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Set-membership computation of integrals with uncertain endpoints

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Numerical integration is one of the fundamental tool of scientific computation. Providing a reliable result to such problem is important for validated simulation [1] or for global optimization with continuous objective function [2]. An important work on inclusion methods for integral equations can be found in [3]. We propose an efficient **guaranteed** method for the computation of the **integral** of a nonlinear continuous function f between two **interval endpoints** $[x_1]$ and $[x_2]$, define by:

$$\int_{[x_1]}^{[x_2]} f(x)dx = \left\{ \int_{x_1}^{x_2} f(x)dx : x_1 \in [x_1], x_2 \in [x_2] \right\} \quad (1)$$

Two cases can occur whether the intervals $[x_1]$ and $[x_2]$ intersect or not. If the two intervals do not intersect, we produce an outer approximation of the set of possible values for the integral proving the following proposed theorem:

Theorem 1. *Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function. For $[x_1] = [x_1, \bar{x}_1]$ and $[x_2] = [x_2, \bar{x}_2]$ two intervals in \mathbb{IR} . Then the minimum and the maximum of set (1) can be defined by*

$$\int_{[x_1]}^{[x_2]} f(x)dx = \left[\min \int_{\mathcal{X}_{1-}^* \cup \{\bar{x}_1\}}^{\mathcal{X}_{2+}^* \cup \{x_2\}} f(x)dx, \max \int_{\mathcal{X}_{1+}^* \cup \{\bar{x}_1\}}^{\mathcal{X}_{2-}^* \cup \{x_2\}} f(x)dx \right] \quad (2)$$

using

$\mathcal{X}_{i-}^* = \{x \in [x_i] : f(x) = 0, f'(x) < 0\}$, $\mathcal{X}_{i+}^* = \{x \in [x_i] : f(x) = 0, f'(x) > 0\}$.

When the two intervals intersect, Second Fundamental Theorem of Calculus is applied. An implementation of the computation of the bounds (2) is introduced using interval analysis [4] and tested on several examples.

References

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