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On the modelling of wavepacket scattering noise with coherence effects

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1 An investigation of a wavepacket model for free-jet and jet-surface interaction noise
 2 was conducted. The source term for the axisymmetric mode was extracted from a
 3 Mach 0.9 jet Large Eddy Simulation (LES) and employed to adjust the parameters
 4 of a simple source model. Streamwise coherence decay, in particular, was considered.
 5 The source model was propagated with both the free-field and tailored Green's func-
 6 tion for a semi-infinite flat plate positioned at a distance of $r/D = 1$ from the jet
 7 axis. First, a model with radial content was considered. Significant deviations were
 8 observed in the prediction of the low-angle directivity of the isolated jet as well as
 9 in the reproduction of the characteristics of the source field. However, the effects
 10 of trailing edge noise were well reproduced. The installed jet case, (Added: at the
 11 region dominated by trailing-edge scattering), showed very little sensitivity to the co-
 12 herence decay, a crucial feature in the isolated jet case. (Deleted: Through the use
 13 of a line source, involving greater ease in the adjustments, the models were capable to
 14 predict the acoustic fields in all studied configurations in low Strouhal number.) The
 15 modelling of installed jets proved to be much simpler, since the results were much
 16 less sensitive to the characteristics of the source.

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I. INTRODUCTION

The adverse effects arising from the integration of engines with the aircraft structure have gained importance with the rise of stricter noise regulations. Among the sources of noise due to this integration are the interaction of the jet flow with wings and flaps. In this case the problem is intensified by the close coupling between the engine and the wing, due to the use of high bypass ratio engines.

The effects caused by the presence of solid surfaces near turbulent regions have been known for decades. Curle¹ presented an extension of Lighthill's theory² in order to account for the influence of solid boundaries on the acoustic field. In addition to the jet quadrupoles, surface pressure leads to the appearance of dipole sources, which have stronger far-field contribution at low Mach numbers. Furthermore, Ffowcs Williams and Hall³ studied the case involving semi-infinite plates using a tailored Green's function. In this case, the deduced edge scattering source is even more efficient than both the free turbulence in Lighthill's theory or compact surfaces in Curle's work.

Wavepackets are intermittent, advecting disturbances that are correlated over distances far exceeding the integral scales of turbulence⁴. Several experimental and theoretical studies highlighted the importance of such structures in jet noise^{5–8}. Theoretical models based on wavepackets have been shown to capture many of the main characteristics of the far-field noise for isolated^{8,9} and installed jet cases^{10–12}. However, inconsistencies are present as the absolute noise levels are under predicted by linear dynamic models, such as the Parabolised Stability Equations. These linear models have unit coherence in contrast with turbulent

flows, in which coherence decays with distance. This issue was investigated by Cavalieri and Agarwal¹³ who show the match of the coherence of the original source to be fundamental for correctly predicting the sound field. Studies showed that the coherence decay found in turbulent jets, when included in the linear models, raise the radiating efficiency of the source^{13–15}.

Regarding installation effects, Cavalieri *et al.*¹⁰ modelled a jet-plate interaction case using a wavepacket source model along with a tailored Green’s function and a boundary element method. The scattering of the non-compact wavepackets in the jet near field was found to be responsible for the amplification of the noise due to the presence of the plate. The used model was identified from far-field measurements of free jet noise. Further studies are needed to evaluate the pertinence of the model, and it would be important to use data from jet turbulence to construct a source. Moreover the role of jitter/coherence decay is apparently weaker in installed jet noise according to Nogueira *et al.*¹², but the cited work dealt with a model problem. Again, the use of turbulence data may help addressing this question in a more definite way. (Added: The use of model sources and Lighthill’s equation for predicting installed jet noise has been also explored by Refs. 16, 17 and 18.)

The objective of this paper is to analyse wavepacket source models, obtained from numerical flow data, for the prediction of free-jet and jet-plate interaction noise. We first investigate simplifications on the Lighthill’s source term and their relevance to the total sound field using the LES data from Ref. 19. Subsequently, the same numerical data is used to obtain parameters of kinematic wavepacket source models, and the produced noise field

was obtained using both the free-field Green's function and a tailored Green's function for a semi-infinite flat plate.

This work is organized as follows. In section II, we describe the used geometries and the main equations. In section III, we present the acoustic results from a simplified source term extracted from the LES data. Results for the model source are presented in section IV. Finally, section V presents the main conclusions.

