A new dynamical repulsive fractional potential for UAV in 3D dynamical environment

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LabRI / IMS

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Abstract

1 Introduction

2 Attractive potential field

3 Repulsive potential field

4 Dynamical fractional repulsive field

5 Conclusion
Introduction

Field of application

- Delivery
- First aid
- Surveillance
Introduction

Field of application

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- First aid
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Introduction

Path planning

Path planning: motions to bring the system from an initial state to an end state, with a time reference.

- Offline algorithms
  - Optimal
  - Static environment
- Online algorithms
  - Dynamic environment
  - Not optimal
**Introduction**

**Scenario**

Fundamental principle of dynamics:

\[ M_{\text{ego}} a_{\text{ego}} = F_{\text{att}} + F_{\text{rep}} \]

\[ s^2 M_{\text{ego}} X(s) = \]

\[ F_{\text{att}}(s) + \sum_{i=1}^{N} F_{\text{rep}}(s) \]

\[ X(s) = \frac{F_{\text{att}}(s) + \sum_{i=1}^{N} F_{\text{rep}}(s)}{s^2 M_{\text{ego}}} \]
First potential field for mobile robots (Khatib 1985) $^1$

- The robot is attracted to the target
- $U_{\text{att}}(p) = \frac{1}{2}k \|p_{\text{tar}} - p_{\text{ego}}\|^2$

1. Khatib, O. (1985), Real-time obstacle avoidance formanipulators and mobile robots, IEEEInternational Conference on Robotics and Automation
Introduction

The first potential field

- First potential field for mobile robots (Khatib 1985)

\[ U_{\text{rep}}(p) = \frac{k}{2} \left( \frac{1}{\|p_{\text{obs}} - p_{\text{ego}}\|} - \frac{1}{\rho_{\text{min}}} \right)^2 \]

Introduction

The first potential field

- First potential field for mobile robots (KhATIB 1985)

\[ U_{\text{tot}}(p) = U_{\text{att}}(p) + U_{\text{rep}}(p) \]

**Introduction**

The first potential field

- First potential field for mobile robots (Khatib 1985)

  - Movement is generated by moving down the field following the lowest gradient
  - \( F(p) = -\text{grad} \left( U_{\text{tot}}(p) \right) \)

Attractive potential field

Ge & Cui method

- A method in a dynamic environment is proposed by GE et Cui 2002
- Taking speed into account

2. Ge, SS S et YJ J Cui (2002), Dynamic Motion Planning for Mobile, Electrical Engineering
A method in a dynamic environment is proposed by Ge et Cui 2002

Taking speed into account
Attractive potential field

Attractive potential

\[
F_{\text{att}} = F_{\text{att}_p} + F_{\text{att}_v} = \alpha_p (p_{\text{tar}} - p_{\text{ego}}) + \alpha_v (v_{\text{tar}} - v_{\text{ego}})
\]
An analogy by a control loop is proposed by Metoui et al. 2009\textsuperscript{3}

\[ F_{\text{att}}(s) = (\alpha_p + \alpha_v s) E(s) \]

\footnotesize
\textsuperscript{3} Metoui, B. et al. (2009), Robust path planning for dynamic environment based on fractional attractive force, 6th International Multi-Conference on Systems, Signals and Devices
An analogy by a control loop is proposed by Metoui et al. 2009

\[ F_{\text{att}}(s) = (\alpha_p + \alpha_v s) E(s) \]
Attractive potential field

For causality reasons, the proportional derivative (or lead-phase) controller above is put under a proper form, namely (Receveur 2019⁴)

\[
F_{\text{att}} = \frac{\alpha_p + \alpha_v s}{1 + \frac{s}{\omega_c}} E(s) = \alpha_p \left(\frac{1 + \frac{\alpha_v}{\alpha_p} s}{1 + \frac{s}{\omega_c}}\right) E(s) = C_0 \left(\frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_h}}\right)
\]

⁴ Receveur (2019), New interpretation of fractional potential fields for robust path planning, Fractional Calculus and Applied Analysis
Attractive potential field

