Forcing a dynamic model for oil production and EROI evolution: The Oil Game

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Abstract

Since 1940, many attempts to model world oil production have been proposed. These approaches, using increasing complexity, consider the growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by modelling a dynamic dependency between oil production and its Energy Return on Energy Invested (ERoEI), based on mass and energy conservation. The ERoEI equation is derived according to the second principle. It leads to a Lotka-Volterra set of equations, which can be applied to all extracted liquid fossil fuels. The model obtained, after comparison with oil extraction and ERoEI evolution on the period 1960-2010, illustrates the production dynamic and the existence of an external, controlling parameter: the investment rate, which account for the re-investment in newly operating liquid fuel sources. The evolution of this parameter provides some possible explanations about the progress of the oil shocks and also some possible explanations about the peak prediction issues of the classical Hubbert model. Studying this evolution also suggests an attempt to control the liquid fuel production in order to obtain a linear time evolution on the period 1960-2010 through an apparently linearly growing investment rate: the oil game. Unfortunately, in order to keep a linearly growing production at long time scale, the investment rate has actually to evolve exponentially: the linear growth is in fact a short time scale approximation of the control required to play the oil game. The model also allows to highlight a major issue in liquid fuel production: even if the gross product can be controlled and keeps growing linearly, the net product, which account for the energy delivered by the oil industry to the world, is falling down faster and faster, due to the decrease of ERoEI. At some point, the net energy benefit will be equal to zero and liquid fuel production will stop, except if energy is given to the oil industry to keep extracting oil. In anyway, liquid fuels would become an energy sink instead of an energy source. Based on the model presented in this study, this will happen between 2027 an 2033. Production of liquid fuels could therefore keep growing linearly until this point, where a quick collapse is expected. Hence production will be strongly asymmetric regarding the peak, contrary to the prediction suggested by Hubbert's model.

Keywords: Oil production, ERoEI, dynamic model, investment rate

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Introduction

Models that account for oil production have been published from 1962 [1], with increasing complexity ([2, 3, 4], citing only very few of them). These models rely on a production dynamic with constant parameters. The aim of this study will be to evaluate how the parameters could evolve in time, based on a coupling between oil production (all extracted liquid fuels) and its Energy Return on Energy Invested (ERoEI) at the wellhead, as defined in [5] or [6]. Through this dependency, it is expected to explain why the prediction of peak is always delayed. For simplicity, "all liquid fuels" in the following refers to "all extracted liquid fuels" or "all liquid fossil fuels".

The model suggested is based on mass and energy conservation for the production equation and derived in accordance with the second principle for the ERoEI equation. It is worth noticing that the structure of the obtained ODE set is equivalent to a Lotka-Volterra set of equations, linking oil production of all liquid fuels Q with its mean ERoEI. In this model, Q appears to be the ERoEI predator as the production "feeds" on ERoEI to grow. It is in line with former use of Lotka-Volterra equations to model dynamic systems in ecology [7] or in economy [8].

The article is organised as follow: A first part is dedicated to a presentation of an assumption on the oil distribution as a function of ERoEI, suggesting why the model applies to the production of all liquid fossil fuels. It also presents a discussion on a production averaged ERoEI and its consequence in term of production modelling. The set of equation is then derived and the prey-predator analogy is presented. A fitting of the model parameters based on historical evolution of oil production and mean ERoEI is then performed. An analysis of the investment rate, a forcing parameter, is done, suggesting a control of the investment to keep a linearly growing production: the oil game.

A second part is dedicated to the the study of net liquid fuel production, relative energetic benefit and relative investment rate. This analysis suggests some possible explanations for the evolution of world economy in the beginning of the 80's, recent recessions and a future collapse of liquid fuel production rather than a slow, progressive decline. It also gives an estimation of the remaining "reachable" liquid fuels. Finally, a short discussion on oil price evolution is presented.

1. Modelling the interaction between oil production and ERoEI

This section is dedicated to the description of the interactions between production of all liquid fuels Q (in Gbbl) and ERoEI at the wellhead, as defined in [5] or [6].

1.1. An assumption about a mean, production averaged, ERoEI at the wellhead

The global modelling of all liquid fuels requires the assumption that the ERoEI considered is representative of the mean ERoEI of all liquid fossil fuels at a given time. Considering N liquid fuel sources in the world, $ERoEI_i$ the ERoEI of a given source and Q_i its production, it is here suggested to take $ERoEI = \frac{1}{Q}\sum_{i=1}^{N} Q_i \cdot ERoEI_i$. This will allow to derive a single equation for the production of all liquid fuels, instead of having a set of N equations.