II. METHODS

A. Geometry

The study presented herein is based on the experiments conducted at PPRIME Institute, Poitiers, France¹¹. The experiment consists of a nozzle of diameter $D = 50$ mm with a flat plate positioned at its vicinity, as shown in the sketch of Fig. 1. (Added: The plate has a chord of $9D$ and a span length of $15D$.) A total of 324 acoustic field measurements were taken on a cylindrical surface of radius $R = 14.2D$. The studied configuration had a jet acoustic Mach number $M_a = 0.9$ and Reynolds number $Re_j = \rho_j U_j D / \mu_j \approx 1 \times 10^6$. The trailing-edge position at the center of the plate is $x/D = 4$ and $r/D = 1$ is the position relative to the nozzle axis; here we consider a cylindrical coordinate system, with origin at the nozzle exit, given by (x, r, ϕ) , corresponding to streamwise, radial and azimuthal coordinates, respectively.

FIG. 1. Scheme of the experimental tests used as reference in this analysis.

B. Large eddy simulation database

The large eddy simulation (LES) database used corresponds to the same jet conditions and nozzle geometry, considering only the free-jet case. (Added: The jet is isothermal in the simulation.) The simulation was performed using the compressible flow solver “Charles” developed at Cascade Technologies, and reproduces the configuration from the companion experiment conducted at the PPRIME Institute, Poitiers, France.

The LES methodology, grid resolution study and validation with experiments are presented in details in Brès *et al.*¹⁹. The instantaneous values of the primitives variables were interpolated from the original unstructured LES grid onto structured cylindrical grids in the jet plume and in the nozzle pipe. These cylindrical grids were designed such that the resolution approximately corresponds to the underlying LES resolution. For the jet plume, the cylindrical grid extends to $0 \leq x/D \leq 30$, $0 \leq r/D \leq 6$, with $(n_x, n_r, n_\phi) = (626, 138, 128)$, where n_x , n_r and n_ϕ are the number of points in the streamwise, radial and azimuthal direction, respectively. The points are equally-spaced in the azimuthal direction to enable simple azimuthal decomposition in Fourier space.

C. Eduction of the source term from LES

The source term is based on the Lighthill² equation given by,

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \quad (1)$$

where the Lighthill's tensor, with viscous terms neglected, is defined as, $T_{ij} = \rho U_i U_j + (p - c^2 \rho) \delta_{ij}$, where U_i is the velocity vector, p the pressure, ρ the fluid density, c the speed of sound and δ_{ij} the Kronecker delta.

We now apply several simplifications to the source term, which nonetheless are expected to retain the main source features for sound generation^{8,14}. The entropy term is not considered for an isothermal jet and source density variations are substituted by the mean ρ_0 . Under the hypothesis of the predominance of the momentum term T_{xx} , separating mean and fluctuating parts of the streamwise velocity $U_x = \overline{U}_x + u_x$ and taking only the linear part, the source term in cylindrical coordinates is given by:

$$S^t = \frac{\partial^2 T_{xx}}{\partial x^2} = \rho_0 \frac{\partial^2}{\partial x^2} 2\overline{U}_x u_x(x, r, \phi, t). \quad (2)$$

The source is decomposed into azimuthal Fourier modes, as we are interest in just the axisymmetric component:

$$S^t(x, r, m, t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S^t(x, r, \phi, t) e^{im\phi} d\phi, \quad (3)$$

105 and only $m = 0$ is considered,

$$S^t = \frac{\partial^2 T_{xx}}{\partial x^2} = \rho_0 \frac{\partial^2}{\partial x^2} 2\overline{U_x} u_x(x, r, m = 0, t). \quad (4)$$

106 (Added: By neglecting T_{rr} and T_{xr} radiation is expected to be predominant towards low
 107 angles²². Also, non-linear terms should be significant only at sideline angles as studied by
 108 Freund[?]. Furthermore, mode 0 is expected to be dominant at at polar angles close to 30° ,
 109 while the contributions of modes 1 and 2 are more significant at high angles^{8,24}.)

110 The superscript t implies the quantities in the time domain. This source term is Fourier
 111 transformed with using the following convention

$$S = \int_{-\infty}^{\infty} S^t e^{-i\omega t} dt. \quad (5)$$

112 The source distribution is obtained from the LES database, which is for a free jet. It
 113 will be used in calculations of free and installed jet noise, as will be presented in the next
 114 sections. For the installed jet calculations, there is an implicit assumption that the presence
 115 of a neighbouring plate does not modify substantially the source field. The accuracy of the
 116 calculation may serve as an *a posteriori* check of this assumption.

