Sampled-Data Sliding Mode Control Design of Single-Link Flexible Joint Robotic Manipulator

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Abstract – **In this paper, we have designed a sampled data control law for the single-link flexible joint robotic manipulator using two different approaches. First, we have designed the sampled-data sliding mode control (SMC) based on the continuous time system. In the second approach, we have obtained the approximate discrete model of the system and then designed discrete sliding mode control based on this approximate system. Simulation results have been obtained and a performance comparison has been presented for both techniques.**

Keywords – **Sampled-data control, Sliding mode control, Robotic manipulator.**

I. Introduction

Robotic manipulators are widely being used in industry are highly nonlinear systems which often suffer from unmodeled dynamics and uncertainties. Recently control engineers have designed controllers for flexible joint robots but it is still very less as compared to the rigid robots. However, in many applications, the robotic manipulators have some flexible joints as well. At the same time, the flexibility of joints may limits the performance and robustness of the controllers designed for robots or may lead to instability [1]. Also, the flexibility of joints can be considered a step for the flexible robot link [2] so to by studying the flexible joint manipulator can lead us to the flexible link which helps to have link lighter in weight that ultimately results in fast robotic motions. If the flexibility of joint is modeled by the spring, dynamic model of flexible joints can be obtained. Some advantages of flexible joints robotic manipulators over rigid manipulators are less control effort, fast motion, light weight and smaller dimension [3]-[6].

To design sliding mode controller for the flexible joint manipulator, state variables of each joint, velocities, acceleration and jerks of the link must be known [5]. Since last two decades, dynamic modeling and non-linear control of robotic manipulators (flexible) is the area of research which received notable consideration. To design a control topology that is efficient and robust, the first and fundamental step is to develop an accurate dynamic model of a flexible joint robot. In industrial and space applications, we need the controller that can reduce the disturbance effects and can cope with modeling uncertainties. For flexible joints, a robust controller should be designed for the stabilization or tracking purpose that has negligible link vibration.

For this purpose, different techniques have been suggested for last two decades. For example, linear quadratic regulation (LQR) control technique is widely used for such control [7], another technique which is vastly used is adaptive output feedback controller based on a back stepping [8]-[10]. In the case of nonlinear control, feedback linearization is the most renowned technique that is used for robotic manipulators [11]. The integral control technique described in [12] and [13], robust control that uses PD control and H∞ control, PD fuzzy and optimal control and sliding mode control [14]. In [15], an adaptive second order terminal sliding mode controller for robotic manipulators is proposed.

Almost all the research for control of robotic manipulators is carried out in the continuous time domain. However, due to the increasing availability of microprocessor hardware, control algorithms are now usually implemented in digital controllers. In this paper, we have focused on sampled data sliding mode control, as SMC is nonlinear control and is more robust as compared to most of the conventional control techniques.

In this paper, we have focused on the stabilization of flexible joint robotic manipulator in discrete time domain. Sliding mode control is designed using sampled data control techniques and their comparison is carried out on the basis of simulation results. This paper is divided into five sections. In section 2 dynamic modeling of single link flexible joint robot is discussed. In section 3, sliding mode controller is designed that would stabilize the robotic manipulator. In section 4 sampled data control techniques are discussed. Using these techniques SMC is designed for the system, simulation results are presented to show the comparison of sampled data control techniques. Section 5 discusses the conclusion on the basis of these simulation results.

II. Dynamic-Modelling of Flexible Joint Single-Link Robotic Manipulator

In this work, we have focused upon the flexible joint single link robotic manipulator. The dynamic model of single-link robotic manipulator with the flexible joints is derived using the Euler-Lagrange equations and is given by [16], [17]

$$I\ddot{q}_1 + MgL\sin q_1 + k(q_1 - q_2) = 0$$
$$J\ddot{q}_2 - k(q_1 - q_2) = \tau$$

Fig. 1 shows the model of single link flexible joint robotic manipulator [18]. q_1, q_2 are the link positions, I describes the link inertia whereas J is the actuator inertia, the nominal load is denoted by MgL, k represents the joint stiffness and input torque is denoted by u.

