not statistically validated and can thus only give an indication concerning students’ attitudes towards the intervention.
The setting of the evaluation is as a repeated measures design (Field & Hole, 2003, p. 183ff) with interdependent data, which necessitates an analysis of variance (Rasch et al., 2010). Nineteen students participated in all three questionnaires (n=19). Shapiro-Wilk tests allow the assumption of normally distributed results for all dimensions except tension-pressure (p<0.05) in Q2. Levene’s test allows to assume homogeneity of variances except in the dimension of perceived competence.

**Results**

The needs dimension of interest-enjoyment is a measure of perceived intrinsic motivation. Results for this dimension are thus very interesting, as they refer directly to how students perceived the tasks. A non-significant Mauchly-test (p = 0.53) and the clearly significant analysis of variance (F_{(2,36)} = 7.80; p < 0.01) justify the use of t-tests. Bonferroni-corrected values show significant (p < 0.01) differences between Q1 and Q2 and between Q2 and Q3. The boxplot in Figure 6 shows a positive shift of means from the first (M₁ = 3.63; SD = 1.22) to the second assessment (M₂ = 5.51; SD=0.83. The third mean M₃=4.39;SD=1.06 lies between the others. Interest and enjoyment decreased in f(u)-luru puzzles compared to the Uluru puzzles. However, the histograms of questionnaires 1 and 3 (Figures 7 and 8) also show a considerable decrease of very negative evaluations (scale range 1–2).

These results can be interpreted as a positive tendency that students preferred the f(u)-luru puzzles over classic mathematical tasks. However, this tendency needs to be treated with care as working arrangements were different for the mathematical task and for the f(u)-luru puzzles.

For the dimension of perceived competence, homogeneity of variances cannot be assumed. A closer look at the data shows that this can be explained by the high mean and small standard deviation in Q2. A comparison of means of questionnaires Q1 (M₁ = 3.93; SD = 1.85), Q2 (M₂ = 5.94; SD = 0.77, and Q3 M₃=4.4;SD=1.75 shows an increase in perceived competence for the Uluru-puzzles, followed by a decreased for f(u)-luru; the pairwise differences between Q1 and Q2 and Q2 and Q3 are significant (p<0.05). The means for perceived competence did not increase significantly between Q1 and Q3. However, as in the dimension of interest-enjoyment, the distributions of students for the lower scores are instructive. In the classic mathematical task covered in Q1, eight students perceived themselves as scarcely competent (scale range 1–2), whereas only two students gave this response in Q3 for the f(u)-luru puzzles. Taking into consideration that the latter were comparatively harder, this can be taken as an indication that students who otherwise trust little in their abilities might have benefited from the intervention.

Results for effort-importance show significant differences (p < 0.01) between Q1 (M₁ = 3.8; SD = 0.77) and the means of Q2 and Q3 (M₂ = 4.95; SD = 1.15 and M₃ = 4.7; SD = 1.02; Mauchly-
test $p = 0.38$, ANOVA $F_{(2,30)} = 7.80$ $p < 0.01$). The students appear to have put greater effort into both, solving Uluru-puzzles and f(u)-luru puzzles. These results could point to an increase in students’ readiness to get deeply involved in a given task.

Results do not allow to assume normally distributed results for the dimension of tension-pressure in Q2. However, this dimension was rated low in all assessments ($M_1 = 2.88$; $M_2 = 2.01$; $M_3 = 3.06$). The students apparently did not perceive themselves under pressure during the intervention.

The students were asked in Q3 to comment on whether they had liked f(u)-luru puzzles better than classic problems in mathematics. Some notable answers:

- I liked the f(u)-luru puzzles more, because is was possible to visualize and understand better.
- Yes, because it was something different. Not so “dry”, but another method to do math problems.
- I liked it better because for some reason I was more motivated.

The majority of students responded positively. Only one student replied: “I personally liked the Uluru puzzles better, because you didn’t need to show mathematical understanding like in the f(u)-luru puzzles. The f(u)-luru puzzles were just as tiresome as normal mathematical problems.” While this answer shows that the intervention did not manage to motivate all students equally to engage in mathematics, it underlines the positive effects of the second step of the intervention.

In addition to the IMI-questions, Q3 also covered items aimed at a more direct evaluation of the intervention, which were rated on the same 7-point Likert scale. The students’ answers to these questions show a positive evaluation. Table 3 shows the items and the means.

<table>
<thead>
<tr>
<th>Item</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>I had fun playing Uluru (the game)</td>
<td>6.65</td>
</tr>
<tr>
<td>I had fun solving Uluru puzzles</td>
<td>6.15</td>
</tr>
<tr>
<td>I had fun solving f(u)-luru puzzles</td>
<td>4.9</td>
</tr>
<tr>
<td>I was motivated by the game Uluru to make an effort in Uluru puzzles</td>
<td>5.55</td>
</tr>
<tr>
<td>I was motivated by Uluru puzzles to make an effort in f(u)-luru puzzles</td>
<td>4.53</td>
</tr>
</tbody>
</table>

Table 3 Evaluation items in questionnaire 3 and their means

**Discussion**

The three-step intervention described in this paper covered only a brief timespan of five double lessons, so only small effects may be expected. Furthermore, different working arrangements at different stages of the intervention must be considered when examining the results. However, the results presented here may be taken as an indication that game-based approaches may have motivational benefits for students. The quantitative evaluation of students’ answers yields a significant and noticeable increase of motivation for the Uluru-puzzles. While results for the f(u)-luru puzzles are not as unequivocally positive, the distribution of student responses compared to their assessment of the mathematical task in the dimensions of interest-enjoyment and perceived
competence allows for the tentative assumption that the f(u)-luru puzzles are more likely to get a larger number of students involved in mathematical thinking than classic problems. Further research is needed, both to determine the effect of the different conditions in this study (e.g. working arrangements, difficulty of the tasks in Q1 and Q3) and to look at potential long-term effects.

While this paper shows potential motivational benefits of game-based approaches, this study did not include an evaluation of students’ argumentation quality in the Uluru puzzles or in the f(u)-luru puzzles. Therefore, it is unclear whether the intervention managed to actually improve mathematical argumentation skills. This question needs to be tackled in future research. For an evaluation in this regard, a consideration of Hintikka logic can prove helpful (Soldano & Arzarello, 2017).

Interventions like the one presented in this paper can be first steps towards a better integration of more students into argumentation discourse in the classroom. Observations in class and student responses to Q3 show that students worked on all given tasks in a highly concentrated way, which naturally included arguments and justifications. Clearly positive reactions of students to the game and the game-based puzzles show that most students like to engage in logical thinking. Exploiting logical games and puzzles as a stepping stone for more students to get involved in mathematical argument seems a promising path towards more equally distributed student participation and might provide a means of tackling existing gaps in students’ access to argumentation discourse.

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