117 **D. Computation of the acoustic pressure at observer positions**

118 The source distribution obtained from the LES database, with the simplifications de-
 119 scribed in the previous section, allow an estimation of the radiated sound in free- and
 120 installed-jet configurations. The calculation is done in the frequency domain, based on
 121 the integral solution of the inhomogeneous Helmholtz equation, resulting from the Fourier

122 transform of Lighthill's equation. As we are dealing with turbulent properties, and so non
 123 square-integrable functions, it is more appropriate to work with two-point statistics. In this
 124 sense, the Power Spectral Density (PSD) of the sound pressure field can be obtained from
 125 the Cross Spectral Density (CSD) of the source field by:

$$\begin{aligned}
 \langle p(\mathbf{x}, m, \omega) p^*(\mathbf{x}, m, \omega) \rangle = \\
 \int_V \int_V \langle S(\mathbf{y}, m, \omega) S^*(\mathbf{z}, m, \omega) \rangle G(\mathbf{x}, \mathbf{y}, \omega) G^*(\mathbf{x}, \mathbf{z}, \omega) d\mathbf{y} d\mathbf{z}
 \end{aligned} \tag{6}$$

126 For the free jet case, $G = G_0$ is the free-field Green's function given as,

$$G_0 = \frac{e^{-ikR}}{4\pi R} \tag{7}$$

127 where R is the distance between source and observer and $k = \omega/c_0$ the acoustic wavenumber.

128 For the installed jet case, the problem is modelled as the jet scattering from a semi-
 129 infinite plate. This approach has been used with wavepacket models and was shown to be
 130 representative of the problem, although some deviations occur due to the differences to the
 131 real case (finite plate)^{10–12}. The tailored Green's function from Ffowcs Williams and Hall³
 132 is used,

$$4\pi G_t = \frac{e^{\frac{1}{4}i\pi}}{\sqrt{\pi}} \left\{ \frac{e^{-iR}}{R} \int_{-\infty}^{u_R} e^{-iu^2} du + \frac{e^{-iR'}}{R'} \int_{-\infty}^{u_{R'}} e^{-iu^2} du \right\} \tag{8}$$

133 where,

$$U_R = 2 \left(\frac{krr_0}{D+R} \right)^{\frac{1}{2}} \cos \frac{\theta - \theta_0}{2}, \tag{9}$$

$$u_{R'} = 2 \left(\frac{krr_0}{D + R'} \right)^{\frac{1}{2}} \cos \frac{\theta + \theta_0}{2}, \quad (10)$$

$$D = \{(r - r_0)^2 + (z - z_0)^2\}^{\frac{1}{2}}. \quad (11)$$

The tailored Green's function on its original form, implies a cylindrical coordinate system with axial coordinate aligned with the trailing edge. The position of the source point in this coordinate system is (r_0, θ_0, z_0) and the observer (r, θ, z) . R is then the distance between source and observer and R' is the distance between the source's image and observer.

As numerical derivatives of the turbulent data generated undesirable noise in the results, the derivatives were transferred to the Green's function. The derivatives were performed analytically and the derivation from Nogueira *et al.*¹² is used for the tailored Green's function.

The whole available LES time data was used, with a total non-dimensional time of $tc_0/D = 2000$ and a sampling rate of $\Delta tc_0/D = 0.2$. (Added: The source domain extended $0 \leq x/D \leq 30$, $0 \leq r/D \leq 3$, with $(n_x, n_r) = (328, 58)$ points.) The Cross Spectral Densities (CSDs) were calculated using Welch's method²⁰ using block size of Nfft= 256 timesteps, overlap of 75%. Sensitivity tests for the window function were conducted. Acoustics results obtained from the CSD calculated with rectangular window presented an undesirable dependence with the block size. Nevertheless, results with Hanning window showed much lower sensitivity to the block size. Based on these results, the Hanning window was chosen for further analyses.

E. Model sources

Model sources used in this work are inspired by the wavepacket line source model presented by Crow²¹ (see Crighton²²). This source and variations were used in Refs. 8, 10–12, some of them introducing radial information with the insertion of radial profiles obtained via PSE solutions. Based on the coherence modelling proposed by Cavalieri and Agarwal¹³ the following source model is employed:

$$\begin{aligned} \langle S(\mathbf{x}_1, \omega) S^*(\mathbf{x}_2, \omega) \rangle = & A \exp \left(-\frac{(x_1 - X_c)^2}{L_x^2} - \frac{(x_2 - X_c)^2}{L_x^2} \right) \\ & \operatorname{erfc} \left(\frac{\alpha(x_1 - X_c)}{\sqrt{2}L_x} \right) \operatorname{erfc} \left(\frac{\alpha(x_2 - X_c)}{\sqrt{2}L_x} \right) \exp \left(-\frac{(r_1 - R_c)^2}{L_r^2} - \frac{(r_2 - R_c)^2}{L_r^2} \right) \\ & \exp [-ik_h(x_1 - x_2)] \exp \left(-\frac{|\mathbf{x}_1 - \mathbf{x}_2|^2}{L_c^2} \right), \end{aligned} \quad (12)$$

such that the *CSD* of the source is represented by 7 global parameters. A represents the maximum *PSD* amplitude, whose streamwise decay is represented by an asymmetric Gaussian profile centered at X_c with characteristic length L_x and α as the skewness parameter. The radial profile is represented by a symmetric Gaussian profile with envelope length L_r . Centering this second Gaussian at $R_c = 0.25$ was taken as a reasonable assumption. Streamwise phase difference is defined by the hydrodynamic wavenumber k_h , related to the phase speed $U_c/U_J = 2\pi St/k_h$.

