Gyro-kinetic theory and global simulations of the collisionless tearing instability: the impact of trapped particles through the magnetic field curvature

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The linear instability of the tearing mode is analysed using a gyro-kinetic approach within a Hamiltonian formalism, where the interaction between particles and the tearing mode through the wave-particle resonance is retained. On the one hand, the curvature of the magnetic field is shown to play no role in the linear instability when only passing particles are present in the plasma. On the other hand, the presence of trapped particles leads to an overall increase of the growth rate. Gyro-kinetic simulations using the state-of-the-art GKw code confirm these findings and are further used to investigate the impact of the magnetic field curvature and the temperature gradient on tearing modes including the effect of trapped particles. Without temperature gradient, wave-particle resonance with the trapped electrons tends to stabilize the tearing mode, while with finite temperature gradient, magnetic curvature tends to destabilize the tearing mode, suggesting an interchange mechanism. The balance of these two stabilizing/destabilizing effects leads to a threshold in temperature gradient beyond which the magnetic curvature plays a destabilizing role. This opens the way to a deeper understanding and control of the tearing instability in fusion plasmas.

I. INTRODUCTION.

In magnetised laboratory and space plasmas, a special class of instabilities, called tearing instability, can occur in the presence of non-ideal effects (such as resistivity or inertia) and can tear apart the magnetic field lines resulting in their reconnection. This process leads to a modification of the magnetic topology, characterised by the formation of magnetic islands. In space conditions they are considered to be important for dynamics and energy release in the magnetotail¹. In laboratory magnetically confined plasmas, tearing modes enhance particle and energy transport, degrading this way the overall confinement. In particular, they have been proved to result in fast ion losses^{2,3} and even lead to catastrophic disruptions⁴. Therefore, the understanding of the tearing mode is of prime importance in space and laboratory plasmas.

The tearing mode instability has been extensively studied in the framework of MagnetoHydroDynamics (MHD) theory^{5,6}. For many situations in space conditions and high temperature tokamak plasmas, regimes can be considered as weakly collisional and kinetic considerations are therefore required^{1,7,8}. In the tearing mode theory, usually the outer (ideal) and inner (inertial and resistive) regions can be separated. The stability of large scale islands in the MHD regime is determined by the global current profile effects, which introduce the so-called Δ' parameter determined from the outer region, and which has to be positive for the instability. The modifications of Δ' by magnetic profiling and kinetic effects due to trapped particles were considered in Ref. 9. The other mechanisms, related to the temperature gradients and collisions, are important locally (in the inner inertial-resistive layer). They have been studied in various settings^{7,10–12} and were recognized as important destabilizing mechanisms for $\Delta' < 0$. In collisionless regimes, the electron inertia replaces

the resistivity to support the instability^{7,13,14}.

Future tokamaks are expected however to operate at very low collisionality and various collisionless effects need to be considered together with magnetic field geometry. Toroidal effects due to magnetic field curvature have been studied in fluid theory^{6,15}. However, gyro-kinetic theory provides the most comprehensive framework for studies of kinetic effects in low collisional regimes and complex magnetic geometries and has been widely used for tearing mode studies^{16–23}.

Within the framework of gyro-kinetic theory, the trajectories of the guiding-centres in tokamaks can be analysed. They have complex structures, characterised by different frequencies and scales. In general, particles can be classified into two groups: (1) trapped particles, whose trajectories are localized on the low field side of the torus in the magnetic mirrors formed by toroidicity and (2) passing particles, which can circulate fully along the magnetic field lines and complete a poloidal turn. Special role of trapped particles on tearing mode stability has been recognized in analytical²⁴ and numerical studies^{25,26}. It was found that trapped particle resonances contribute to the destabilization of microtearing modes in low collisionality regimes^{25–29}. In this paper we investigate resonant effects of passing and trapped particles on tearing modes in collisionless regimes. This is done with analytic theory and with global gyro-kinetic simulations, through which we also perform an extensive parametric study of the tearing instability with emphasis on the effects of the magnetic field curvature and particle resonances.

The remainder of the paper is structured as follows. In section II, we derive, within the gyro-kinetic approach, the perturbed parallel current using a Hamiltonian formalism. Section III is devoted to the derivation of the tearing mode dispersion relation. This is done by integration of Ampère's law, under the so-called constant- ψ approximation and sep-

arating the radial domain into an ideal MHD (outer) region where kinetic effects are negligible and where we calculate numerically the Δ' stability parameter, and a non-ideal (inner) region where kinetic effects are retained through the wave-particle resonance. In section IV the different resonances between the tearing mode and the particles are studied analytically and the presence of trapped particles is shown to increase the growth rate of the instability. We also show that the magnetic curvature does not play any role in the tearing instability when only passing particles are considered. Section V is devoted to the numerical analysis using Gkw simulations. In particular, we confirm the analytic results and study in detail the impact of trapped particles, curvature and temperature gradient. In section VI we report on the different responses of particles to the tearing instability through an energy exchange diagnostic and evidences of the interaction between trapped particles and the tearing mode are provided.

II. HAMILTONIAN FORMALISM TO CALCULATE THE PERTURBED PARALLEL CURRENT WITHIN A GYRO-KINETIC APPROACH

The tearing instability can be described at the simplest level using Ampère's law

$$\nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{j} \quad (1)$$

where $\mathbf{A}$ is the vector potential, $\mathbf{j}$ is the current and μ_0 is the vacuum permeability. We assume that we have only parallel component of $\mathbf{A}$. Therefore $\mathbf{A} = A_{\parallel} \mathbf{b}$, where $\mathbf{b}$ is the unit vector along the magnetic field lines. Projecting onto the parallel direction straightforwardly gives an equation for the perturbed parallel current $j_{\parallel}$

$$\nabla_{\perp}^2 A_{\parallel} = -\mu_0 j_{\parallel} \quad (2)$$

that has to be integrated to obtain the dispersion relation. In Eq.(2), $\nabla_{\perp}^2$ is the Laplacian operator in the direction perpendicular to the equilibrium magnetic field.

In the following, we assume that the main response is that of electrons. Therefore, in order to calculate the perturbed parallel current, we will first of all calculate the exact linear response of electrons, which will yield the perturbed parallel current after integration in velocity space. The details of this calculation are given in Appendix A, where we follow closely the formalism used in Ref. 30. The idea is to start with the Vlasov equation in conservative form using a Hamiltonian

formalism

$$\frac{\partial F}{\partial t} - [\mathcal{H}, F] = 0 \quad (3)$$

where $[X, Y]$ represents the Poisson brackets between X and Y , i.e. $[X, Y] = \partial_{\mathbf{x}} X \partial_{\mathbf{p}} Y - \partial_{\mathbf{p}} X \partial_{\mathbf{x}} Y$, where $\mathbf{x}$ and $\mathbf{p}$ are the position and momenta, forming a set of canonical variables satisfying Hamilton's equations. We can introduce another system of canonically conjugated variables, satisfying Hamilton's equations, especially suitable for the analysis of particles in a tokamak, since they reflect the three directions of periodicity of the trajectories. These are the angle-action variables $\alpha, \mathbf{J}$, with $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and $\mathbf{J} = (J_1, J_2, J_3)$, where α_1 represents the gyro-phase, α_2 represents the periodicity angle in the poloidal direction and α_3 represents the periodicity angle in the toroidal direction. The actions are motion invariants associated to each direction of the particle trajectory. In particular, $J_1 = m_e \mu / (e Z_e)$, where e is the elementary charge, Z_e is the charge number (in the special case of electrons, $Z_e = -1$) and $\mu = m_e v_{\perp} / (2B)$ is the magnetic moment, with $v_{\perp}$ the projection of the electron velocity onto the direction perpendicular to the magnetic field and B the modulus of the magnetic field. The second action, J_2 , is written in terms of the toroidal flux of the magnetic field Φ as $J_2 = e Z_e \Phi + \frac{1}{2\pi} \oint m_e v_{\parallel} d\ell$, where ℓ is the coordinate along the magnetic field line and $v_{\parallel}$ is the component of the guiding-centre velocity parallel to the magnetic field. Finally, the third action J_3 is the toroidal canonical momentum $J_3 \equiv P_{\varphi} = -e Z_e \psi + m_e R v_{\varphi}$, with ψ the poloidal flux of the magnetic field, R is the major radius and v_{φ} is the projection of the electron velocity onto the toroidal direction.

