Dispersive wave emission from wave breaking

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Compiled June 4, 2013

We show that pulses undergoing wave-breaking in nonlinear weakly dispersive fibers radiate, owing to phase-matching, assisted by higher-order dispersion, of linear dispersive waves with the shock wave front. Our theoretical results perfectly explain the radiation recently observed from pulses propagating in the normal dispersion (i.e. non solitonic) regime.

OCIS codes: (060.4370) Nonlinear optics, fibers; (190.5530) Pulse propagation and temporal solitons

Bright solitons propagating in standard or photonic crystal fibers close to the zero-dispersion wavelength (ZDW) are known to emit radiation in the region of normal group-velocity dispersion (GVD). The underlying mechanism is the resonant coupling with linear dispersive waves (DW) induced by higher-order dispersion [1, 2]. DW emission plays a prominent role in supercontinuum generation [3, 4], and is still attracting a lot of interest due to the richness of the possible scenarios which range from Raman-induced DW trapping [5] as opposed to dynamics dominated by soliton recoil in Raman-free settings [6], to the demonstration of new frequencies generated via coupling to negative frequencies [7]. More recently the idea that DW emission would be strictly related to solitons has been challenged by experimental results that have demonstrated DW emission occurring from a pump pulse propagating in the normal GVD regime [8], where the nonlinearity enforces the dispersive broadening of the pulse. The aim of this letter is to give a theoretical foundation to this observation by showing that, in this regime, DW emission stems from the wave-breaking phenomenon [9–15]. In particular, we show that the dispersive shock waves (DSW, similar to those arising in the spatial case [14, 15]), which develop in the regime of weak dispersion, resonantly amplify DW at frequencies given by phase-matching selection rules, where the DSW velocity plays a determinant role.

Specifically, we investigate the regime of pulse durations and powers considered in Ref. [8], where wave-breaking is driven, as we show below, by the Kerr effect in the regime of weak dispersion. Nevertheless, for sake of completeness, we describe the nonlinear pulse propagation by means of the Generalized Nonlinear Schrödinger Equation (GNLSE) [4]:

$$i\partial_z A + d(\partial_t)A + \gamma\left(1 + i\tau_s\partial_t\right)\left(A\int R(t')|A(t-t')|^2\right) = 0, \tag{1}$$

where $d(\partial_t) = \sum_{n\geq 2}\frac{\beta_n}{n!}(i\partial_t)^n$ is the dispersion operator, γ is the nonlinear coefficient, $R(t) = (1 - f_R)\delta(t) + f_R h_R(t)$ includes both instantaneous (Kerr) and Raman response ($f_R = 0.18$), $\tau_s \approx 1/\omega_p$ is the self-steepening time, ω_p is the pump carrier frequency, around which $d(\partial_t)$ is expanded, and t is the retarded time in the frame traveling at natural group velocity $V_g = V_g(\omega_p) = \beta_1^{-1}$. For definiteness we consider a standard telecom fiber (Corning MetroCor) with nonlinear and dispersion parameters as follows: $\gamma = 2.5$ $\mathrm{W^{-1}km^{-1}}$, $\beta_2 = 6.4$ $\mathrm{ps^2/Km}$, $\beta_3 = 0.134$ $\mathrm{ps^3/Km}$, and $\beta_4 = -9\times 10^{-4}$ $\mathrm{ps^4/Km}$ (higher-order terms are negligible), which gives a ZDW $\lambda_{ZDW} = 1625$ nm [8].

The shock formation process can be described, neglecting for the time being Raman ($f_R = 0$) and self-steepening ($\tau_s = 0$) that play a minor role (see below), by applying a Madelung transform $A(z,t) = \sqrt{\rho(z,t)}\exp[-i\int^t u(z,t')dt']$. In the small dispersion (or highly nonlinear) regime [12–15], we derive the following system of conservation laws of hydrodynamic type $\mathbf{q}_z + \mathbf{f}_t(\mathbf{q}) = 0$, with $\mathbf{q} = (\rho, \rho u)$ standing for equivalent mass and momentum of the flow [13]:

$$\rho_z + \left[\beta_2\,\rho u + \frac{\beta_3}{2}\,\rho u^2 + \frac{\beta_4}{6}\,\rho u^3\right]_t = 0, \tag{2}$$

$$(\rho u)_z + \left[\beta_2\,\rho u^2 + \frac{\beta_3}{2}\,\rho u^3 + \frac{\beta_4}{6}\,\rho u^4 + \frac{\gamma}{2}\rho^2\right]_t = 0. \tag{3}$$

The derivation of Eqs. (2-3) implies to ignore higher order derivatives or averaging over the fast time oscillations of the solutions of Eq. (1), which corresponds to the WKB procedure. The goodness of this approximation is measured by the smallness of the parameter $\varepsilon = \sqrt{L_{nl}/L_d}$, being $L_{nl} = (\gamma P_0)^{-1}$ and $L_d = T_0^2/\beta_2$ the nonlinear and dispersion lengths associated with the input peak power P_0 and pulse duration T_0 [14, 15].