1.2. A dynamic model for oil production and ERoEI evolution

1.2.1. Production equation

In order to derive the production equation, a mass balance is considered over the whole set of liquid fuel sources, based on a one year time laps ($\Delta t = 1$ year). The ODE is then derived taking $\Delta t \to dt$. For simplicity, the balance is based on gross product Q_g and the net product Q_n is deduced from Q_g afterwards:

On a given year n, a gross product Q_g^n is extracted from the N liquid fuel sources. A fraction of this production k_0 (an investment rate, in year^{-1}) is used to extract liquid fuels from new sources. Let us consider Δh an energy density contained in the liquid fuel (similar to a heat of combustion).

The work W_{ex}^n available for extraction in the new sources is then $W_{ex}^n = k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$ with η being the efficiency of all the processes needed to turn the extracted liquid fuel into work. This includes: transportation of liquid fuel to refinery, refining, combustion and transformation of heat into work, but also the exploration and structure development (such as wells and platforms) required to get this amount of liquid fossil fuel. This efficiency has been studied for crude oil and is equal to 0.2045 according to Hill [9]. For all liquid fuels, due to the use of production averaged quantities, this value should be about the same.

According to the definition of ERoEI at wellhead, with $ERoEI^n$ being the mean ERoEI on year n, this work allows to get the following amount of energy at the next time laps: $ERoEI^n \cdot k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$, corresponding to an increase in production $\Delta^+ Q_g^{n+1}$ which follows $\Delta^+ Q_g^{n+1} \cdot \Delta h = ERoEI^n \cdot k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$.

From the initial gross product Q_g^n, it remains $(1 - k_0 \cdot \Delta t) \cdot Q_g^n$, therefore, considering only the increase in production due to the newly exploited sources, one gets : $Q_g^{n+1} - Q_g^n = Q_g^n \cdot k_0 \cdot \Delta t \cdot (\eta \cdot ERoEI - 1)$.

During the same time laps, the producing fuel sources show a decline which follows the model described in Sorrell [10]: Considering k_1 as the mean oil source decline rate (in year^{-1}), the associate decrease in production is equal to $Q_g^{n+1} - Q_g^n = -k_1 \cdot \Delta t$.

Both phenomena occur at the same time, during the same time laps. Since they are linear, it is possible to use superimposition to get: $Q_g^{n+1} - Q_g^n = Q_g^n \cdot [k_0 \cdot (\eta \cdot ERoEI - 1) - k_1] \cdot \Delta t$. Taking $\Delta t \to dt$ leads to:

$$\dot{Q}_g = k_0 \cdot Q_g \cdot (\eta \cdot ERoEI - 1) - k_1 \cdot Q_g \,. \tag{1}$$

Now, based on ERoEI definition, it is possible to derive net product Q_n from gross product Q_g: Since $ERoEI = \frac{Q_g}{W_{ext}}$, one can evaluate Q_{ext}, the amount of liquid fuel used for extraction: $ERoEI = \frac{Q_g}{\eta \cdot Q_{ext}}$. Since $Q_{ext} = Q_g - Q_n$, one gets: $Q_n = \frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI} Q_g$

1.2.2. ERoEI equation

Based on Eq.(1) and using a prey-predator analogy, Q_g seems to "feed" on ERoEI to grow. More precisely, according to Eq.(1) structure, the "natural" prey of Q_g is $\eta \cdot ERoEI - 1$. Following this analogy, the prey should be decreasing proportionally to $\eta \cdot ERoEI - 1$ and Q_g, and should be growing due to the renewal of fossil fuels. This is neglected since it can be considered as happening at geological times. Considering a decline rate

k_2 (in (Gbbl.year)$^{-1}$) for ERoEI, this rational leads to the following equation: $\eta \cdot \dot{ERoEI} = -k_2 \cdot Q_g \cdot (\eta \cdot ERoEI - 1)$, equivalent to $\dot{ERoEI} = -k_2 \cdot Q_g \cdot ERoEI + \frac{k_2}{\eta} \cdot Q_g$. It is interesting to notice that if $k_2 \cdot Q_g = A$ with A being a constant (as it is the case here, according to Fig.2), this equation reads: $\dot{ERoEI} = -A \cdot ERoEI + \frac{A}{\eta}$. It leads to the inverse of the logistic function, which is the solution obtained for ERoEI evolution in the work of Hill [9]. The Lotka-Volterra approach allows to get the equation that lead to the solution obtained using the second principle, it can be considered as equivalent. This emphasizes the relevancy of the prey-predator analogy for liquid fuel product and its ERoEI.