We anticipate that we will calculate the parallel current for highly passing electrons, for which J_2 is closer to a function of the radial coordinate than J_3 , i.e. we can make the approximation $J_2 \approx e Z_e \Phi(r)$, which will introduce the gradients of the equilibrium with respect to the radial coordinate. This is why we perform a change of variables $\mathbf{J} \rightarrow \mathbf{I} = (J_1, J_2, \mathcal{H}_{\text{eq}})$, where $\mathcal{H}_{\text{eq}}$ is the equilibrium Hamiltonian, representing the energy in the absence of any perturbation. Following Hamilton's equations, the time derivative of the angles is $\dot{\alpha} = \partial_{\mathbf{J}} \mathcal{H} \equiv \Omega$, where we have introduced the three frequencies of motion, satisfying the ordering $\Omega_1 \gg \omega, \Omega_2, \Omega_3$, with ω the frequency of the tearing mode. The distribution function and the Hamiltonian are decomposed into equilibrium and perturbed parts. By definition, the equilibrium quantities do not depend on the angles, but only on the actions. This allows us to linearize the Vlasov equation and, owing to the periodicity of the perturbations with respect to the angles α , we can write the perturbed distribution function and Hamiltonian as Fourier series $\{\delta F, \delta \mathcal{H}\} = \sum_{\omega, \mathbf{n}} \{\delta F_{\mathbf{n}, \omega}, \delta \mathcal{H}_{\mathbf{n}, \omega}\} \exp(i(\mathbf{n} \cdot \alpha - \omega t))$. After some algebra, it can be shown that the exact linear solution of the Vlasov equation is written as

$$\delta F_{\mathbf{n},\omega} = \frac{\partial F_{\text{eq}}}{\partial \mathcal{H}_{\text{eq}}} \delta \mathcal{H}_{\mathbf{n},\omega} + \frac{1}{B} \frac{\partial F_{\text{eq}}}{\partial \mu} \delta \mathcal{H}_{\mathbf{n}',\omega} - \frac{\omega \partial \mathcal{H}_{\text{eq}} F_{\text{eq}} + n_2 \partial J_2 F_{\text{eq}}}{\omega - \mathbf{n}^* \cdot \boldsymbol{\Omega}} \delta \mathcal{H}_{\mathbf{n}^*,\omega} \quad (4)$$

where $\mathbf{n}' = (n_1 \neq 0, n_2, n_3)$ and $\mathbf{n}^* = (n_1 = 0, n_2, n_3)$.

The perturbed parallel current is calculated as

$$j_{\parallel} = e Z_e \int d^3 \mathbf{p} (v_{\parallel, \text{eq}} \delta F + \delta v_{\parallel} F_{\text{eq}}) \quad (5)$$

where the perturbed parallel velocity is expressed in terms of the parallel vector potential

$$\delta v_{\parallel} = -\frac{e Z_e}{m_e} A_{\parallel} \quad (6)$$

The perturbed parallel velocity proportional to the vector potential plays an essential role in cancelling the adiabatic response of electrons in the calculation of the perturbed parallel current, as can be observed in (B10). This is the so-called *cancellation problem*, often invoked within the framework of PIC simulations³¹. In order to continue further with analytic calculations we assume an equilibrium distribution function

$$F_{\text{eq}} = \frac{n_{\text{eq}}}{(2\pi T_{\text{eq}}/m_e)^{3/2}} e^{-\frac{\mathcal{H}_{\text{eq}} - \mathcal{U}_{\text{eq}}}{T_{\text{eq}}}} \quad (7)$$

$$j_{\parallel}(\mathbf{x}, t) = \rho_* e n_0 v_{\text{th}} \frac{2}{T_{\text{eq}}} \sum_{m,n,\omega} \left\langle \hat{v}_{\parallel}^2 \frac{\hat{\omega} - \hat{\omega}_{*g}}{\hat{\omega} - \hat{k}_{\parallel} \hat{v}_{\parallel} - \hat{\omega}_D} \right\rangle \hat{A}_{\parallel m,n,\omega}(\mathbf{x}) e^{i(m\theta + n\varphi - \omega t)} \quad (8)$$

where m and n are the poloidal and toroidal mode numbers, respectively. Note that, in general, the previous expression implies an infinite sum over all the wave-numbers and frequencies, but in the following we will simply project onto one single mode. The angles have been replaced by θ and φ , representing the poloidal and toroidal angles, respectively. The symbol $\hat{\cdot}$ indicates normalized quantities, the parallel wave vector $k_{\parallel}$ is given by $k_{\parallel} = (m/q + n)/R_0$ and the magnetic drift frequency ω_D is given by $\omega_D = 2qn/r (m_e v_{\parallel, \text{eq}}^2 + \mu B_0) / (e Z_e B_0 R_0)$, with q the safety factor characterising the helicity of the magnetic field lines and R_0 and B_0 the major radius and the modulus of the magnetic field, respectively, evaluated both at the magnetic axis. The notation $\langle \cdots \rangle$ has been introduced to represent an average over gyro-centre equilibrium velocity space weighted by the equilibrium distribution function

$$\langle \cdots \rangle = \int \mathcal{J} dv_{\parallel, \text{eq}} d\mu \cdots F_{\text{eq}} \quad (9)$$

with $\mathcal{J}$ the jacobian of the transformation. Notice that $k_{\parallel}$

which satisfies $\partial \mathcal{H}_{\text{eq}} F_{\text{eq}} = -F_{\text{eq}}/T_{\text{eq}}$. In the previous expression, $\mathcal{U}_{\text{eq}}$ is a function which depends on the motion invariants and represents the departure from the zero-mean-velocity Maxwellian equilibrium. In this paper, in order to generate an equilibrium parallel current, $\mathcal{U}_{\text{eq}}$ is an odd function of $v_{\parallel}$. The perturbed parallel current will be written in normalized units. For this purpose we normalize the velocities to the thermal velocity of electrons v_{th} , the distances to R_0 , the frequencies to the transit frequency $\omega_t = v_{\text{th}}/R_0$, the equilibrium distribution function to n_0/v_{th}^3 , with n_0 some normalizing density, the parallel vector potential to $B_0 R_0 \rho_*^2$ and the temperature to a normalizing temperature defined as $T_0 = m_e v_{\text{th}}^2/2$. Using the perturbed distribution function given by Eq. (4) and considering only deeply passing electrons, we show in Appendix B that the perturbed parallel current can be expressed as

vanishes on the rational surface defined as $q = -m/n$. In the remainder of this paper, the *eq* subscript for the parallel velocity will be dropped for the sake of simplicity. Note that ρ_* gives the typical ordering between the equilibrium and the perturbed distribution function and $e n_0 v_{\text{th}}$ is the normalization for the current. In this expression we have introduced the generalized diamagnetic frequency $\omega_{*g} = m T_{\text{eq}} \partial J_2 \log F_{\text{eq}}$, which includes the spatial dependence of the equilibrium. For thermal passing particles, at lowest order in ρ_* , one can write $J_2 \approx e Z_e \Phi$. Therefore, the derivative with respect to J_2 can be reduced to a derivative with respect to the radial position.