For causality reasons, the proportional derivative (or lead-phase) controller above is put under a proper form, namely (RECEVEUR 2019)

\[ F_{\text{att}} = \frac{\alpha_p + \alpha_v s}{1 + \frac{s}{\omega_c}} E(s) = \alpha_p \left( \frac{1 + \frac{\alpha_v s}{\alpha_p}}{1 + \frac{s}{\omega_c}} \right) E(s) = C_0 \left( \frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_h}} \right) \]

4. Receveur (2019), New interpretation of fractional potential fields for robust path planning, Fractional Calculus and Applied Analysis
For the simulation, $m_{\text{ego}} = 1.5\, \text{kg}$, the desired time response is $t r_{5\%} = 3\, \text{s}$, which gives $\omega_{cg} = 1\, \text{rad/s}$. And a stability degree of phase margin $M_\phi = 60^\circ$.

<table>
<thead>
<tr>
<th>parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>$60^\circ$</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>$0.05, \text{rad/s}$</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>$20, \text{rad/s}$</td>
</tr>
<tr>
<td>$C_0$</td>
<td>$0.075$</td>
</tr>
</tbody>
</table>
The repulsive fields will be considered as disturbances in the control loop.
Repulsive potential field

Ge & Cui's repulsive force is divided into two forces, one in position and the other in speed
\[ F_{rep} = -\nabla_p U_{rep}(p, v) - \nabla_v U_{rep}(p, v) \]

- \[ F_{repu} = -\eta \frac{(\rho_s(p, p_{obs}) - \rho_m(v_{RO}))^2}{(\rho_s(p, p_{obs}) - \rho_m(v_{RO}))^2} \left(1 + \frac{v_{RO}}{a_{max}}\right)^n_{RO} \]
- \[ F_{repp} = \frac{\eta v_{RO} v_{RO} \rho_s(p, p_{obs}) a_{max} (\rho_s(p, p_{obs}) - \rho_m(v_{RO}))^2}{(\rho_s(p, p_{obs}) - \rho_m(v_{RO}))^2} n_{RO} \]
### Repulsive potential field

#### Repulsive Ge & Cui method

<table>
<thead>
<tr>
<th>Method</th>
<th>Ge &amp; Cui</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>good</td>
</tr>
<tr>
<td>Energy</td>
<td>good</td>
</tr>
<tr>
<td>Distance</td>
<td>average</td>
</tr>
<tr>
<td>Dynamic environment</td>
<td>good</td>
</tr>
<tr>
<td>Dangerousness of the obstacle</td>
<td>bad</td>
</tr>
<tr>
<td>Smooth path</td>
<td>bad</td>
</tr>
</tbody>
</table>
The potential field is generalized by Poty 2004 \(^5\)

\[
U_{rep}^n(\rho) = \begin{cases} 
\rho^n - \rho_{max}^{n-2}, & \forall n \in ]0; 2[ \cup ]2; +\infty[, \\
\rho_{min}^{n-2} - \rho_{max}^{n-2}, & \forall \rho \in [\rho_{min}; \rho_{max}] \\
\frac{\ln(\rho_{max}) - \ln(\rho)}{\ln(\rho_{max}) - \ln(\rho_{min})}, & n = 2, \forall \rho \in [\rho_{min}; \rho_{max}] \\
0, & \forall \rho > \rho_{max}
\end{cases}
\]

5. Poty, A. (2004), Dynamic path planning for mobile robots using fractional potential field
Repulsive potential field

Influence distance of the normalized fractional potential based on Weyl’s definition:

\[ U_{n}^{rep}(\rho) = \begin{cases} \frac{\rho^{n-2} - \rho_{max}^{n-2}}{\rho_{min}^{n-2} - \rho_{max}^{n-2}}, & \forall n \in ]0; 2[ \cup ]2; +\infty[, \\ \frac{\ln(\rho_{max}) - \ln(\rho)}{\ln(\rho_{max}) - \ln(\rho_{min})}, & n = 2, \forall \rho \in [\rho_{min}; \rho_{max}] \\ 0, & \forall \rho > \rho_{max} \end{cases} \]
**Repulsive potential field**

Compare method

<table>
<thead>
<tr>
<th>Method</th>
<th>Weyl potential</th>
<th>Ge &amp; Cui</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>average</td>
<td>good</td>
</tr>
<tr>
<td>Energy</td>
<td>bad</td>
<td>good</td>
</tr>
<tr>
<td>Distance</td>
<td>bad</td>
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<tr>
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</tr>
<tr>
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<td>good</td>
<td>bad</td>
</tr>
</tbody>
</table>
Dynamical fractional repulsive potential definition