However, the obtained equation cannot fit the purpose here, since its structure is decoupling Q_g and ERoEI. In order to keep this coupling, the following form is retained:

$$\dot{ERoEI} = -k_2 \cdot Q_g \cdot ERoEI \, . \tag{2}$$

Parameter k_2 is expected to decrease in time, according to the natural distribution of oil as a function of its availability on earth. In order to model k_2, the following dependency is proposed: $k_2 = C/(t - t_0)$ where C is a constant (in (Gbbl)$^{-1}$) and t_0 (in year) is a time offset. C can be interpreted as the effect of oil distribution as a function of ERoEI, regarding the rate at which oil is extracted. It suggests that the largest amount of oil on earth is available at the lowest ERoEI.

The work of Hill [9] nevertheless suggests that the “natural” physical coupling between ERoEI and Q_g is established through a distribution $ERoEI = f(Q_g)$, which is the expression of the Etp equation, derived by Hill [9] based on the second principle. Therefore, this distribution is now studied to see how it could be a surrogate to Eq.(2).

In [9], the ERoEI(t) function is derived based on a production which follows a Hubbert's curve for crude oil only (cumulative product $Q_p = 2357, 15$ Gbbl, peak at $t_m = 2001$, $Q_m = Q(t = t_m) = b \cdot Q_p/4$). Based on the previous remarks, $Q_g = f^{-1}(ERoEI)$ can be explicitly derived for a Hubbert like extracting scenario:

$$Q_g(ERoEI) = b \cdot Q_p \frac{\left(\frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI_m - 1}\right)^{b/a}}{\left[1 + \left(\frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI_m - 1}\right)^{b/a}\right]^2} . \tag{3}$$

With $ERoEI_m = 13.3$ being the value of ERoEI at time t_m and $a = 0.0537$ the parameter of the ERoEI solution (the inverse of a logistic function) calculated in [9]. Eq.(3) is general, but the values of Q_p and b are representative of a production which follow the Hubbert's scenario presented earlier. For all liquid fuels, this scenario is not realistic, at least for the values of Q_p and b previously suggested. Therefore, in order to estimate function f^{-1} for all liquid fuels, an adapted scenario has to be establish. This will be discuss later in this article, and distribution $ERoEI = f(Q_g)$ will be calculated for all liquid fuels.

1.3. Fitting the model parameters on the period 1960-2010

k_1 represents the oil sources mean decline rate. This parameter should be extracted from experimental measurements, using inverse methods. Based on the results of [10], the mean value lies in the range $4.1 - 6.7\%$ but is increasing with the exploitation of new non-conventional sources. Therefore, k_1 is set equal to 6% (a mean value based on previous remark) and k_0, k_2 and $k_2 \cdot Q_g$ can be fitted. Now, based on Eq.(1) and (2), setting a value for k_1, it is possible to plot k_0 and k_2 time evolution, based on historical data of Q_g and ERoEI:

$$k_0 = \frac{\dot{Q}_g}{Q_g \cdot (\eta \cdot ERoEI - 1)} + \frac{k_1}{(\eta \cdot ERoEI - 1)} , \tag{4}$$

$$k_2 = -\frac{\dot{ERoEI}}{Q_g \cdot ERoEI} . \tag{5}$$

The oil production data is extracted from [11]. Unfortunately, there is no available data for the mean ERoEI defined in section 1.1. The thermodynamic model suggested by [9], which fits the requirements described in section 1.1 and presents values at different times which are consistent with actual, measured values of active oil sources is used. Using the values of [9] to calibrate this model will certainly not allow to estimate precisely the investment rate time evolution, but it should allow to evaluate its global trend over the last decades to estimate its evolution in the forthcoming decade(s).

The analysis is performed on the period 1960-2010 using a three-year averaging on Q_g and a second order upwind method to calculate Q_g and ERoEI derivatives. The evolution obtained for k_2 is presented in Fig.1. The continuous line represents the model $k_2 = C/(t - t_0)$, with $C = 0.06036$ Gbbl^{-1} and $t_0 = 1950.5$ year. The model seems to fit adequately the data, with a mean relative error of 1.05%. The evolution shows two periods: The first one, before the first oil shock, corresponds to a rapid and smooth evolution of k_2. The second one, after the second oil shock, shows some jumps which could correspond to the exploitation of fossil fuels that were not exploited before due to their low ERoEI, in comparison with the mean ERoEI of the moment. When these sources become of interest and start being exploited, the value of k_2 suddenly drops because exploiting these sources does not affect much the mean ERoEI.