III. GYRO-KINETIC DISPERSION RELATION OF THE TEARING INSTABILITY

In order to derive the dispersion relation of the tearing instability, we make use of Ampère's law, given by Eq. (2), where we introduce the expression (8) for the perturbed parallel current. Ampère's law can also be written using normalized quantities and projecting onto one single (m, n, ω)

Fourier mode

$$\delta_e^2 \hat{\nabla}_\perp^2 \hat{A}_{\parallel m, n, \omega}(\mathbf{x}) = \frac{2}{\hat{T}_{\text{eq}}} \left\langle \hat{v}_\parallel^2 \frac{\hat{\omega} - \hat{\omega}_{*g}}{\hat{\omega} - \hat{k}_\parallel \hat{v}_\parallel - \hat{\omega}_D} \right\rangle \hat{A}_{\parallel m, n, \omega}(\mathbf{x}) \quad (10)$$

where $\hat{\delta}_e = \delta_e/R_0$ and $\delta_e = \sqrt{m_e/\mu_0 e^2 n_0}$ is the electron skin depth. The dispersion relation of the tearing mode is obtained integrating equation (10), but for this purpose we need to give convenient equilibrium quantities that will allow us to perform analytic calculations.

Regarding the equilibrium distribution function F_{eq} , we assume that it is given by the expression (7), where the $\mathcal{U}_{\text{eq}}$ term represents a shift in the parallel velocity, which leads to an equilibrium parallel current. In normalized units it takes the form

$$\hat{F}_{\text{eq}} = \frac{\hat{n}_{\text{eq}}}{(\pi \hat{T}_{\text{eq}})^{3/2}} \exp \left(-\frac{\hat{v}_\parallel^2 + \hat{v}_\perp^2 - 2v_\parallel \hat{u}_e}{\hat{T}_{\text{eq}}} \right) \quad (11)$$

where we assume that $\hat{u}_e$ depends only on P_φ and verifies the ordering $u_e^2 \ll v_\parallel^2$ (small shift in $v_\parallel$). In addition, we assume circular concentric magnetic surfaces. The magnetic field is therefore given by the simplified expression $\mathbf{B} = B_0 R_0 (r/q(r) R_0 \mathbf{e}_\theta + \mathbf{e}_\varphi)/R$, with $\mathbf{e}_\theta$ and $\mathbf{e}_\varphi$ unit vectors in the poloidal and toroidal directions, respectively, and $q(r)$ the safety factor as a function of the minor radius r . The poloidal flux is then written as

$$\psi(r) = B_0 \int_0^r \frac{r'}{q(r')} dr' \quad (12)$$

and the toroidal flux satisfies the equation

$$\frac{d\Phi}{d\psi} = -q(r) \quad (13)$$

Therefore, the expression for the generalized diamagnetic frequency can be written as follows

$$\omega_{*g} = \frac{m T_{\text{eq}}}{e} \frac{1}{r B_0} \left[\frac{1}{n_{\text{eq}}} \frac{dn_{\text{eq}}}{dr} + \frac{1}{T_{\text{eq}}} \frac{dT_{\text{eq}}}{dr} \left(\frac{v_\parallel^2}{v_{\text{th}}^2} + \frac{\mu B}{T_{\text{eq}}} - \frac{2v_\parallel u_e}{v_{\text{th}}^2} - \frac{3}{2} \right) + 2 \frac{v_\parallel}{v_{\text{th}}^2} \frac{du_e}{dr} \right] \quad (14)$$

Note that we keep the electron mean parallel velocity, together with its radial gradient. However, in the following, the assumption is made that $u_e \ll v_{\text{th}}$, so that the term multiplying the temperature gradient simplifies to the standard $E/T_{\text{eq}} - 3/2$. Nevertheless, the radial derivative du_e/dr is kept. Therefore, the diamagnetic frequency can be split into standard density and temperature terms, ω_{*n} and ω_{*T} , respectively, and the term ω_{*u} coming from the radial gradient of the parallel velocity, i.e.

$$\omega_{*g} = \omega_{*n} + \omega_{*T} + \omega_{*u} \equiv \omega_* + \omega_{*u} \quad (15)$$

where the notation ω_* has been used to gather both density and temperature gradients. Using normalized quantities, each of these frequencies read

$$\hat{\omega}_* = \rho_* \hat{T}_{\text{eq}} \frac{m}{2\hat{r}} \left[\frac{1}{\hat{n}_{\text{eq}}} \frac{d\hat{n}_{\text{eq}}}{d\hat{r}} + \frac{1}{\hat{T}_{\text{eq}}} \frac{d\hat{T}_{\text{eq}}}{d\hat{r}} \left(\hat{E} - \frac{3}{2} \right) \right] \quad (16)$$

$$\hat{\omega}_{*u} = \rho_* \frac{m}{\hat{r}} \hat{v}_\parallel \frac{d\hat{u}_e}{d\hat{r}} \quad (17)$$

The mean parallel velocity must be related to the safety factor by the Ampère's law applied to the equilibrium. In the cylindrical limit, Ampère's law in normalized units can be approximated as

$$\frac{\rho_e R_0}{\delta_e^2} \hat{j}_{\text{eq}} = \frac{1}{\hat{r}} \frac{d}{d\hat{r}} \left(\frac{\hat{r}^2}{q} \right) \quad (18)$$

where the equilibrium current is obtained from the integration of the equilibrium distribution function

$$J_{\text{eq}} = e Z_e \int \mathcal{J} dv_\parallel d\mu v_\parallel F_{\text{eq}} \quad (19)$$

with $\mathcal{J} = 2\pi B/m_e$. The equilibrium parallel current is then $J_{\text{eq}} = e Z_e n_{\text{eq}} u_e$ and in normalized units

$$\hat{j}_{\text{eq}} = Z_e \hat{n}_{\text{eq}} \hat{u}_e \quad (20)$$

Unless stated otherwise, in the following, the $\hat{\cdot}$ symbol will be dropped for the sake of simplicity and all quantities are assumed to be normalized. We assume that the gradient of the equilibrium parallel current is mainly due to the gradient of the mean electron parallel velocity. Therefore, one can integrate the normalized Ampère's law to obtain the expression of the safety factor in terms of the integrated mean velocity

$$q(r) = \frac{\delta_e^2}{\rho_e R_0 Z_e n_{\text{eq}}} \frac{r^2}{\int_0^r dr' r' u_e(r')} \quad (21)$$

Therefore, the differential equation (10) can be written using only the radial profile of the safety factor, which yields the linear dispersion relation of the tearing mode. Nevertheless, solving analytically this dispersion relation in the whole radial domain for general profiles of density, temperature and electron velocity is rather arduous. Therefore the tearing mode dispersion relation is solved by splitting it into two linear differential equations that will be solved in two different regions of the radial domain: an ideal outer region and a narrow resonant layer. For each region we will make some assumptions. In the resonant layer, $|k_\parallel| \ll 1$ and therefore the whole resonance $\omega - k_\parallel v_\parallel$ must be kept leading to the ordering $\omega \approx k_\parallel v_{\text{th}}$, however in the ideal region, $|\omega| \ll |k_\parallel v_\parallel|, |\omega_{*g}|$. The two solutions are matched by considering the so-called constant- ψ approximation⁵ that assumes that the perturbed parallel scalar potential of the magnetic field is constant inside the non-ideal layer.