Finally the coherence decay is represented by Gaussian profiles of characteristic length L_c . The parameters were obtained by minimizing the sum of the squares of the absolute error between the numerical source and the proposed model.

(Deleted: In order to further simplify the model, a line source approach, similar to Ref. 13 is derived. For a free-jet case, at low St number, with observer at the far-field with ($r_{obs} \gg r_s$) it is acceptable to assume radial compactness, leading to an independence of the Green's function with the radial distance. In this way parting from the eq. 6 becomes,)

(Deleted: with the CSD of the line source defined as)

(Deleted: This line source can be parametrised in a similar manner to the volumetric source in eq. 12, leading to)

III. SOUND RADIATION FROM LES SOURCE TERMS

A. Isolated jet

The linearised, $m = 0$, T_{xx} source term extracted from the LES was first propagated by the integration with the free-field Green's function (Eq. 6). As already known^{8,23,24}, the axisymmetric mode is dominant at low polar angles, while higher order modes are important at high angles. Only $m = 0$ experimental data is show for the free-jet case, while the complete set of azimuthal modes will be used for the installed case.

Results obtained by the integration with the free-field Green's function are shown in Fig. 2 (LES source) for $St = 0.2$, 0.3 and 0.75 . The results show good agreement to experimental data for $m = 0$ at low angles and St numbers, reproducing the axisymmetric super directive features⁸. Although the agreement is indeed very good at $St = 0.2$ (Fig. 2(a)), levels are overpredicted at 20° and 30° for the other St .

FIG. 2. Comparison of free jet results for the three considered St numbers.

Neglecting the entropy term in the definition of the source term could be an important issue for these observed deviations, since Bodony and Lele²⁵ identified a cancellation effect between the entropy and momentum terms for St below 0.3 at this radiation direction. However, we should keep in mind that the analysed jet is isothermal, unlike the one considered in the cited study. Here, the disparities become larger with growing St , and which is also aligned with the growing importance of refraction effects. These deviations are expected to reduce once the complete Lighthill's tensor is taken into account.

By looking at angles higher than 60° , we see that levels were underpredicted by about 10 dB. At these directions both T_{rr} and T_{xr} and possibly non-linear terms have considerable contribution to total noise levels, but were not considered in this study²². As already expected, more terms and more azimuthal Fourier modes should be accounted for when trying to predict noise at high angles for an isolated jet.

B. Installed Jet, plate at $r/D = 1$

Fig. 3(a) and 4(a) depict the results for the installed case (Added: (LES source)) for azimuthal angle $\phi_{obs} = 260^\circ$, at the unshielded side of the plate (See Fig. 1). For the region dominated by the edge scattering (polar angles around 90°), levels were close to the experimental total noise (all modes). This now points out to the dominance of the mode $m = 0$ and T_{xx} for installed jet noise, which is in line with the results of Refs. 10 and 12. This observation supports the study in the following section, where a simplified axisymmetric T_{xx} wavepacket model source is adjusted to predict the installation noise field. (Added: In order to evidence the contribution of the plate to the total noise, the scattered only noise field was included in the same plots.)

FIG. 3. Comparison of installed jet results for $St = 0.2$.

For $St = 0.2$ and 0.3 in Fig. 3(Added: b) and 4(Added: b) at $\phi_{obs} = 220^\circ$, results are underpredicted by about 2 to 4 dB. We should bear in mind that the azimuthal directivity of (Replaced: installed-jet replaced with: edge-scattering) noise is of dipole shape, peaking at $\phi = 90^\circ$ and 270° and with minima at $\phi = 0, 180^\circ$ ²⁶; thus, from $\phi = 260^\circ$ to 220° we are moving to a region of lower installation effects. Furthermore, at $M = 0.9$, (Replaced:

FIG. 4. Comparison of installed jet results for $St = 0.3$.

212 ~~installation~~ replaced with: ~~trailing-edge~~ effects are not so prominent as in lower Mach
 213 number^{11,27}. In this sense, in such radiation direction, the free jet could present noise levels
 214 comparable to the scattered field. Thus, the remaining terms and modes in the Lighthill
 215 source have some contribution at this position. In addition, the differences between the real
 216 finite plate and its representation by the semi-infinite plate may also play a role for the
 217 observed deviations. This is supported by the presence of level oscillations with the polar
 218 angle in the experimental data, potentially caused by the secondary scattering by lateral
 219 edges¹⁰.