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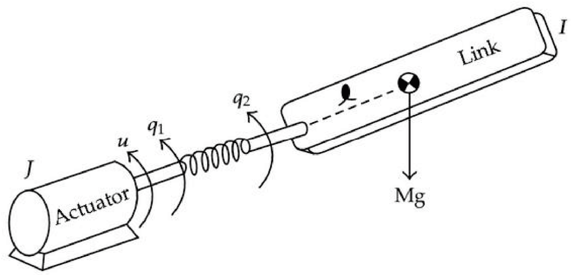


Fig. 1. Model of a single-link, flexible joint manipulator

Using this dynamic model state space model of this robotic manipulator is derived. State space model of manipulator is given by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -(a\cos x_1 + b + c)x_3 + a(x_2^2 - c)\sin x_1 + bdu$$

where $x_1 = q_1$ the constants are given by $a = \frac{MgL}{I}$, $b = \frac{k}{I}$, $c = \frac{k}{j}$, $d = \frac{1}{J}$ and the input torque is $\tau = u$. we have designed a sliding mode controller using this state space model for the stabilization of this system in next section.

III. Stabilization using Sampled Data SMC Based on Emulation

Sampled-data control refers to the control of a continuous system with discrete controller. There are two main techniques for sampled data control [19] [20].

- Emulation design
- Approximate discrete model based control design

Emulation design is considered as a simpler method to design a controller for sampled-data systems. In terms of system stability and performance, it is also considered as inferior design technique as compared to other methods. In designing controller by this technique sampling is completely overlooked. The continuous controller is discretized, in next step implementation is done using sample and hold devices.

Fig. 2 shows the block diagram of steps that are followed during the emulation design. In this technique of control design, the control law derived for the continuous time system is discretized. The classical sliding mode control suffers from chattering. Chattering is a severe problem especially in the mechanical system because it causes wear and tear in the system. There are different methods developed to reduce chattering. A well-known method is the replacement of the discontinuous signum function with the saturation function [17]. In this method, the real sliding is replaced with the sliding in a small vicinity of the discontinuous surface. The chattering issue can be overcome if we use saturation function instead of signum function. Consider the sliding surface

$$S(x) = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \tag{3}$$

The following control law has been proposed for the stabilization of the single link flexible joint robotic manipulator.

$$u[k] = \frac{1}{bd}[-k_1 x_2[k] - k_2 x_3[k] - k_3 x_4[k] - a(x_2^2[k] - c)\sin x_1[k]) - \beta(x) sat(S[k])]$$

where

$$\beta(x) = -(a\cos x_1 + b + c)x_3 + \beta_0 ; \beta_0 > 0 \tag{4}$$

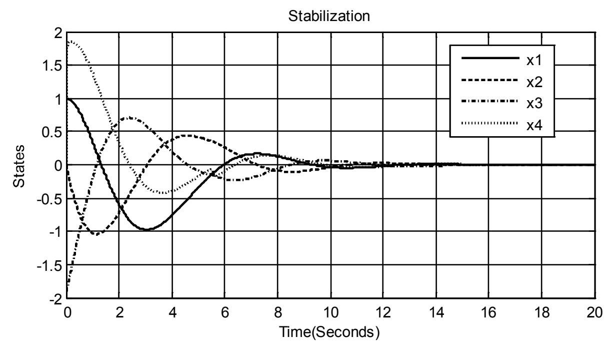


Fig. 3. States Stabilization using Sampled Data SMC based on Emulation

In Fig. 3, state stabilization based on Emulation design is shown. These simulation results are generated by using MATLAB/SIMULINK. State x_1 is the position of the link and we can see this state stabilizes to the origin within 12 sec. All other states also stabilize to the origin within a certain time. This shows that we have designed efficient control law using emulation that stabilizes all the system states.

IV. Stabilization using Sampled Data SMC Based on Approximate Discrete Model Design

In approximate discrete model based control design technique, the controller is designed by using the approximate discrete model of the plant in discrete time domain. For non-linear systems, exact modeling of the system is almost impossible so approximate discrete model of the plant is used to design a controller for nonlinear systems. Then, this discrete controller is implemented using sample-and-hold devices. In this paper, we have discussed the controller design for sampled data control.