A. The tearing mode equation in the outer (ideal MHD) region

In the ideal region where $\omega_{*g} \gg \omega$ and $k_{\parallel} v_{\parallel} \gg \omega, \omega_D$ the expression in between brackets of equation (10) reads

$$\left\langle v_{\parallel}^2 \frac{\omega - \omega_{*g}}{\omega - k_{\parallel} v_{\parallel} - \omega_D} \right\rangle = \left\langle v_{\parallel}^2 \frac{\omega_{*}}{k_{\parallel} v_{\parallel}} \right\rangle \quad (22)$$

Since ω_{*} (resp. ω_{*u}) is an even (resp. odd) function in $v_{\parallel}$, the only remaining term in the numerator of expression (22) is ω_{*u} . This leads to the differential equation

$$\delta_e^2 \nabla_{\perp}^2 A_{\parallel m, n, \omega}(\mathbf{x}) = -\rho_{*} n_{\text{eq}} \frac{m}{r k_{\parallel}} \frac{du_e}{dr} A_{\parallel m, n, \omega}(\mathbf{x}) \quad (23)$$

Using the relation between the safety factor and the mean electron parallel velocity given by equation (21) and assuming again that the density gradient does not affect significantly the parallel current gradient we can write

$$\frac{d^2 A_{\parallel}(r)}{dr^2} + \frac{1}{r} \frac{dA_{\parallel}(r)}{dr} - \frac{m^2}{r^2} A_{\parallel}(r) = \Lambda(r) A_{\parallel}(r) \quad (24)$$

where we have noted $A_{\parallel} \equiv \delta A_{\parallel m, n, \omega}$ for simplicity and

$$\Lambda(r) = \frac{-3s + 2s^2 - r^2 q''(r)/q(r)}{r^2 (1 + q(r) \frac{m}{n})} \quad (25)$$

where s is the magnetic shear defined as $s = r q'(r)/q(r)$, with q' and q'' representing respectively, the first and second derivatives of the safety factor with respect to r . We consider that $q > 1$, $q' > 0$ and $q'' > 0$ for all r . We will also suppose that there is a radial position r_s such that $q(r_s) = -m/n$. This means that for $r < r_s \Rightarrow q(r) < -m/n$ and for $r > r_s \Rightarrow q(r) > -m/n$. Then, for $r < r_s$ (resp. $r > r_s$) we have the inequality $\Lambda(r) < 0$ (resp. > 0), which means that for $A_{\parallel} > 0$ the second derivative must be negative (resp. positive) and therefore the solution is concave (resp. convex). In addition, note that $\lim_{r \rightarrow r_s^{\pm}} \Lambda(r) = \pm \infty$. This means that the first derivative is discontinuous at the position $r = r_s$, which allows us to define the parameter Δ' ⁵

$$\Delta' = \lim_{\epsilon \rightarrow 0} \frac{A'_{\parallel}(r_s + \epsilon) - A'_{\parallel}(r_s - \epsilon)}{A_{\parallel}(r_s)} \quad (26)$$

where $A'_{\parallel}$ denotes the derivative of $A_{\parallel}$ with respect to r . This parameter represents the jump of the solution across the resonant surface and it will be used as a matching parameter between the solution in the inner region and the solution in the outer region. In normalized units we have $\Delta' = R_0 \Delta'$. In the following, only the normalized values of the Δ' parameter are used.

Equation (24) can be solved using the shooting method³² for a general q profile, but in the following we use a q profile of the Wesson-type³³

Figure 1: The tearing mode eigenfunction for different values of ν , $q_a = 3.5$ and $\epsilon = 1/3.3$.

$$q(r) = q_a \frac{r^2/\epsilon^2}{1 - (1 - r^2/\epsilon^2)^{\nu+1}} \quad (27)$$

where q_a is the value of the safety factor at the position $r = a$ and ν is a parameter that controls the current density peaking. The solution of the tearing mode equation in the outer region is presented in Fig. 1 for three different values of the parameter ν , and fixing $q_a = 3.5$ and the aspect ratio $R_0/a = 0.3$. The boundary conditions $A_{\parallel}(r_{\min}) = A_{\parallel}(r_{\max}) = 0$ are used, and at the resonant surface $r = r_s$, we fix a normalized value $A_{\parallel}(r_s) = 1$.

In Fig. 2 we show the contours of Δ' in solid black lines scanning over q_a and ν . The dotted magenta lines denote the position of the resonant surface. The dashed blue line presents the limit $q(r_{\min}) = 1$. The domain of interest for the analysis of the $(m, n) = (2, -1)$ mode relies above that limit. Beneath it, the mode would co-exist with the $(m, n) = (1, -1)$ mode. In the same figure, the asterisks represent the simulations that will be analysed in section V, which are localised in the region $q(r_{\min}) > 1$, exploring increasing values of Δ' . In the next section we assume that the value of Δ' is known and determined from the outer solution, and we use it to solve the tearing equation in the inner layer.

B. The tearing mode equation in the inner region

In the region around the resonant surface $k_{\parallel} \ll 1$ and therefore the resonance in the expression in between brackets of Eq. (10) is kept as it is. Due to parity reasons, the contribution from ω_{*u} in the numerator can be neglected, so that the

Figure 2: Contours of Δ' (solid black) and the resonant surface position r_s (dotted magenta) as a function of the q profile parameters ν and q_a . The blue dashed line denotes the limit $q(r_{\min}) = 1$ and the blue asterisks denote where on this diagram the simulations shown in section V lie. Δ' is normalized to R_0^{-1} and r_s is normalized to R_0 .

tearing mode equation reads

$$\delta_e^2 \nabla_{\perp}^2 A_{\parallel m, n, \omega}(\mathbf{x}) = \frac{2}{T_{eq}} \left\langle v_{\parallel}^2 \frac{\omega - \omega_{*}}{\omega - k_{\parallel} v_{\parallel} - \omega_D} \right\rangle A_{\parallel m, n, \omega}(\mathbf{x}) \quad (28)$$

In this expression, the parallel wave vector can be expressed as $k_{\parallel}(x) = k_y/L_s x = k'_{\parallel} x$, with L_s the magnetic shear scale length and $k_y = m/r$, and the diamagnetic frequency is expressed as

$$\omega_{*} = \omega_{*n} \left[1 + \eta_e \left(v^2 - \frac{3}{2} \right) \right] \quad (32)$$

where $v^2 = v_{\parallel}^2 + v_{\perp}^2$, $\omega_{*n} = \rho_{*} T_{eq} m / (2r n'_{eq} / n_{eq})$ and $\eta_e = (n'_{eq} / n_{eq}) / (T'_{eq} / T_{eq})$.

IV. ANALYTICAL SOLUTION OF THE TEARING MODE DISPERSION RELATION

It is not straightforward to compute the integral (31). In order to shed some light of the underlying physics, we make

We assume that the width of the resonant layer is 2Δ . Integrating the equation over this region yields

$$\delta_e^2 \int_{-\Delta}^{\Delta} \frac{d^2 A_{\parallel}}{dx^2} dx = A_{\parallel}(0) \int_{-\Delta}^{\Delta} \Lambda dx \quad (29)$$

where we have again noted $A_{\parallel} \equiv A_{\parallel m, n, \omega}$ and the so-called constant- ψ approximation has been made, assuming that the solution is constant in the resonant layer. In addition, to perform the integration in the resonant layer the assumption of slab geometry is made, such that $\nabla_{\perp}^2 \equiv d^2/dx^2$, where $x = (r - r_s)/R_0$ represents the distance to the rational surface at the radial position r_s , normalized to R_0 . This assumption is justified due to the small width of the resonant layer and is applied in this paper only to simplify the analytic calculations. Moreover, the coefficient Λ is now given by the more general expression

$$\Lambda = \frac{2}{T_{eq}} \left\langle v_{\parallel}^2 \frac{\omega - \omega_{*}}{\omega - k_{\parallel} v_{\parallel} - \omega_D} \right\rangle \quad (30)$$