220 Minor scattering effects are expected in directions parallel to the trailing edge in the
 221 case of the semi-infinite plate. As consequence the results for $\phi_{obs} = 180^\circ$ (Fig. 3(c)), polar
 222 directivity trends are more similar to those observed for the free jet and so, the missing
 223 modes and momentum terms are also important.

IV. SOUND RADIATION FROM MODEL WAVEPACKET SOURCE

We now benefit from the presented LES data to adjust a simple kinematic wavepacket source model, such as the ones proposed by^{8,10,13,21}. One of the objectives here is to explore the effects of the coherence decay in the isolated and installed jet configurations. The resulting model contributes both to the understanding of the underlying physics of the problem, highlighting the relevant wavepacket parameters to predict installation effects.

A. Two-dimensional source model

The model parameters were obtained by minimising the sum, (Added: over the discretized domain,) of the squares of the absolute error between the LES source (Added: CSD) and the proposed model (Eq. 12) (Added: $(\sum^N \sum^N (||CSD_{LES} - CSD_{model}||^2))$). The domain for the fitting was limited between $x = 0.5D$ and $x = 15D$ and $r = 0.2D$ and $r = 1D$ (Added: with a total number of N points). (Added: The process is carried out with the Nelder-Mead Simplex Algorithm²⁸). The parameters found for each St are summarized in table I. The total relative error ($\Delta = \sum^N \sum^N ||CSD_{LES} - CSD_{model}|| / \sum^N \sum^N ||CSD_{LES}||$) is shown in the same table. For the noise computation, the model source was defined on a grid extending from $x = -7L_x$ to $x = 7L_x$ and from $r = 0$ to $r = 1D$, with 25 points in the radial and 256 points in the axial directions. Results were verified to be grid independent with these values. Acoustic results are shown in Fig. 2(a) ($St = 0.2$), Fig. 2(b) ($St = 0.3$) and Fig. 2(c) ($St = 0.75$) for the free jet.

Levels were overpredicted by about 7 dB for $St = 0.2$ and 4 dB for $St = 0.3$ at 20° . For all considered St , levels were highly underpredicted over 60° . The low-angle directivity for the axisymmetric mode could only be reasonably captured for $St = 0.2$. Results for $St = 0.75$ reasonably reproduced the directivity trends, however, noise levels were overpredicted by about 10 dB.

Some deviations from the expected low-angle directivity occurred and seem to increase with St . One possible cause are fitting errors. The failure of the model on capturing the sideline noise may not be unsettling, since the axisymmetric mode and even the T_{xx} term do not have the main contributions to the total noise. However, at low angles, the difficulties in predicting both the magnitude and directivity of the sound field do represent an important issue.

With the plate at $r/D = 1$, at polar angles dominated by the edge scattering noise and $\phi_{obs} = 260^\circ$ (Fig. 3 and 4), levels were much closer to the reference experimental and LES data. Deviations were less than 1 dB for $St = 0.2$ and about 2.5 dB for $St = 0.3$. Trailing-edge effects are not expected to be important at $St = 0.75$ so installed results were omitted for this case.

In Fig. 5, we can see the comparison between experimental and model results for all the measured directivity range, with the plate at $r/D = 1$, $St = 0.3$. The directivity trends could be well captured for azimuthal angles close to the plane perpendicular to the plate, say between $\phi_{obs} = 50^\circ$ and 140° and between $\phi_{obs} = 220^\circ$ and 320° . With the results up to this point, we can note that although the wavepacket model showed some deviations in the prediction of free jet noise, specially at high angles and high St , a close match is obtained

265 wherever edge scattering is dominant. These results are aligned with the observations from
 266 section III and with Cavalieri *et al.*¹⁰.

FIG. 5. (Color online) Model source SPL results compared to the experimental data with tailored Green's function at all the measured positions ($St = 0.3$).

267 We can note that the relative errors shown in table I are high, specially for $St = 0.75$,
 268 and imply a limitation of the used fit. In order to help understanding the possible causes of
 269 the found deviations, we now look at the plots of the source region. A comparison between
 270 the LES and model sources can be observed in Fig. 6 for the real part of the CSD, for each
 271 of the analysed St and for two fixed radial reference positions at $x_{ref} = X_{max}$, where X_{max} is
 272 the streamwise position of the peak power spectral density. The fitting technique provided a
 273 good representation of the maximum amplitude at $r = R_c$ and source extension around the
 274 maximum. Nevertheless, there is a clear mismatch between the radial profiles up to $x/D = 5$
 275 and in the jet shear-layer. The use of simple Gaussian profiles for the radial amplitudes and
 276 coherence contributed to these deviations, which become larger at higher St .