Since Eqs. (2-3) turn out, for small $\beta_{3,4}$, to be hyperbolic, they admit weak solutions in the form of classical shock waves, i.e. traveling jumps from left (ρ_l, u_l) to right (ρ_r, u_r) values, whose velocity V_c can be found from the so-called Rankine-Hugoniot condition $V_c(\mathbf{q}_l - \mathbf{q}_r) = [\mathbf{f}(\mathbf{q}_l) - \mathbf{f}(\mathbf{q}_r)]$ [16]. However, the jump is regularized by GVD in the form of a DSW, i.e. an expanding fan filled with oscillations described in terms of a modulated nonlinear periodic wave [10, 14, 15]. In this regime, the shock velocity can be identified with the velocity V_s of the steep front near the deepest oscillation (DSW leading edge), which differs from V_c and can be determined only numerically.

This scenario is illustrated in Fig. 1, that shows numerically evaluated profiles from an input pulse $A_0(t) = \sqrt{P_0}\mathrm{sech}(t/T_0)$ with $P_0 = 600$ W and $T_0 = 850$ fs, after 20m of propagation. Integration of Eqs. (2-3) by means of a Lax-Wendorff scheme [16] shows the formation of traveling jumps (green dots). The process is asymmetric on the leading and trailing edges of the pulse due to the effect of β_3.

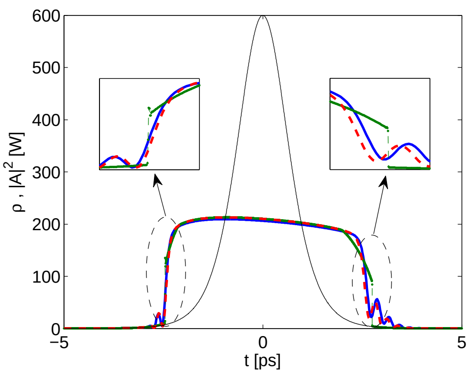

Fig. 1. (Color online) Power profiles $\rho = |A|^2$ at $z = 20$m, comparing numerical solutions of Eqs. (2-3) (green dots) with those of the full GNLSE Eq. (1) (solid blue curve), and GNLSE with only β_3 perturbation ($\tau_s = f_R = \beta_4 = 0$, dashed red curve). Thin black curve is the input sech-shaped pulse at $\lambda_p = 1568.5$ nm. Here $\varepsilon = 0.077$.

In particular $z = 20$m nearly corresponds to the breaking distance of the leading edge, whereas on the trailing edge breaking occurs at $z \simeq 13$m (at $z = 20$ the different velocity V_s of the DSW front with respect to V_c starts to become visible in Fig. 1). This dynamic is essentially reproduced by Eq. (1) [see solid blue line], as expected due to the small value $\varepsilon = 0.077$. Beyond wave-breaking, Eqs. (2-3) cease to be valid, and the jumps are replaced by oscillating fronts (see insets). The dashed red line in Fig. 1, obtained by retaining in Eq. (1) only $\beta_{2,3}$ terms ($\tau_s = f_R = \beta_4 = 0$) proves that the phenomenon is essentially driven by the Kerr effect.