The integral on the left hand-side of Eq. (29) gives $\Delta' A_{\parallel}(0)$. For the integral on the right-hand side we perform a change of variable $v'_{\parallel} = -v_{\parallel}$ and $x' = -x$ to the negative domain of the space integral. In addition, since the perturbed parallel current is localised in a very narrow region around the resonant surface, we can extend the integral with respect to the radial distance to infinity without introducing significant errors. Under these assumptions, the collisionless tearing mode dispersion relation in the magnetic limit is expressed in normalized units as

$$\delta_e^2 \Delta' = \frac{8n_{eq}}{T_{eq}^{5/2} \sqrt{\pi}} \int_0^{+\infty} dx \int_{-\infty}^{+\infty} dv_{\parallel} \int_0^{+\infty} v_{\perp} dv_{\perp} v_{\parallel}^2 e^{-\frac{v_{\perp}^2 + v_{\parallel}^2}{T_{eq}}} \frac{\omega - \omega_{*}}{\omega - k_{\parallel}(x)v_{\parallel} - \omega_D} \quad (31)$$

first of all the assumption that all the particles are deeply passing. Afterwards, we assume that only a fraction of the electron population is deeply passing and then analyse its effect on the tearing instability.

A. Solution of the dispersion relation with only passing particles

As a first step, we neglect the curvature and gradient of the magnetic field in the magnetic drift, such that $\omega_D = 0$ and only the $k_{\parallel} v_{\parallel}$ resonance is taken into account, i.e. the resonance with the parallel motion $\omega - k_{\parallel} v_{\parallel}$. In this case, since the denominator does not depend on the perpendicular velocity, we can first of all perform the integral with respect to $v_{\perp}$. Secondly, the velocity space integral can be performed using the plasma dispersion function and its derivatives as

defined in Ref. 34.

After some algebra, the dispersion relation of the tearing mode reads

$$i\delta_e^2\Delta' = \frac{4n_{eq}\sqrt{\pi}}{\sqrt{T_{eq}}|k'_\parallel|} \left[\omega - \omega_{*n} \left(1 + \left(2T_{eq} - \frac{3}{2} \right) \eta_e \right) \right] \quad (33)$$

Writing $\omega = \omega_r + i\gamma$, where ω_r is the real frequency and γ is the growth rate, the solution of Eq. (33) reads⁸

$$\gamma = \frac{1}{4\sqrt{\pi}} \frac{\sqrt{T_{eq}}}{n_{eq}} \delta_e^2 |k'_\parallel| \Delta' \quad (34)$$

$$\omega_r = \omega_{*n} \left[1 + \left(2T_{eq} - \frac{3}{2} \right) \eta_e \right] \quad (35)$$

Strictly speaking, if one chooses the normalization constant T_0 as the value of the temperature at the resonant surface, in this case $T_0 = T_{eq}$, i.e. $\hat{T}_{eq} = 1$. Then the background density and temperature profiles do not change the growth rate of the mode, however they additionally make the mode oscillate with a real frequency that scales linearly with the density and temperature gradients. It is important to note that we have taken into account density and temperature gradients in the local approximation. This means that although we consider radial profiles of the background quantities, they are evaluated locally. Therefore, they enter the equation as constant values on the resonant surface. Of course, a more rigorous approach is to consider the radial dependence of the profiles. However this makes the x -integral too complicated to be solved analytically. The effect of profiles will be evaluated more adequately in the next section through gyro-kinetic simulations using Gkw code.

We look now into the effects of the inhomogeneity of the magnetic field on the stability of the tearing mode. This means that the term containing the toroidal effects are kept in the resonant term of Eq. (31). Of course, the concept of toroidicity in slab geometry is quite contradictory. In fact, several assumptions are made for the magnetic drift ω_D . First, we consider the local approximation, meaning that the x -dependence is omitted but the derivative with respect to x can be non-zero. Therefore, the geometry term $\mathbf{b} \times \nabla B$ responsible for the curvature of the magnetic field can be treated as a parameter in the equation. Second, all oscillations in the perpendicular direction are neglected, we assume to be at the low field side of the tokamak where $\theta = 0$. Thus the toroidal coupling between modes is not considered.

Integrating first over the radial direction allows to make no assumption on the dependence of ω_D on the velocity space. Then using the Gaussian integrals for the velocity space, we obtain the same expressions for the frequency and growth rate as those given by Eq. (34). A more accurate calculation can be performed by considering that the magnetic drift is actually a differential operator in the poloidal angle, which introduces nonlocal effects due to mode-mode coupling. It

can be shown that even when these effects are taken into account, the mode frequency and growth rate are not modified. The details of the calculations are not given here for the sake of readability and clarity, but the interested reader can find the complete calculation in Appendix D.

Therefore, when considering only passing particles, and in the collisionless magnetic limit, the drift of particles due to the magnetic field inhomogeneity and curvature does not have an impact on the stability of the mode.

B. Correction due to the presence of trapped electrons

To consider the impact of trapped electrons on the stability of the tearing mode, we need to use the resonant response given by expression (4), which is the one valid for both passing and trapped electrons, since no assumption on their orbits was made at that step. For trapped particles, $\Omega_2 \gg \omega, \Omega_3$. Therefore, for a resonance to occur the only possibility is $n_2 = 0$. This is mathematically equivalent to performing a bounce-average of the Vlasov equation. The resonance then reduces to $\omega - n_3\Omega_3$. In Appendix C we show that for trapped particles, the precessional frequency is proportional to the magnetic drift (see Eq. (C30)). In the absence of magnetic drift the response of trapped particles to the tearing mode is purely non resonant. Therefore, trapped particles do not contribute to the growth rate and the response of the whole electron population is reduced to the response of deeply passing electrons. Note that even if the response was resonant, it can be shown (see Appendix E) that the Hamiltonian of deeply trapped particles in the absence of magnetic drift vanishes. Thus, the velocity integral in Eq. (31) must be performed only in the passing domain $|v_\parallel| > \sqrt{2\varepsilon}|v_\perp|$ and the growth rate and frequency finally read

$$\gamma = \frac{1}{4\sqrt{\pi}} \frac{\sqrt{T_{eq}}}{n_{eq}} \delta_e^2 |k'_\parallel| \Delta' (1 + 2\varepsilon) \quad (36)$$

$$\omega = \omega_{*n} \left[1 + \left(2T_{eq} - \frac{3}{2} \right) n_{eq} \right] \quad (37)$$

Therefore, when a fraction of the electron population is magnetically trapped, the growth rate of the tearing mode is increased by a factor $1 + 2\varepsilon$. Physically, this can be understood as follows²⁴. The free energy for the tearing mode to be excited comes from the radial gradient of the equilibrium parallel current. This free energy is encapsulated in the stability parameter Δ' . Therefore, the free energy comes from the ideal region outside the resonant layer. This energy is subsequently transferred to particles within the resonant layer, where kinetic effects must be retained. The difference between the free energy that the mode takes from the ideal region and the energy that the mode transfers to the particles in the resonant layer is the energy available for the mode to grow. Since deeply trapped electrons do not contribute to

this transfer of energy, there is an increase of energy available for the mode to grow, resulting in an increased growth rate.

When the magnetic drift is taken into account, a resonance can occur between the tearing mode and the precessional motion of trapped particles, opening a new channel of energy transfer from the mode to the particles and therefore stabilizing the mode. The analytic calculation of the modification of the growth rate in the presence of trapped particles and magnetic drift is not straightforward. Therefore, we use a gyro-kinetic code for the remaining analysis.