277 As depicted Fig. 6, at $St = 0.2$ and reference position $r_{ref} = 0.3$, the model reproduces
 278 the main characteristics of the source up between $x/D = 5$ and 10. Greater divergence can

be observed up to $x/D = 5$ around the lipline. When the reference point is at the lipline,
 the disparities are even more evident. Similar conclusions can be drawn for the other St ,
 with even greater disparities at $r_{ref} = 0.5D$. Deviations from the reference data seem to
 increase with St , as well as the sound-field results.

FIG. 6. Comparison between plots of real part of the CSD obtained from the LES data and from the model source with $x_{ref} = X_{max}$ at $St = 0.2$ (top), $St = 0.3$ (middle) and $St = 0.75$ (bottom).

FIG. 7. Comparison between coherence plots obtained from the LES data and from the model source with $x_{ref} = X_{max}$ and $St = 0.3$ and two radial reference positions $r_{ref} = 0.3D$ (top) and $r_{ref} = 0.5D$ (bottom).

The comparison of coherence plots is depicted in Fig. 7 for reference points at $r/D = 0.3$ and 0.5. The radial coherence decay is obviously not well reproduced by the model. At $r_{ref} = 0.3D$, the streamwise coherence could be reasonably represented. However, at $r_{ref} = 0.5D$, a much shorter coherence length is required. This range of coherence length scales in different regions of the source field is not easy to represent with a simple kinematic model.

The content of the Green's functions in the wavenumber space are different^{12,13}, which would permit source wavenumbers that are evanescent in the free-field case to generate far-field noise when the jet is close to a flat plate. Slight changes in wave-packet envelope or coherence decay may influence free-jet noise substantially¹³, and lead to little changes for installed jets¹². That is possibly why the results with the plate were in good agreement with the reference data, despite of the problems observed in the free-jet case.

Although it is not straightforward to tell which are the causes of errors in the acoustic prediction by simply looking at the source field, it is evident that some considerable deviations are present between the fitted wavepacket model and the LES field. These were possibly caused by the use of simple functions for both the amplitudes and coherence, as well as by the neglect of the radial phase differences and radial coherence. However, we are working in the framework of simple kinematic models, and so a parametrisation that would be too complex is not the intention of this work. Furthermore, is important to emphasize that, even with the mentioned simplifications, the model could correctly represent the expected noise characteristics the phenomenon (Added: of trailing-edge scattering, one of the main phenomena of installed jets) (Deleted: for the installed case). Using the same 7-parameter

model, the next section aims at characterising the sensitivity of the results to the coherence length.

B. Sensitivity to the coherence length

As already mentioned, the coherence length is an imperative parameter in wavepacket models for isolated jet prediction^{13,15}. We now assess the effects of the variation of this parameter based on the range of values identified in the LES data at $x = X_c(St)$ and $St = 0.3$. Three values were defined: the value at the lipline ($L_c^C = 0.94$), the radial average of L_c until $r = 0.5D$ ($L_c^B = 2.66$) and the fitted value from the previous section ($L_c^A = 3$). The unit coherence source, with $L_c^0 \rightarrow \infty$, was also considered.

By looking at the resulting free-jet far-field noise for $St = 0.3$ in Fig. 8, we can see a clear change in the noise directivity with the different L_c values. The source with the lipline L_c produces very high noise levels in comparison with the LES source. Although reasonable level agreement at low angles was only achieved when considering the larger, original L_c , only results with L_c^C reproduced the low-angle directivity trends. As expected^{13,15}, the L_c^0 source resulted in levels at least 12 dB lower than the reference data.

(Added: Fig. 8 also show the results for the total noise from the jet-plate configuration.)

The significant changes in level and directivity with L_c , found for the free jet, does not occur in the installed case at $\phi_{obs} = 260^\circ$, over $\theta_{obs} = 50^\circ$. In this case, the results are much less dependent on the coherence parameter, and agreement at the edge-scattering-dominated region is good even for the unit coherence source. (Added: The total noise for L_c^C for the same observer positions differs from the others by about 3 4 dB at $\theta_{obs} = 100^\circ$, however we

should not that these levels are very similar to those of the free-jet with the same L_c value. However, we cannot observe such differences when looking at the separated scattering field, shown in the same figure.) By looking at (Added: the total noise at) $\phi_{obs} = 220^\circ$, we can note that this sensitivity becomes higher as the observer moves to a direction parallel to the plate trailing-edge, where the scattering effects are reduced. (Added: The plots for the scattering-only noise in Fig. 8 confirms that the sensitivity is higher at $\phi_{obs} = 220^\circ$.) Indeed, the sensitivity to coherence seems to be much lower wherever edge scattering effects are dominant. Results for $St = 0.2$ showed similar trends to $St = 0.3$ and were omitted here for conciseness.