The evolution obtained for $k_2 \cdot Q_n$ is presented in Fig.2, showing the relevancy of the previous remark on the ERoEI ODE and on the prey-predator analogy.

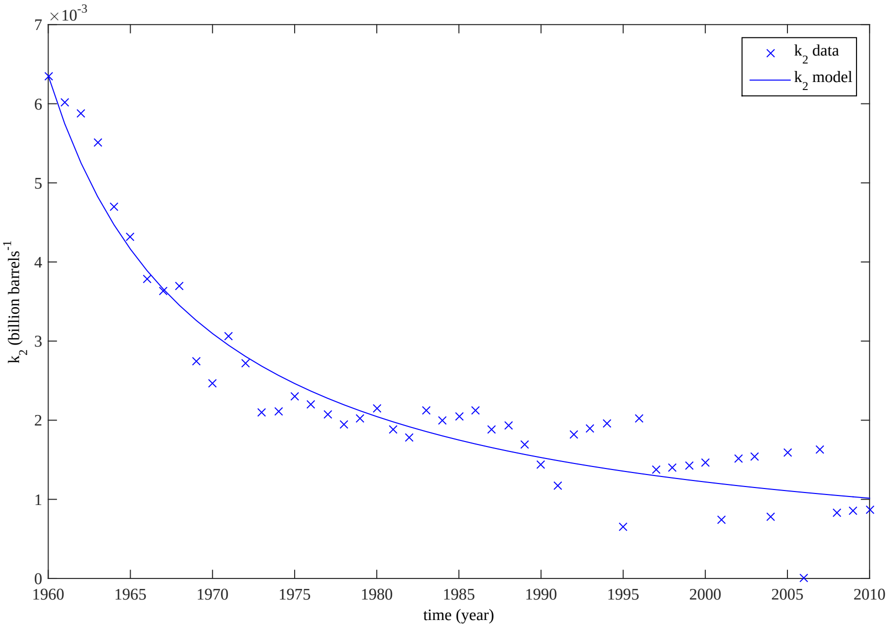

Figure 1: k_2 time evolution

1.4. Studying the investment rate

1.4.1. History of the investment rate

The investment rate can be evaluated through the value of k_0, which lies in the range $[0;1[$. It is nevertheless suggested to study $k_{eff} = k_0 \cdot \frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI}$

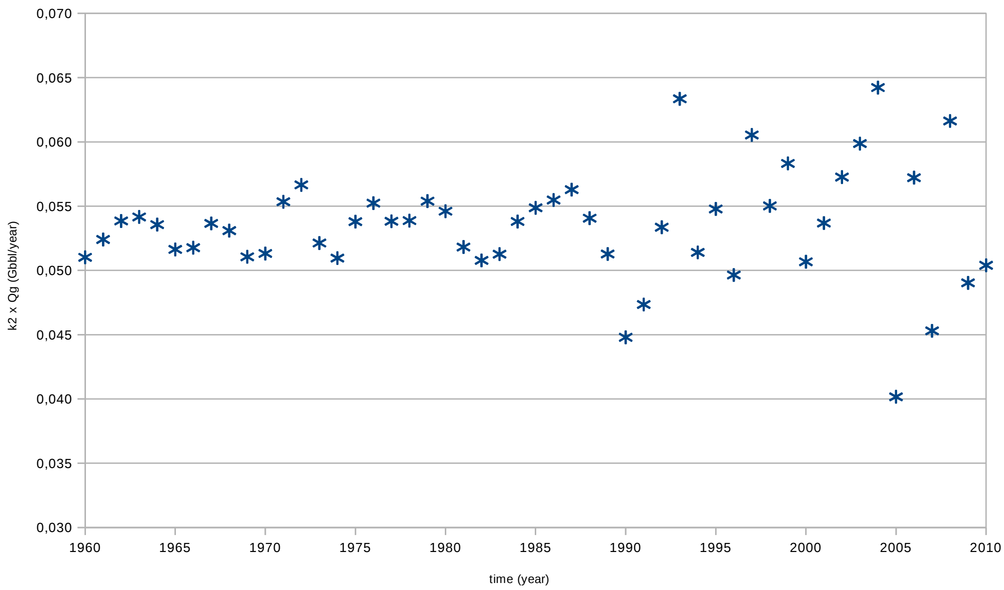

Figure 2: $k_2 \cdot Q_g$ time evolution

instead of k_0, for function fitting requirements. The parameter k_{eff}, which represents the forcing of the system is plot over time in Fig.3. Its analysis provides some possible characteristics of the oil extraction strategy, which are presented below.