V. LINEAR GYROKINETIC SIMULATIONS OF COLLISIONLESS TEARING MODES WITH GWK

The self-consistent treatment of tearing modes requires radial profiles. For this reason, the global version of the gyrokinetic code GWK³⁵ is used. In this section we provide a brief description of the code and then we present the implementations required for the excitation of the tearing mode in GWK.

A. The gyrokinetic model in GWK

The code GWK solves the gyrokinetic equations. The full details can be found in Ref. 35 and references therein. Here we outline the basic set of linear equations that are solved. As in our theoretical approach presented in the previous sections, the δF approximation is used. The equation for the perturbed distribution function, for each species is,

$$\frac{\partial g_s}{\partial t} + (v_{\parallel} \mathbf{b} + \mathbf{v}_D) \cdot \nabla \delta F_s - \frac{\mu_s}{m_s} \frac{\mathbf{B} \cdot \nabla B}{B} \frac{\partial \delta F_s}{\partial v_{\parallel}} = S(F_{\text{eq},s}) \quad (38)$$

where $g_s = \delta F_s + (Z_s e / T) v_{\parallel} \langle A_{\parallel} \rangle F_{\text{eq},s}$, $\mu = m_s v_{\perp}^2 / 2B$ is the magnetic moment, $v_{\parallel}$ is the velocity along the magnetic field, and m_s and Z_s are, respectively, the particle mass and charge number for species s . The drift velocity due to the gradient and curvature of the magnetic field is $\mathbf{v}_D = (1/Z_s e)(m_s v_{\parallel}^2 + \mu_s B) / B \mathbf{B} \times \nabla B / B^2$. Note that the nonlinear term $(\mathbf{v}_{E \times B} \cdot \nabla) g_s$, where $\mathbf{v}_{E \times B}$ is the $\mathbf{E} \times \mathbf{B}$ velocity) is not taken into account in this linear study. $S(F_{\text{eq},s})$ is the source term which is determined by the equilibrium distribution function of species s . The last term of the left-hand side of Eq. (38) is responsible for the trapping of particles (the so-called μ -gradB or mirror term). The temperature and density profiles have the radial form,

$$n(r) = n_0 \exp \left[-\frac{R}{L_n} w \tanh \left(\frac{r - r_0}{w} \right) \right] \quad (39)$$

$$T(r) = T_0 \exp \left[-\frac{R}{L_T} w \tanh \left(\frac{r - r_0}{w} \right) \right] \quad (40)$$

where the distances are normalised to R_0 , w is the width of the region where the gradient is localised, R/L_n , R/L_T , n_0

and T_0 are the logarithmic density and temperature gradients and the density and temperature, respectively, all evaluated at the reference radius r_0 . All the normalizations in the code are consistent with the ones used so far in the present paper. The electrostatic potential is calculated from the gyrokinetic quasineutrality equation given by,

$$\sum_s Z_s e \int (J_0 \cdot \delta F_s)^{\dagger} \mathcal{J}_s dv_{\parallel} d\mu_s + \sum_s \frac{Z_s^2 e^2}{T_s} \int ((J_0^2 \cdot \delta F_s)^{\dagger} - \phi) F_{\text{eq},s} \mathcal{J}_s dv_{\parallel} d\mu_s = 0 \quad (41)$$

where the first term is the perturbed gyro-centre charge density and the second is the polarisation density which is only calculated from the local Maxwellian. In this expression, J_0 is the gyro-average operator and the dagger symbol represents its complex conjugate. Similarly, the parallel vector potential is calculated from parallel Ampère's law

$$-\nabla^2 A_{\parallel} + \sum_s \frac{\mu_0 Z_s^2 e^2}{T_s} \int v_{\parallel}^2 (J_0^2 \cdot A_{\parallel})^{\dagger} F_{\text{eq},s} \mathcal{J}_s dv_{\parallel} d\mu_s = \sum_s \frac{\mu_0 Z_s e}{T_s} \int v_{\parallel} (J_0 \cdot g_s)^{\dagger} \mathcal{J}_s dv_{\parallel} d\mu_s \quad (42)$$

B. Simulations set-up to analyse the tearing instability in GWK

The tearing mode is linearly excited in the code²¹ by introducing an electron flow u_e in the equilibrium distribution function. The generated tearing mode is driven due to the inertia of electrons when we neglect resistivity through collisions. The mean velocity of electrons is related to the imposed q profile following the expression (21). We use the same q profile as for our analytic calculations, given by the Eq. (27). As an example, a linear collisionless simulation of a self-consistent tearing mode has been run for the following parameters: aspect ratio $R_0/a = 3.33$, normalized ion Larmor radius $\rho_* = \rho_i/R_0 = 0.002815$, the safety factor at the edge $q_a = 3.5$, current peaking parameter $\nu = 2.6$. This corresponds to a tearing mode with $\Delta' \approx 17.31$, calculated using the shooting method to solve the tearing equation outside the resonant layer, as presented in section III A. We represent in Fig. 3 the safety factor profile described by Eq. (27) (bottom) for these parameters, the radial eigenfunction of the parallel vector potential (middle) and its second radial derivative (top) which is highly localized around the resonant layer at the $q(r/R_0) = 2$ surface, represented by a vertical dashed line together with the label $r = r_s$. For linear simulations, a single $n = 1$ toroidal mode is considered and the numerical resolution is $N_r \times N_s \times N_{v_{\parallel}} \times N_{\mu} = 512 \times 30 \times 64 \times 15$, where N_r is the number of radial grid points, N_s is the number of points in the parallel direction, $N_{v_{\parallel}}$ is the number of points in parallel velocity and N_{μ} is the number of points in the magnetic moment direction. The selected number of points in

Figure 3: The radial profiles of the imposed safety factor q (bottom), the parallel vector potential (middle) and its radial second derivative (top) as calculated by GKW in toroidal geometry for density and temperature gradients $R/L_n = R/L_T = 2.2$. The most unstable mode locates at the $q = 2$ surface, indicated by a vertical dashed line and labeled by $r = r_s$, where the parallel current is highly localized.

the radial direction is enough to describe the physics around the thin resonant layer²¹. In our collisionless simulations we can artificially suppress the electrostatic potential, the mirror term and the curvature of the magnetic field. This capability allows us to run the code in the same limit as the one where the analytic theory has been derived. When radial gradients are used in the simulations, the reference position r_0 is the same as the position of the resonant surface, i.e. $r_0 = r_s$, and the width of the region where the gradient is localized is set to $w = 0.03$.

C. Parametric study of the collisionless tearing mode

Analytically, we have shown previously that, when particle trapping is taken into account, the growth rate is increased by a factor $1 + 2\epsilon$. The position of the resonant surface is inversely proportional to Δ' . This means that including trapping effects will amplify the growth rate by a ~ 1.44 factor

Figure 4: The ratio between the growth rate with and without trapping in terms of Δ' , in solid black line as calculated numerically with GKW and in red dashed line as predicted by our analytic theory.

that should decrease with Δ' . In Fig. 4, we represent the ratio between the tearing mode growth rate with and without trapping as a function of the stability parameter Δ' , calculated using the shooting method and the q profile used in GKW. The scan on Δ' has been performed by changing the q_a and ν parameters in the q profile. The points used in this scan are indicated by asterisks in Fig. 2. The analytical prediction in dashed red line agrees with the numerical calculation from GKW simulations, represented by solid black line, within an error of $\sim 1 - 5\%$. All the simulations used for Fig. 4 have been run by switching off the term $\mathbf{v}_D \cdot \nabla$ in the gyro-kinetic equation, which represents the magnetic gradient and curvature drift. It is to be noted that GKW this term can be switched on and off independently of the mirror term $\mathbf{b} \cdot \nabla B$, responsible for the trapping of particles. This mirror term is the one that has been switched on and off to obtain the ratio $\gamma_{\text{trapping}}/\gamma_{\text{no trapping}}$. One can conclude that in the absence of magnetic drift, magnetic trapping due to the mirror term destabilizes the mode by a factor that depends on the position of the resonant surface of the mode.