The strong spectral broadening that accompanies steep front formation can act as an efficient seed for DW which are phase-matched to the shock in its moving frame at velocity V_s. In such frame, linear waves have wavenumber $\tilde{k}(\omega) = [k(\omega) - \omega/V_s]$ ($k(\omega) = \omega n(\omega)/c$ is the full dispersion) whereas the pump has wavenumber $\tilde{k}_p = k(\omega_p) - \omega_p/V_s$. By expanding $\tilde{k}(\omega)$ around ω_p and setting $\Delta\omega = \omega - \omega_p$, we find that the phase-matching condition $\tilde{k}(\omega) = \tilde{k}_p$, is fulfilled at frequency detunings $\Delta\omega = \Delta\omega_{DW}$ that solve the explicit equation

$$\sum_{n=2}^{4} \frac{\beta_n}{n!} \Delta\omega^n - \Delta\omega\Delta k_1 = k_{NL}, \qquad (4)$$

where $\Delta k_1 = V_s^{-1} - V_g^{-1}$ is the inverse velocity mismatch (with respect to V_g), and we included also the nonlinear contribution k_{NL} due to the pump. Assuming β_4 and k_{NL} to be negligible in Eq. (4), we obtain explicitly

$$\Delta\omega_{DW} = \frac{-3\beta_2}{2\beta_3}\left(1 + \sqrt{1 + \Delta k_1 \frac{\beta_3}{6\beta_2^2}}\right), \qquad (5)$$

which, in the limit of negligible velocity mismatches (i.e., $|\Delta k_1| \ll \frac{6\beta_2^2}{|\beta_3|}$), reduces to the simple formula [8]

$$\Delta\omega_{DW} \approx -3\frac{\beta_2}{\beta_3}. \qquad (6)$$

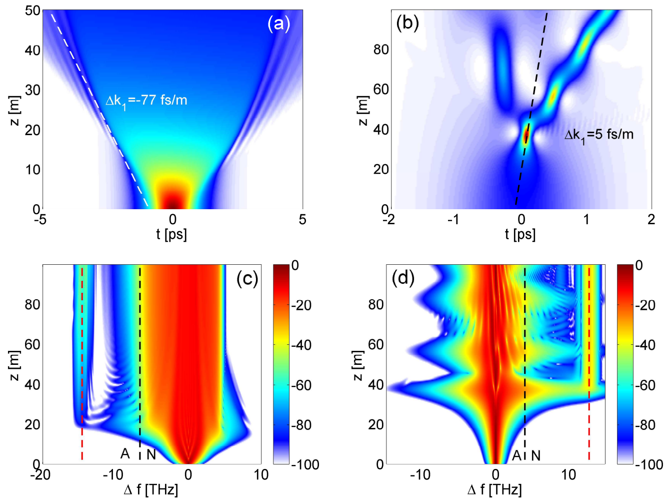

Fig. 2. (Color online) Temporal (a) and spectral (c) evolution of an input sech pulse $P_0 = 600$ W, $T_0 = 850$ fs, at $\lambda_p = 1568.5$ nm (normal GVD). (b,d) Same with anomalous GVD ($\lambda_p = 1661$ nm), $P_0 = 40$ W. In (c,d) A/N labels anomalous/normal GVD regions, and the dashed red lines stand for the DW detuning predicted by Eq. (4) with inverse velocity Δk_1 (of shock and soliton, respectively) given by oblique dashed lines in (a,b).

We show that, while k_{NL} can be safely neglected in the regime considered below, Eq. (6) describes with good accuracy only DW emitted by solitons, characterized by $\Delta k_1 \approx 0$ [2]. Conversely, DSWs possess always non-zero velocity mismatches Δk_1 that strongly affect the determination of $\Delta\omega_{DW}$, thus requiring to use Eq. (4).

Figure 2 contrasts the two typical situations encountered when pumping in the normal (a-c) or anomalous (b-d) GVD regime, respectively. The DSW forming on the leading pulse edge in the temporal evolution in Fig. 2(a) is responsible for the resonant amplification of the DW at an activation distance nearly coincident with the breaking distance at which maximal spectral broadening also occurs [see Fig. 2(c)]. The velocity V_s of the DSW leading edge (dashed white line) turns out to be significantly different from V_g. By accounting for such mismatch ($\Delta k_1 = -77$ fs/m) in Eq. (4), we are able to accurately predict the DW frequency through the phase-matching curve shown in Fig. 3(a) [see also the corresponding dashed red line in Fig. 2(c)]. Although also the DSW on the trailing edge can in principle radiate, Eq. (4) predicts for positive and large Δk_1, a DW so strongly detuned ($\Delta\omega_{DW} \simeq -23$ THz in this case) that the seeding mechanism is no longer effective.

When the pump is a higher-order soliton (anomalous GVD) the onset of the DW occurs at the distance of maximal temporal compression (or spectral broadening), $z \simeq 40$m in Fig. 2(b)-(d). The substantial difference, in this case, is that the maximally compressed pulse has a (local) inverse velocity 15 times smaller than the previous case. As a result Δk_1 turns out to be totally negligible in the phase-matching curve in Fig. 3(b) and in the determination of $\Delta\omega_{DW}$ [also shown as a dashed red line in Fig. 2(d)].