On the period 1960-1968, k_{eff} shows a relatively linear behaviour. This period corresponds to an evolution of oil extraction that begin to behave as exponential around 1965. Due to the laws of market, and the oil price at this period which is rather low, keeping an exponential growth for Q_g could have been responsible for an important decrease in oil price. In order to keep a decent benefit without using too much of their resources, producers have to reduce Q_g, by reducing k_{eff}. This strategy begins in 1969, according to Fig.3. However, due to the behaviour of k_2 at this time, the system shows a great inertia and damping k_{eff} is not sufficient to control instantaneously Q_g. Any reasons could have been sufficient to suddenly reduce k_{eff} and adapt Q_g. Three years after the first inflection of k_{eff}, the first oil shock happen and k_{eff} is adapted.

After the first shock, k_{eff} is surprisingly constant, with a linear time evolution for Q_g. The second shock corresponds to another, longer drop of k_{eff}.

After the second shock, k_{eff} seems to evolve (globally) linearly, with raises and plateaus during the period 1985-2000. The solid line fits the data with a mean relative error of less than 1%. This behaviour allows Q_g to grow linearly in time. Also, since the system inertia has evolved in time with k_2, the plateaus are responsible every time for a slow damping of Q_g, which corresponds to past predictions of a nearby peak, using Hubbert's curves. This phenomenon leads every time to an economical recession and a raise in oil price (this can be shown in comparing Fig.3 with an oil price chart), at the moment where producers need to increase their investment to keep k_{eff} close to the solid line that ensure a linearly growing Q_g. The origin of this raise/plateaus dynamic can be explained the following way: With time, the production of an oil source eventually decreases, meaning that exploration is firstly needed to extract more oil. It means that if exploration does not suggests new sources to exploit, the production stagnates because it is not possible to invest in new sources, therefore k_{eff} is constant or slightly decreasing. When new sources become available, the investment rate can quickly increase until the new sources become less available and then exploration has to start again.

1.4.2. Projections based on the constant 1985-2000 dynamic

Following this line using raises and plateaus, allow to optimize the oil benefit and production: It could be compared to a game where k_{eff} should be kept on this line to optimize benefits. This strategy can then be extended to forthcoming years. One can observe that k_{eff} begins to deviate on the period 2000-2010. It seems that, in order to keep a constant derivative for Q_g, k_{eff} should not follow the same trend any more. The data of [12] is an extension of [11] data. It is used to evaluate the evolution of this slope. On the period 2000-2020, the slope seems to be different from the one observed on the period 1985-2000. Instead of plateaus, between 2005 and 2010, drops are required on k_{eff} to fasten the effect on Q_g, and the mean slope has to be higher than before.

1.4.3. The limit of the 2000-2020 dynamic

The extension of that game actually shows the real rule: in order to keep a linearly growing Q_g, k_{eff} has to evolve exponentially in time. To