We have subsequently performed a parametric study of the linear instability of the tearing mode to further study the impact of the curvature, radial gradients and particle trapping. For this purpose, we have run a set of global simulations, with and without temperature gradient, with and without particle trapping and with and without magnetic curvature. Note that the density gradient is always set to $R/L_n = 0$. This is due to the fact that the equilibrium parallel current depends linearly on the density. Therefore, in the absence of density gradient the stability parameter Δ' is not modified.

Figure 5: The growth rate of a collisionless tearing mode for flat background profiles as a function of Δ' (top) and $|k'_\parallel| \Delta'$ (bottom) given by GKW simulations, for several cases of a population consisting of only passing particles, and passing and trapped particles and neglecting/considering magnetic drift.

The growth rate in the absence of temperature gradient is presented in Fig. 5. The calculated growth rate is plotted as a function of Δ' on the top panel and $|k'_\parallel| \Delta'$ on the bottom panel. The black curves represent the case of no particle trapping, neglecting (solid) and taking into account (dashed) the curvature of the magnetic field. These curves are conveniently labeled in the figure. For the sake of readability, the same labels, colours and symbols are used in the following. As predicted by theory, in the absence of particle trapping,

Figure 6: The growth rate of a collisionless tearing mode for a flat density gradient and a non zero temperature gradient ($R/L_T = 6.9$) as a function of Δ' given by GKW simulations, for several cases of a population consisting of only passing particles, and passing and trapped particles and neglecting/considering magnetic drift.

the two curves with/without magnetic curvature overlap in the case of flat density and temperature profiles. When the mirror term $\mathbf{b} \cdot \nabla B$ inducing magnetic trapping is considered, the collisionless tearing mode is more unstable, which is consistent with Fig. 4. In that case, it is observed that adding the magnetic drift $\mathbf{v}_D \cdot \nabla$ stabilizes the mode. The reason for this effect is analysed in section VIB.

Regarding the dependence on the stability parameter, our analytic theory predicts a linear scaling of the growth rate with Δ' , regardless of the pressure profile. We observe that the trend of the curve with Δ' is linear only for small values of Δ' and departs from the linear behaviour when increasing Δ' . Actually, when modifying Δ' , the position of the resonant surface is also modified and so is the value of $|k'_\parallel|$, which implies that the proportionality coefficient is modified. However, when looking at the dependence of the growth rate on $|k'_\parallel| \Delta'$, a linear behaviour is observed, which is in better agreement with the analytical relation found in section IV.

Note nonetheless, that a nonlinear behaviour of the growth rate was reported in Ref. 36, where the growth rate was found to scale as $\sqrt{\Delta'}$. This behaviour was explained by the short wavelength effects taken into account in the Bessel function $J_0(k_\perp \rho_e)$. The Bessel function was then expanded for large arguments, and a scaling of the growth rate with $\sqrt{\Delta'}$ and $(\rho_e a)^{1/4}$ was found. The latter is a finite Larmor radius effect. However when we eliminate the finite Larmor radius effect in the code, by not computing the gyroaverage, we have not observed any change in the growth rate of the mode. The effect

of variations of the wave vector in the radial direction cannot be verified *per se* in our simulations since a Fourier representation is used for the binormal coordinate (perpendicular to the field) and we consider only one binormal mode. One would need to consider more than a single mode to check this effect, which is left for a future publication.

When temperature gradient is included, the growth rate is amplified with respect to the case $R/L_T = 0$ only when considering the particle trapping, which is in agreement with analytic calculations²⁴. Note that the opposite result can be obtained in different regimes, as was found in δF PIC gyrokinetic simulations in the slab geometry limit³⁷, where the temperature gradient tends to stabilize the tearing mode. In order to make quantitative comparisons between our results and those of Ref. 37 we should perform further simulations in slab geometry, which is beyond the scope of the present paper. If particle trapping is neglected, the impact of the temperature gradient is negligible. This is shown in Fig. 6, where the dependence of the growth rate on the stability parameter Δ' is plotted considering a temperature gradient $R/L_T = 6.9$. It is observed that the effect of the magnetic drift frequency is opposite to that in the absence of temperature gradient, i.e. the magnetic curvature tends to destabilize the collisionless tearing mode. This is in agreement with recent analytic results based on a fluid approach³⁸, which suggests an interchange-like destabilization of the collisionless tearing mode, i.e. a destabilization due to the temperature gradient when the magnetic curvature is taken into account. Nevertheless, further quantitative analysis for comparison between fluid and gyro-kinetic theories should be done, which is beyond the scope of the present paper.

We have seen that, in the presence of particle trapping, for $R/L_T = 0$ (resp. $R/L_T = 6.9$), the magnetic curvature stabilizes (resp. destabilizes) the mode. This implies that there must exist a temperature gradient at which the interchange destabilization³⁸ compensates for the stabilization observed in Fig. 5. To analyse this more in detail we have selected one single Δ' and we have performed a scan on the temperature gradient R/L_T . The result is shown in Fig. 7. It is observed that, when neglecting the particle trapping, the temperature gradient does not have any significant impact on the tearing mode stability. However, when some particles are trapped, the temperature gradient destabilizes the mode. This destabilization is further increased by the magnetic field curvature only beyond an R/L_T -threshold around $R/L_T \approx 2$.

For the sake of completeness, we have run simulations with flat density and temperature profiles, but considering the electrostatic potential by solving the quasi-neutrality equation, coupled to the gyrokinetic Vlasov-Ampère system. We show the results in Fig. 8. We confirm that including trapping effects destabilizes the mode even in the presence of electrostatic potential. We confirm also a stabilizing role for the curvature in the case of zero gradients, even when the destabilizing effect³⁹ of the electrostatic potential is considered.

Figure 7: The growth rate of a collisionless tearing mode as a function of R/L_T for flat density profile and fixed Δ' .

Figure 8: The growth rate of a collisionless tearing mode as a function of Δ' for flat temperature and density profiles, but considering the electrostatic potential ($\phi \neq 0$).

We can see that the role of magnetic field curvature through particle trapping is to stabilize the mode when no gradients are considered, and to destabilize it when temperature gradients are considered, beyond an R/L_T -threshold. This threshold occurs when a stabilizing effect due to trapped particles is compensated by the interchange destabilization. To get a better understanding of the underlying physics, a quantitative analysis of the energy exchange between the particles and

the mode has been made by using a diagnostic that computes the variation the kinetic energy of particles in the code.

VI. ENERGY EXCHANGE DIAGNOSTIC FOR WAVE-PARTICLE INTERACTION

The following diagnostic is used to identify in velocity space the various contributions of passing and trapped particles to the linear growth rate of the most unstable mode. Note that in our case we focus on the interaction between the tearing mode and electrons. The diagnostic is based on the energy conservation property of the Vlasov-Poisson system of equations which allows to directly link the mode growth rate to the work done by the perturbed electric field on the particles. Strictly speaking, this method is only applicable if the set of equations solved and their numerical implementation conserves energy.