~~(Deleted: One extra featured to be observed is that the installed-jet levels are very similar to the free jet for the lowest L_c^C at high angles, say above 80° . The possible reasons are that the resulting free jet noise is so high, that it masks the scattering noise and that the source is approaching the compact limit, such that scattering effects are reduced, as observed by Nogueira *et al.*¹². The main conclusion from these results is that the scattered field demonstrates low sensitivity to the coherence decay, and seems to depend more strongly on other model parameters.)~~

(Added: When moving the plate away from the jet, say $r/D = 1.5$, reduction of the scattering effects is expected. Fig. 8 depicts the results with the plate at at this position. Again, the sensitivity to coherence seems to be very low when edge scattering effects are dominant. As already commented the levels obtained for L_c^C are influenced by the very high free jet noise.)

(Added: Finally, the plate was kept at $r/D = 1$ and its trailing-edge was moved to other three axial positions ($x_{TE} = 2D$, $x_{TE} = 6D$ and $x_{TE} = 8D$). Results are shown in Fig. 10 for $\phi_{obs} = 260^\circ$. As experimental results are not available at these plate positions, only the LES source results are used as reference. For $x_{TE} = 2D$, the noise levels show low sensitivity to the coherence parameter at angles in which edge-scattering is dominant. However, when the plate is positioned very close to the peak source ($x_{TE} = 6D$) and further downstream of the peak source ($x_{TE} = 8D$), noise results becomes more sensitive to the variation of L_c . Results from the unit coherence source, for instance, were about 6 dB/ St lower in comparison with the sources with finite L_c values for ($x_{TE} = 8D$). This is an interesting finding that suggest that the results become more sensitive to the coherence parameter as the trailing edge is moved downstream of the peak source. It is also possible to observe that the SPL variation decreases as the L_c value decreases.)

(Added: The variation of x_{TE} was useful to identify the effects of coherence when the plate is moved downstream of X_{max} . However, the results for $x_{TE} = 8D$, and even $x_{TE} = 6D$, should be observed with care as this configuration probably violates the hypothesis of low influence of the plate over the jet flow field. In this sense, these results may not reproduce well the aspects of the real phenomenon.)

~~(Deleted: Analysis with line source)~~

~~(Deleted: Indeed, the isolated jet case is very sensitive to source variations and the radial profiles carry some complexity regarding the range of length scales, for example. It should be thus much simpler to fit a line source with radially integrated LES data. For doing that, a line source was educed by considering radial compactness at low St . The derivation of the~~

FIG. 8. (Color online) Model source results compared to the experiment and LES source for $St = 0.3$ with Free-field and tailored Green's function and varying L_c .

line source is described in section II E and the model shown in Eq. ???. The source parameters are obtained by using the double radial integral of the LES source and are summarized in table ???. This procedure is supposed to carry less uncertainties in the fitting process.)

(Deleted: Although the approximation sounds valid for the free jet, its application may be not accurate for the installed case, since the Green's function depends strongly on the distances between source and trailing edge, which varies azimuthally. Assuming radial compactness in this case probably leads to larger errors for the installed case. To reduce the effects of these simplifications, a cylindrical surface source, concentrated on the lip line

FIG. 9. (Color online) Model source results compared to the experiment and LES source for $St = 0.3$ with tailored Green's function for the plate at $r/D = 1.5$ and varying L_c .

FIG. 10. (Color online) Model source results compared to the LES source for $St = 0.3$ with the plate at different axial positions and varying L_c . Observers at $\phi_{\text{obs}} = 260^\circ$.

is also used, as defined by Nogueira *et al.*¹². The same parameters used for the line source are used in the cylindrical surface approach.)

(Deleted: Some of the parameters in Table ?? changed significantly from those in Table I.

The axial position X_c appears more upstream for all St . The envelope length L_x now

decreases with St , a different trend from the two-dimensional fit. Higher values of k_h were

found for all St .)

(Deleted: Acoustic results are show in Fig. ??(a) for $St = 0.2$, comparing the line and cylindrical sources. For the isolated jet, results showed a slightly better agreement with reference data in both the maximum amplitude and directivity when compared to the previously used source model. However, noise levels are still overpredicted by about 4 dB. Regarding installed jet cases (Fig. ??(d)), the results for $St = 0.2$ are in very good agreement with the reference data, as well as the volumetric source. Minor differences can be noted between the line and cylindrical surface sources, even for the installed case with plate at $r/D = 1$.)

(Deleted: Free-jet results were also slightly improved for $St = 0.3$ (Fig. ?? (b)). For the installed jet case (Fig. ?? (e)), the model results were about 4 dB higher than the reference data. Besides possible inaccuracies in the fitting process, it is important to emphasize that the use of the radially integrated data can lead to errors in the prediction of the jet-plate noise.)