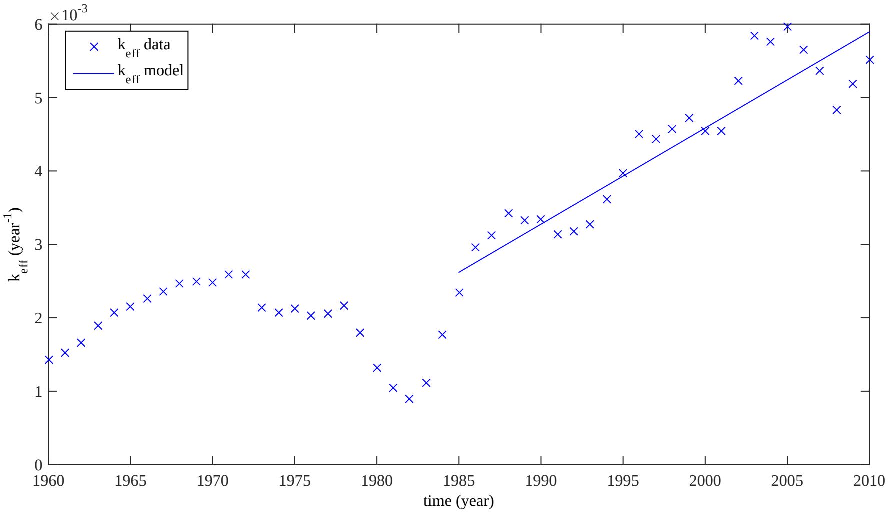

Figure 3: k_{eff} time evolution for $k_1 = 6\%$

keep playing that game the way it started, k_{eff} should follow the equation $k_{eff} = 1.2650 \cdot 10^{-25} \exp\left(t/\tau\right)$ with $\tau = 37.32$ years. Fig.4 shows the extension of k_{eff} on the period 2005-2020 along with a projection using the exponential fit. This projection suggests that the slope has again to be inflected, by strongly inflecting the shoots between the plateaus/drops. It also requires exploration to be more and more efficient, to ensure that the plateaus won't last for too long. It is worth noticing that according to [13] for instance, the exact opposite is expected.

2. Results and analysis

2.1. A business as usual scenario

Based on Eq.(1), the threshold value of $ERoEI$ for which oil production cannot grow any more is $\eta^{-1} \sim 4.89$. Another threshold could be the relative investment rate which cannot exceed 1 (i.e. the net investment cannot exceed the net product). This relative investment k_r reads: $k_r = \frac{Q_g}{Q_n} \cdot k_0 = \frac{\eta \cdot ERoEI}{\eta \cdot ERoEI - 1} \cdot k_0$. Supposing the exponential investment dynamic could be sustained until the mean liquid fossil fuel $ERoEI$ reaches 4.89 or until the relative investment rate reaches 1 represents a business as usual scenario where oil is extracted until the energetic benefit drops to zero or until development of new sources

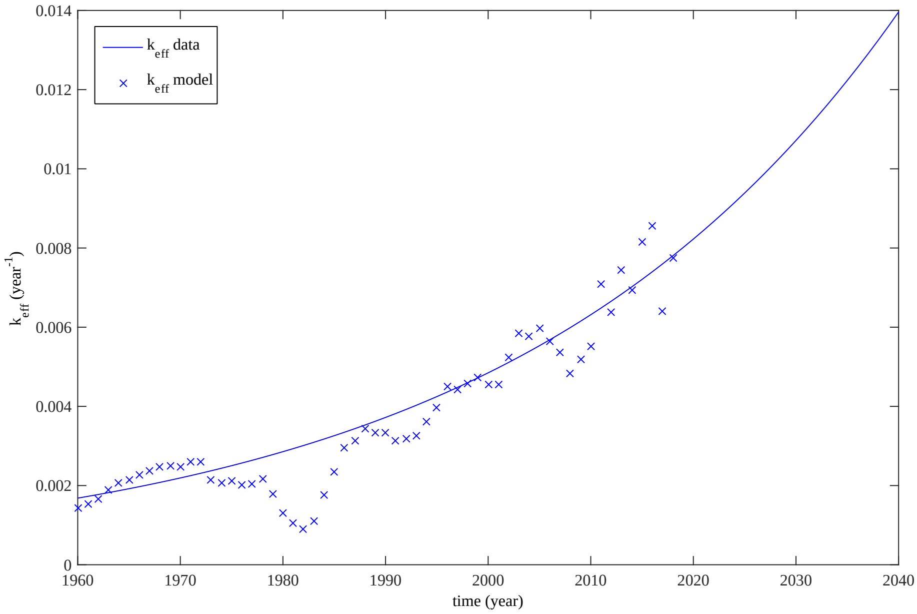

Figure 4: k_{eff} time evolution for $k_1 = 6\%$

becomes impossible. It could allow a linear growth of Q_g until 2040. This linear growth would be followed by a quick collapse of Q_g.

This business as usual scenario would be equivalent, for $t < 2040$ years to a Hubbert's curve with parameters : $Q_p = 5200$ Gbbl, a peak at $t_m = 2040$ years and $b = 0.028$. Please note that $Q_p = 5200$ Gbbl has nothing to do with an amount of recoverable liquid fuels. This aspect will be discussed later. Using these parameters allows, based on the ERoEI model of Hill, to derive the associated distribution $Q_g = f(ERoEI)$. Now, in order to estimate the evolution of Q_g and ERoEI, one can use either Eq.(1) and Eq.(2), either Eq.(1) and Eq.(3) using the above values for Q_p and b.