Definition

\[ U_{\text{rep}}(p, v) = \frac{(\rho_s(p, p_{\text{obs}}) - \rho_m(v_{RO}))^{n-2} - \rho_{\text{max}}^{n-2}}{\rho_{\text{min}}^{n-2} - \rho_{\text{max}}^{n-2}} \]

As for Ge & Cui, the force will be divided into 2 forces

\[ F_{\text{repv}} = \eta \left( n-2 \right) \frac{(\rho_s(p, p_{\text{obs}}) - \rho_m(v_{RO}))^{n-3} v_{RO}}{\rho_s(p, p_{\text{obs}}) a_{\text{max}} \left( \rho_{\text{min}}^{n-2} - \rho_{\text{max}}^{n-2} \right)} v_{RO} \perpendicular n_{RO} \perpendicular \]

\[ F_{\text{repp}} = \eta \left( n-2 \right) \frac{(\rho_s(p, p_{\text{obs}}) - \rho_m(v_{RO}))^{n-3} \left( 1 + \frac{v_{RO}}{a_{\text{max}}} \right)}{\left( \rho_{\text{min}}^{n-2} - \rho_{\text{max}}^{n-2} \right)} n_{RO} \]

the repulsive potential field function takes into account an order \( n \) to manage obstacle avoidance according to its dangerousness and the obstacle speed to operate in a dynamical environment
Dynamical fractional repulsive potential definition

Definition

- $U_{\text{rep}}(p, v) = \frac{(\rho_s(p,p_{\text{obs}}) - \rho_m(v_{RO}))^{n-2} - \rho_{n}^{-2}}{\rho_{\text{min}}^{-n} - \rho_{\text{max}}^{-n}}$

- As for Ge & Cui, the force will be divided into 2 forces
  - $F_{\text{repv}} = \eta \frac{(n-2)(\rho_s(p,p_{\text{obs}}) - \rho_m(v_{RO}))^{n-3} v_{RO}}{\rho_s(p,p_{\text{obs}}) a_{\text{max}} \left(\rho_{\text{min}}^{-n} - \rho_{\text{max}}^{-n}\right)} v_{RO} \perp n_{RO} \perp$
  - $F_{\text{repp}} = \eta \frac{(n-2)(\rho_s(p,p_{\text{obs}}) - \rho_m(v_{RO}))^{n-3} \left(1 + \frac{v_{RO}}{a_{\text{max}}}\right)}{\left(\rho_{\text{min}}^{-n} - \rho_{\text{max}}^{-n}\right)} n_{RO}$

- the repulsive potential field function takes into account an order $n$ to manage obstacle avoidance according to its dangerousness and the obstacle speed to operate in a dynamical environment
**Dynamical fractional repulsive potential definition**

**Definition**

\[ U_{rep}(p, v) = \frac{(\rho_s(p, p_{obs}) - \rho_m(v_{RO}))^{n-2} - \rho_n^{n-2}}{\rho_{min}^{n-2} - \rho_{max}^{n-2}} \]

As for Ge & Cui, the force will be divided into 2 forces

\[ F_{repv} = \eta \frac{(n-2)(\rho_s(p, p_{obs}) - \rho_m(v_{RO}))^{n-3}v_{RO}}{\rho_s(p, p_{obs})a_{max}(\rho_{min}^{n-2} - \rho_{max}^{n-2})} v_{RO} \perp n_{RO} \perp \]

\[ F_{repp} = \eta \frac{(n-2)(\rho_s(p, p_{obs}) - \rho_m(v_{RO}))^{n-3}(1 + \frac{v_{RO}}{a_{max}})}{(\rho_{min}^{n-2} - \rho_{max}^{n-2})} n_{RO} \]

the repulsive potential field function takes into account an order \( n \) to manage obstacle avoidance according to its dangerousness and the obstacle speed to operate in a dynamical environment.
## Simulation

Initial conditions:

<table>
<thead>
<tr>
<th>Element</th>
<th>Position</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGV</td>
<td>[0, 0, 10]</td>
<td>[0, 0, 0]</td>
</tr>
<tr>
<td>Obstacle 1</td>
<td>[20, 25, 10]</td>
<td>[0, 0, 0]</td>
</tr>
<tr>
<td>Obstacle 2</td>
<td>[90, 95, 10]</td>
<td>[0, 0, 0]</td>
</tr>
<tr>
<td>Obstacle 3</td>
<td>[70, 70, 10]</td>
<td>[0, 0, -1]</td>
</tr>
<tr>
<td>Target</td>
<td>[120, 120, 10]</td>
<td>[0, 0, 0]</td>
</tr>
</tbody>
</table>
Dynamical fractional repulsive potential definition

Simulation
New definition

Simulation

Comparative study

<table>
<thead>
<tr>
<th>Method</th>
<th>( n )</th>
<th>Time (s)</th>
<th>Distance (m)</th>
<th>Energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge &amp; Cui</td>
<td>/</td>
<td>74.45</td>
<td>176.73</td>
<td>1.325 \times 10^5</td>
</tr>
<tr>
<td>Weyl potential</td>
<td>0.5</td>
<td>78.70</td>
<td>187.44</td>
<td>1.406 \times 10^5</td>
</tr>
<tr>
<td>dyn. frac. pot.</td>
<td>0.5</td>
<td>72.35</td>
<td>175.36</td>
<td>1.315 \times 10^5</td>
</tr>
</tbody>
</table>
Dynamical fractional repulsive potential definition

Simulation

Test order:
Comparative study

<table>
<thead>
<tr>
<th>Method</th>
<th>$n$</th>
<th>Time (s)</th>
<th>Distance (m)</th>
<th>Energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dyn. frac. pot.</td>
<td>0.2</td>
<td>71.67</td>
<td>174.94</td>
<td>$1.312 \times 10^5$</td>
</tr>
<tr>
<td>dyn. frac. pot.</td>
<td>0.5</td>
<td>72.35</td>
<td>175.36</td>
<td>$1.315 \times 10^5$</td>
</tr>
<tr>
<td>dyn. frac. pot.</td>
<td>0.8</td>
<td>72.93</td>
<td>175.69</td>
<td>$1.318 \times 10^5$</td>
</tr>
<tr>
<td>dyn. frac. pot.</td>
<td>1</td>
<td>73.25</td>
<td>175.85</td>
<td>$1.319 \times 10^5$</td>
</tr>
<tr>
<td>dyn. frac. pot.</td>
<td>1.5</td>
<td>73.71</td>
<td>176.21</td>
<td>$1.322 \times 10^5$</td>
</tr>
</tbody>
</table>
## Conclusion

### Repulsive potential

<table>
<thead>
<tr>
<th>Method</th>
<th>Weyl potential</th>
<th>Ge &amp; Cui</th>
<th>Dynamical fractional potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>average</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>Energy</td>
<td>bad</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>Distance</td>
<td>bad</td>
<td>average</td>
<td>good</td>
</tr>
<tr>
<td>Dynamic environment</td>
<td>bad</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>Dangerousness</td>
<td>good</td>
<td>bad</td>
<td>good</td>
</tr>
<tr>
<td>Smooth path</td>
<td>good</td>
<td>bad</td>
<td>good</td>
</tr>
</tbody>
</table>
3 robustness:

- The robustness because of environmental disturbance (wind, noise)
Conclusion

Perspective

- The robustness of performances through the consideration of parametric variations (mass)
Conclusion

Perspective

- The robustness in signal loss

<table>
<thead>
<tr>
<th>Danger</th>
<th>Action</th>
<th>probable event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Perform the optimal mission</td>
<td>No loss</td>
</tr>
<tr>
<td>1</td>
<td>Perform the mission</td>
<td>GPS loss</td>
</tr>
<tr>
<td></td>
<td>(change method, speed reduction)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>return to the starting point</td>
<td>Loss of perception (Lidar, camera)</td>
</tr>
<tr>
<td>3</td>
<td>Cancel the flight</td>
<td>Loss of the inertial unit</td>
</tr>
</tbody>
</table>
References I


References II


Sources

- https://www.lefigaro.fr/
- https://www.clubic.com/
- https://twitter.com/hashtag/aerones
- https://france3-regions.francetvinfo.fr/
- https://www.sudouest.fr/