Fig. 2. Emulation design

Fig. 4. SMC based on approximate discrete model

Fig. 4 shows the block diagram of steps that are followed during the sampled data control using approximate discrete model. Using this technique SMC is designed in this section. The discretization is done using Euler forward difference method. The Euler model approximation is

$$\dot{x}(t) = \frac{x(k+1) - x(k)}{T}$$

State space model of manipulator (1) is

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -(a\cos x_1 + b + c)x_3 + a(x_2^2 - c)\sin x_1 + bdu$$

Using the approximate discrete model of the manipulator we will design SMC that will stabilize the system. The approximate discrete model is given by

$$x_1[k+1] = x_1[k] + Tx_2[k]$$
$$x_2[k+1] = x_2[k] + Tx_3[k]$$
$$x_3[k+1] = x_3[k] + Tx_4[k]$$
$$x_4[k+1] = x_4[k+1] + T(-(a\cos x_1[k] + b + c)x_3 + a(x_2^2 - c)\sin x_1 + bdu[k])$$

Consider the following sliding surface

$$S[k] = k_1 x_1[k] + k_2 x_2[k] + k_3 x_3[k] + k_4 x_4[k]$$

To stabilize it globally using Sliding mode control law $u[k]$

$$u[k] = \frac{1}{bdT}[-S[k] - k_1 T x_2[k] - k_2 T x_3[k] - k_3 T x_4[k] - T^2 a(x_2^2[k] - c)\sin x_1[k]) - \beta(x)\, sat(S[k])]$$

where

$$\beta(x) = -T^2(a\cos x_1 + b + c)x_3 + \beta_0 \,;\ \beta_0 > 0$$

The constants are $[a\ b\ c\ d] = [2,1.4,0.2,0.5]$.

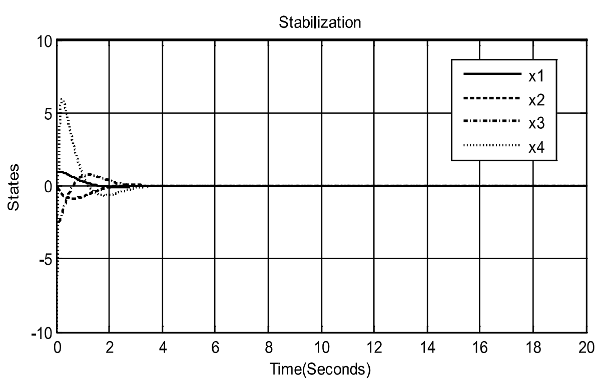


Fig. 5. States Stabilization using sampled data SMC based on approximate discrete model

In Fig. 5, state stabilization based on SMC (approximate discrete model) is shown. These simulations are generated by using MATLAB SIMULINK. State x_1 is the position of the link and we can see this state stabilizes to the origin within 4 seconds. All other states also stabilize to origin approximately within four seconds. This shows that SMC designed using approximate discrete model is more efficient as compared to the previous technique and it also stabilizes all the system states.