In order to evaluate the end of oil extraction based on this scenario, three variables are studied: the net product Q_n, the relative energetic benefit $\epsilon_B = \frac{Q_n}{Q_g} = \frac{\eta EroEI-1}{\eta ERoEI}$ and the relative investment rate $k_r = \frac{k_0}{\epsilon_B}$. These variables are studied with Eq.(1) and Eq.(2), then with Eq.(1) and Eq.(3). A 10% margin on η is also considered, as this value has been fitted for crude oil and is now applied to all liquid fuels.

2.2. Results

The numerical simulations are done with a 0.1 year time step and a Runge-Kutta 4 method, starting in 2010, after a numerical validation on the period 1960-2010. The first remark is that the results are within 3% regardless the set of equations which is used. It suggests that the prey-predator analogy makes sense, with Q_g being the predator of $ERoEI$. It also suggests that this kind of Lotka-Volterra set of equations could applies to other energy sources such as coal, gas and nuclear energy, if data such as $Q_g = f(ERoEI)$ are available, or can be derived based on thermodynamical considerations.

Plots are presented for the most pessimistic calculation (in term of remaining liquid fuel) and all results are presented with a dispersion based on the extreme calculations. Fig.5 shows the net product as a function of time. It highlights that the maximum net energy delivery to the world by the oil industry peaked in 1979. It also shows the real difference in energy delivered by the oil industry to the world before and after the second oil shock, evolving from a continuous increase to a steady state until 2000. The net product begins to fall around 2000, but this is damped by the investment done in oil production at this time. These investments seem to have maintain the net product almost constant until 2005, at which point the net product start to decline linearly, until 2017-2018 where the fall begins to accelerate. All these remarks seem in line with the recessions that occurred since the year 2000.

Fig.6 shows the evolution of the relative investment rate. It highlights the rate at which investment have to increase from now on to keep playing the oil game. It also shows that the study of the net product hides the potential investment issues: at some point it becomes impossible to invest enough energy to keep the gross production increasing even if the net product (or the relative benefit) is still above zero. The relative investment rate is therefore an appropriate metric to evaluate the end of the oil age. Once its value reaches one, it becomes impossible to put new sources in operation; the production relies only on sources which are already operating, hence gross product will decrease by 6% (according to k_1 value) every year from this point. Considering the extreme calculations, this "dead-point" of the oil industry is reached between 2027 and 2033.

These calculations are done for a given scenario, it cannot be done differently due to formulation of the equation set which is solved. The reality of extraction dynamic could be different. However, the amount of "reachable" liquid fuels is fixed by these equations. This study hence suggests that the remaining reachable amount of gross liquid fuels lies between 305 and 560

Gbbl. In term of net product, it represents only 76 to 172 Gbbl. As a comparison, this is equal to the net energy delivered by the oil industry to the world during the last 5 to 10 years.