(Deleted: Data for $St = 0.75$, depicted in Fig. ?? (c) reveals considerable differences between the cylindrical surface and the linear sources. The noise levels were overpredicted by about 4 — 5 dB by the line model and the cylindrical surface model could not correctly reproduce the directivity trends.)

(Deleted: Fig. ?? compares the radially integrated LES data with the fitted model sources with reference point at X_{max} . We can now see that the model can reproduce the main characteristics of the LES source in terms of PSD, CSD and coherence for the chosen St . Indeed it becomes much simpler to fit line information with simple Gaussian functions in comparison to the two-dimensional cases presented on the previous sections. The difficulties

regarding the radial variation of the coherence lengthscale L_c , for instance, are overcome by compacting the coherence information on a line.)

(Deleted: Fig. ?? shows that the asymmetric Gaussian function, with the obtained parameters, is suitable for modelling the PSD amplitudes. The amplitude and phases of the CSD were also well represented by the model at this position, as also shown in Fig. ?. It is important to note, however, that the shapes of both the CSD and coherence are not symmetric, especially for $St = 0.75$. In Fig. ?, for instance, the coherence shows a narrow peak, with rapid decay, followed by a much slower decay at the right side for $St = 0.75$. These features could not be well reproduced by the chosen Gaussian function.)

(Deleted: Even though the asymmetric shapes of both the CSD and coherence could not be completely represented by the used functions, it is clear the information was enough for a good and fast prediction of the trailing-edge scattering noise. More complexity seems to be required for the isolated jet at higher St , both regarding the radial information and the analysis of the coherence function. This is outside the scope of the present study.)

V. CONCLUSION

A study of jet-plate interaction noise was conducted using numerical data as input for wavepacket source models. A simplified source term was extracted from a numerical simulation database and the corresponding acoustic field was obtained for both the free-jet, by using the free-field Green's function, and for the installed jet by using a tailored function for a semi-infinite plate. After analysing the main characteristics of the simplified source term,

the numerical data was used to adjust the parameters of simple source models. The noise prediction capabilities of such models were assessed for both isolated and installed jets.

The main features of the sound field for the isolated jet at low St and low polar angles could be obtained by considering only the linear part of the $T_{xx}(m = 0)$ source term extracted from the LES. Moreover, this term was shown to be the main contributor to the jet-plate scattered field, confirming the observations from 10. Significant sensitivity to the window function used on the calculation of the source CSD was observed. In general, results using rectangular window were not desirable and presented large sensitivity to the block size, in comparison to the Hanning windows. This suggested high sensitivity of the acoustic analogy to errors in the signal processing.

~~(Deleted: Two- and one-dimensional)~~ Source models were built, ~~(Deleted: both)~~ by minimising the squared error between the LES CSD and the proposed models. The sensitivity to the coherence length scales was analysed. By observing the CSD fields, the two-dimensional source approach could not correctly represent the characteristics of the source in every region. That may be the main cause for the found deviations on the free-jet noise results. Anyhow, the installed jet noise ~~(Added: , for configuration in which trailing-edge scattering is relevant,)~~ was seen to be much less sensitive to details in the modelled source.

~~(Deleted: The use of simpler source models, such as line source adjusted with radially integrated LES data, has proven to be more effective. This approach resulted in good agreement with experimental data, a consequence of involving less uncertainties in the fitting process. From the obtained results, we could observe that simplified models such as the ones~~

~~used by 13 and 12 are able to correctly predict the noise levels for both installed and isolated jets, as long as they are parametrised with consistent source data.)~~

In summary, simple wavepacket models adjusted with flow field data, showed great capabilities to reproduce the noise level and directivity, specially for installed jets, with very low computational effort; kinematic wave-packet sources may be useful for parametric studies of jet and wing placement, highlighting configurations with lower sound radiation. The installed jet results were much less sensitive to fitting errors, specially to variations on the coherence parameters. In this way, modelling for installed jets becomes much simpler. (Added: It is also important to mention that the analyses were limited to installed configurations in which the jet flow is not significantly affected by the plate and so trailing-edge scattering would be the main installation effect.)

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TABLE I. Values of the resulting source parameters from the CSD fits. Non-dimensional values.

St	A	X_c	L_x	α	L_r	k_h	L_c	Δ
0.2	1.18×10^3	9.32	5.92	0	0.17	1.67	3.95	0.64
0.3	0.31×10^3	3.26	9.45	-5.06	0.16	2.6	3.00	0.63
0.75	0.07×10^3	1.77	7.91	-22.8	0.11	6.01	3.31	0.84

List of Changes

Added: **at the region dominated by trailing-edge scattering,** on page 2, line 10.