We then performed a systematic study in a 100 m long fiber using the same input pump pulse though varying its

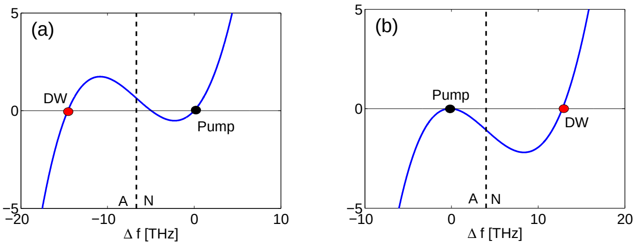

Fig. 3. (Color online) Phase matching curves from Eq. (4): (a) normal GVD regime (λ_p = 1568.5 nm, 7.23 THz from ZDW); (b) anomalous GVD regime (λ_p = 1661 nm, -4 THz from ZDW). The red markers give the DW frequency detuning, and the vertical dashed line stands for the ZDW.

frequency. The results of our simulations are summarized in Fig. 4, where we plot the DW detunings as a function of pump detuning (both referred to ZDW). Note that we increase the power P_0 (as indicated in Fig. 4) as the detuning from ZDW (and hence the effective GVD) grows in order to maintain the wavebreaking and achieve a sufficient spectral broadening to seed the DW. In the anomalous GVD regime (negative detunings), simulation results (solid markers) are reasonably well fitted by the simplified formula (6) (dashed blue line), and the agreement improves when β_4 is account for in Eq. (4) (solid red curve). In fact the inverse velocity of the maximal compressed pulse that sheds the DW is extremely low, as shown in Fig. 5. The scenario is totally different in the normal GVD regime (positive detunings), where neither Eq. (6), nor the inclusion of β_4 fit the simulation results. Whereas, when the inverse velocity shown in Fig. 5 is accounted for, the agreement is nearly perfect. Interestingly enough, the inverse shock velocity scales linearly with pump detuning and is nearly independent on the pump power, a fact that does not simply follow from scaling arguments and need further assessment.

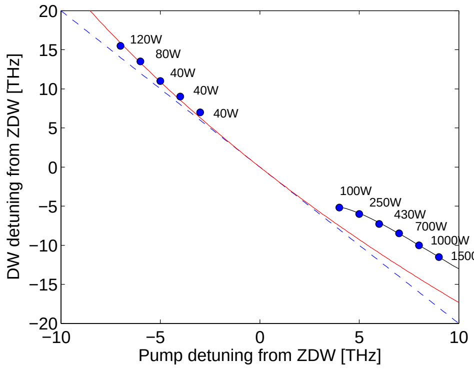

Fig. 4. (Color online) DW detuning as a function of pump detuning (both from ZDW), contrasting numerical simulations (filled circles) with the prediction from Eq. (6) (dashed blue line) and from Eq. (4) with $\Delta k_1 = 0$ (solid red curve). The solid black line refers to the prediction from Eq. (4) with inverse velocity Δk_1 shown in Fig. 5.

In summary, we have shown that the frequency of DW shed by pulses propagating in the normal GVD regime can

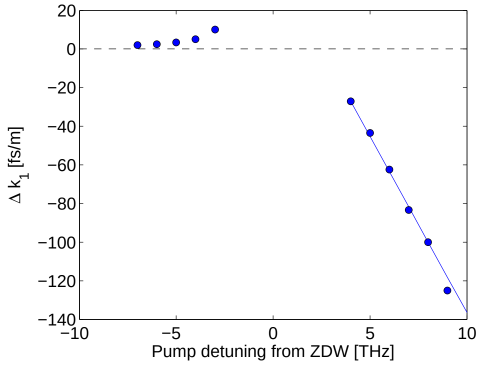

Fig. 5. (Color online) Inverse velocity shifts $\Delta k_1 = V_s^{-1} - V_g^{-1}$ (filled circles) vs. pump detuning from ZDW for the shock (positive detunings) and solitons (negative detunings). Solid blue line: linear best-fit for Δk_1.

be accurately predicted on the basis of phase-matching arguments which involve the velocity of the shock produced via wave-breaking.

Funding from MIUR (grant PRIN 2009P3K72Z) is gratefully acknowledged.

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