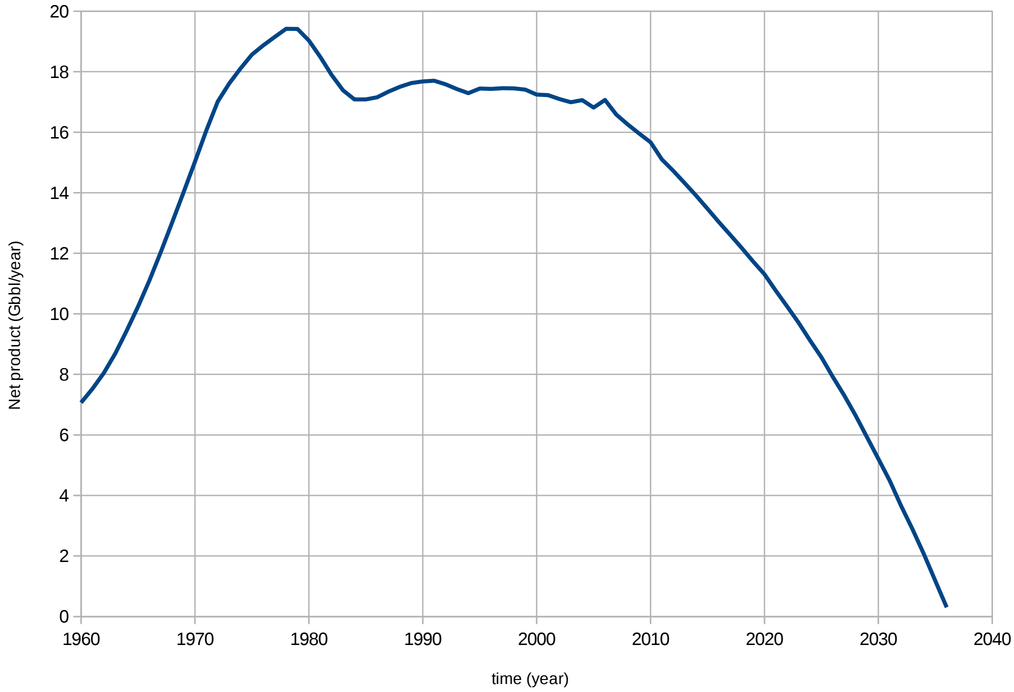

Figure 5: Net product: energy delivered to the world

2.3. *A relation between production and oil price*

The origin of oil price variation is a highly discussed topic. According to some authors as [14], it might not follow the law of market any more. It is also discussed as being a consequence of geopolitical events, as discussed in [15] or [16] for instance. In this article, it is suggested that oil price should be able to pay employees and share-holder of the oil industry only, since the energetic cost has already been taken into account through η. This would suggest that benefit (in money) is proportional to the gross product, while the actual benefit is the oil price multiplied by the net product. The price

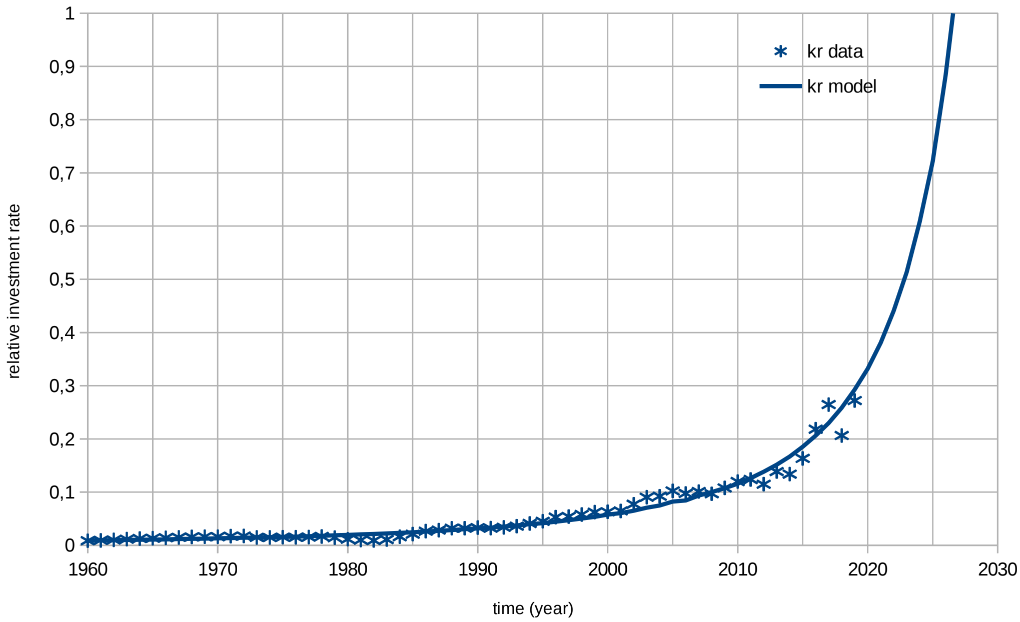

Figure 6: Relative investment rate

P should then be proportional to $1/\epsilon_B$. That would be the price at which oil should be sold to maintain the industry, hence oil price used to follow such behaviour. However, according to [17] for instance, the world cannot pay this price any more and the price should fall, along with the production due to some bankrupts in the oil industry. However, debt could prevent these bankrupts, to keep the system based on oil running as long as possible. Therefore discussing oil price seems out of the scope of this study; a rough approximate of the remaining reachable liquid fuels seems a more reliable metric of liquid fuel depletion.

Conclusion

This study proposes a production averaged model which allow to study the mean extraction dynamic of all liquid fuels. This dynamic seems to follow a typical prey-predator behaviour.

It is in accordance with the results obtained by Hill, with a few years delay. This is due to the fact that Hill's study is focused on crude oil while this study is extended to all extracted liquid fuels.

It shows that, whatever the possible investment of the oil industry, the development will cease between 2027 and 2033 and the net energetic benefit will follow. According to Garrett's theory on GDP and the contribution of liquid fuels in the world energetic mix, it suggests that world GDP will be reduced by roughly 35% in approximatively 10 years.

Finally, the prey-predator analogy suggests this method could also be applied to study the dynamics of other energy source extraction, such as gas, coal or nuclear energy.

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