V. Comparison of Approximate Discrete Model and Emulation Design SMC

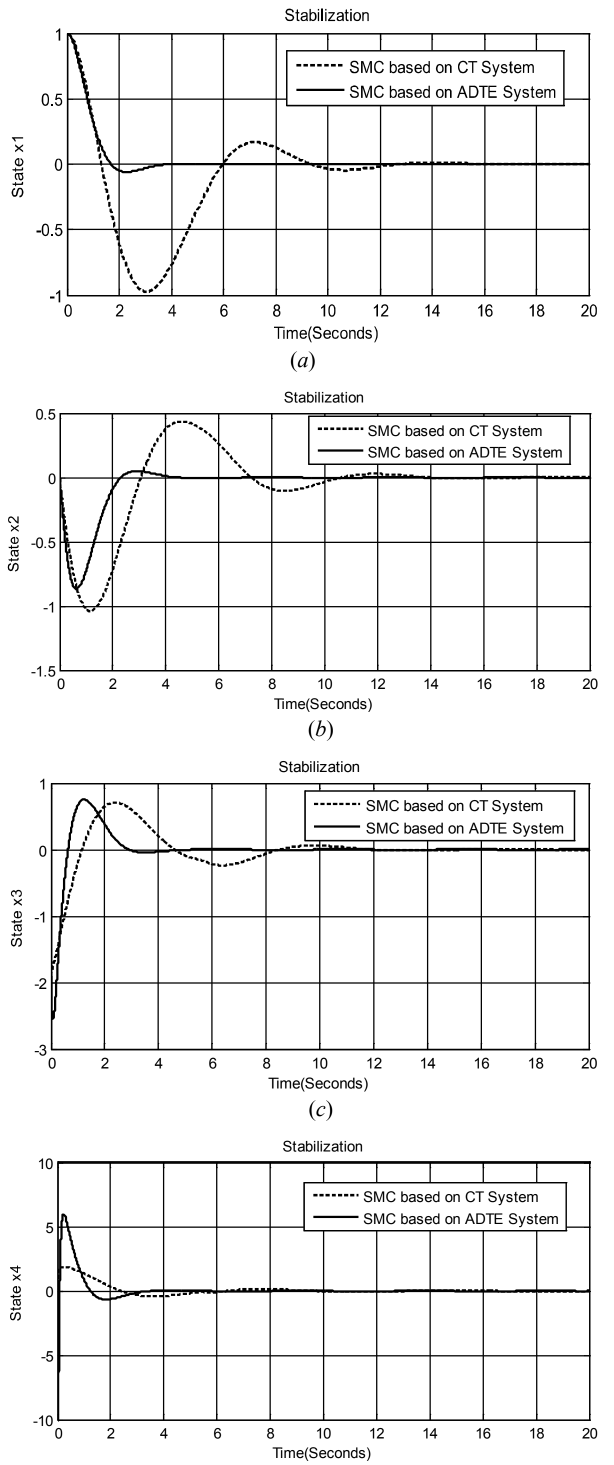
In this section, a comparison of both sampled-data control techniques is presented. The simulation is done using MATLAB/SIMULINK for the implementation of the controller on robotic manipulator. The results of the comparison of both techniques are shown in Fig. 6.

In simulation results, stabilization of Single link flexible joint robotic manipulator using SMC based on emulation design and SMC based on approximate discrete model are compared. It is obvious that stabilization of states using approximate discrete model is quicker than that of SMC based on emulation design. In the case of SMC based on emulation design, there are oscillations of states before settling down to the origin. Also, time taken to stabilize using emulation design is more than that of approximate discrete model design. Results of SMC based on approximate discrete model stabilize the states quickly which shows SMC based on approximate discrete model is better stabilization approach for single link robotic manipulator than that of SMC based on emulation design.

VI. Conclusion

In this paper, SMC feedback control for single-link flexible joint robotic manipulator system is discussed. Two approaches for the design of sampled data sliding mode control are studied and simulation results are presented. It is clear that the SMC based on approximate discrete model is better as it stabilizes the states in less time for the same parametric values of the system. Moreover, by using SMC based on approximate discrete model the region of attraction is increased as compared to emulation design. The main outcome of Sampled Data SMC is the robustness. It (SMC based on approximate discrete model) is more robust against disturbance to conventional techniques used for robotic manipulator control (feedback linearization). SMC based on approximate discrete model has the insensitivity to parameter variations to a larger extent (greater region of attraction), fast dynamic responses and better disturbance rejection which is required for the smooth motions of the robotic manipulator.

Now sampled data control is becoming popular than the continuous time. The sampled data controls that are derived and simulated can be further extended to sampled data observer. The states can be estimated with the help of an observer so we do not need to have sensors on all states. This work can also be extended for the n-link flexible joint robotic manipulator. Based on this work, a robust controller can also be designed for the tracking problem for the robotic manipulator.



(*a*) (*b*) (*c*) (*d*)

Fig. 6. Estimated States Comparison of approximate discrete model, Emulation SMC: (*a*) State X_1 stabilization, (*b*) State X_2 stabilization, (*c*) State X_3 stabilization, (*d*) State X_4 stabilization

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