A. Conservation of energy

The time variation of the space-integrated potential energy of an electromagnetic wave is given by

$$\begin{aligned} \frac{d\mathcal{E}_p}{dt} &= \frac{d}{dt} \int d^3\mathbf{x} \left(\frac{\epsilon_0}{2} \mathbf{E} \cdot \mathbf{E} + \frac{\epsilon_0 c^2}{2} \mathbf{B} \cdot \mathbf{B} \right) \\ &= \int d^3\mathbf{x} \left(\epsilon_0 \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} + \epsilon c^2 \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) \end{aligned} \quad (43)$$

Using Faraday's induction law $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ and Ampère's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial_t \mathbf{E}$, where $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{J}$ are the electric field, the magnetic field, and the current density, respectively, Eq. (43) becomes

$$\frac{d\mathcal{E}_p}{dt} = - \int d^3\mathbf{x} \left[\mathbf{J} \cdot \mathbf{E} + \frac{1}{\mu_0} \nabla \cdot (\mathbf{E} \times \mathbf{B}) \right] \quad (44)$$

Owing to the conservation of the total energy of the system, we have

$$\frac{d\mathcal{E}_k}{dt} = - \frac{d\mathcal{E}_p}{dt} \quad (45)$$

where $\mathcal{E}_k$ is the kinetic energy of particles. This gives the time evolution of the kinetic energy

$$\begin{aligned} \frac{d\mathcal{E}_k}{dt} &= - \sum_s \int d^3\mathbf{x} \int d^3\mathbf{v} Z_s e F_s \mathbf{v} \cdot \left(\nabla \tilde{\phi} + \frac{\partial \tilde{A}_{\parallel}}{\partial t} \mathbf{b} \right) \\ &\quad + \int d^3\mathbf{x} \frac{1}{\mu_0} \nabla \cdot (\mathbf{E} \times \mathbf{B}) \end{aligned} \quad (46)$$

The sign of the time evolution of the kinetic energy indicates the contribution to the growth rate of the mode. If $\dot{\mathcal{E}}_k > 0$, the energy is transferred from the mode to the particles and therefore the mode is damped. In the opposite case,

the mode gains energy and is therefore unstable. It is important to note that in the case of global modes such as tearing modes, a positive variation of the particles kinetic energy at a local point does not necessarily suggest destabilization of the whole mode. Some transport of energy should be taken into account through advection, especially when the electrostatic potential is considered and the $\nabla \cdot (\mathbf{E} \times \mathbf{B})$ term is non zero. In our simulations, we check that even when the electrostatic potential is taken into account that term is negligible compared to the $\mathbf{J} \cdot \mathbf{E}$ term. We have implemented in the code the first term on the right hand-side of Eq. (46) as a function of $(r, v_{\parallel}, v_{\perp})$. The last term on the right hand-side is calculated in a separate diagnostic that computes all the radial flows. The implemented diagnostic reads

$$\dot{\mathcal{E}}_k = -e Z_s \int d\ell F(v_{\parallel} \mathbf{b} + \mathbf{v}_E + \mathbf{v}_D) \cdot (\nabla J_0 \cdot \phi + \partial_t J_0 \cdot A_{\parallel} \mathbf{b}) \quad (47)$$

with ℓ the parallel coordinate.

B. Response of the particles to the tearing mode

We present the results from the described diagnostic, which allows us to characterize the particle response to the collisionless tearing instability in the purely magnetic case, i.e. the electrostatic potential is set to zero and therefore the $\mathbf{E} \times \mathbf{B}$ flux in Eq. (46) vanishes. The local interaction between the waves and particles in the velocity space can thus be described by Eq. (47) with the $A_{\parallel}$ term only.

Fig. 9 illustrates the local exchange of energy between particles and the mode in the velocity space for $R/L_n = R/L_T = 0$. Positive (resp. negative) values imply a transfer of energy from the mode to the particles (resp. from the particles to the mode). The plots are shown for a local point around the resonant position $r = r_s$ where the exchange of energy is the most localized. The dashed line in the middle panel denotes the trapping cone $v_{\parallel} = \sqrt{2} r_s / R_0 v_{\perp}$. As discussed earlier, this relation is valid only at the tokamak midplane where the magnetic field intensity has a maximum value. Therefore, one needs to be aware of the position on the magnetic field when setting the trapping boundary. In our simulations, the amplitude of the mode is localized at $l = 0$, which makes this relation valid. The three different panels represent the three different curves that we have shown in Fig. 5 (actually we have four curves, but when there is no particle trapping the two black curves are the same). It is observed that when trapping is considered without magnetic curvature the effect is the suppression of the particle response inside the trapping cone, as predicted by our analytic theory. Numerical integration in velocity space results in a reduction of 13% of the integrated values of $\dot{\mathcal{E}}_k$ in figure 9b with respect to figure 9a. Although quantitative analysis to compare with the modification of the growth rate would require also integration over the radial position, the result we obtain is qualitatively

Figure 9: Time derivative of the kinetic energy at the resonant position in the presence of a collisionless tearing mode without density/temperature gradients, i.e. $R/L_n = R/L_T = 0$.

consistent with a reduced local transfer of energy from the mode to the particles. Let us remind that the tearing mode gets the energy from the outer region (represented by Δ') and transfers part of this energy to particles in the resonant layer. Therefore, the fact that the tearing mode transfers less energy to the particles in the resonant layer implies that there is more energy available for the mode, which results in an increased growth rate. However, when including magnetic curvature and trapping is allowed, trapped particles respond to the presence of the tearing mode, allowing this way the transfer of energy from the tearing mode to the trapped particles, which results in a decrease of the growth rate. This is evidenced by the bottom panel, where the red curve represents the positions in velocity space at the resonant surface where the precession frequency of trapped particles equals the tearing frequency, i.e. $\omega = n_3 \Omega_3(v_{\parallel}, v_{\perp})$, with $n_3 = 1$. The frequencies of motion are calculated in detail in Appendix C. In particular, we use the expression C30 to plot the red curve. It is observed that the maximum of the response is located on the red curve and corresponds to barely trapped particles. Consequently, one can assess that adding particle trapping in the presence of magnetic curvature can modify the stability of the mode through the resonance between the tearing mode and barely trapped particles.

VII. CONCLUSIONS

In this paper we have studied the stability of a collisionless tearing mode in a curved and inhomogeneous magnetic field. The tearing mode equation is derived in the framework of gyro-kinetic theory with a Hamiltonian approach. The dispersion equation was obtained for the tearing instability, describing both the outer (ideal or non-resonant) and inner (non-ideal or resonant) regions. Using the gyro-kinetic theory in the analysis of the tearing mode allows us to look into the kinetic aspects of the instability and therefore analyzing possible resonances between particles and the mode in the inner region. In the outer region, the fluid limit is used to avoid the resonances. A shooting method has been employed to integrate the equation of the tearing mode for a general safety factor profile and a diagram of the stability parameter Δ' has been obtained.

We have solved analytically the equation of the tearing mode in the inner region, where the kinetic effects are kept and we have recovered the result that the growth rate scales linearly with $|k'_{\parallel}| \Delta'$, where $k'_{\parallel}$ is the radial derivative of the parallel wave-vector evaluated at the resonant surface. We have also found that trapping effects in the absence of magnetic curvature lead to an increase of the growth rate by a factor $1 + 2\epsilon$, where $\epsilon = r/R_0$. This is due to the fact that the mode transfer its energy mainly to passing electrons in the resonant layer. Therefore, introducing trapped electrons increases the energy available for the mode to grow. Moreover, we have predicted that the magnetic field curvature does not

play any role in the stability of the tearing mode when only passing electrons are considered.

Further analysis has been carried out numerically, by means of linear global gyro-kinetic simulations using GKW code. We have verified our analytic results, namely the linear scaling of the growth rate with $|k'_{\parallel}|\Delta'$ and the destabilization due to trapped electrons. We have subsequently performed a parametric study of the stability of the tearing mode and we have obtained two main results. First, in the presence of trapped particles, the temperature gradient plays a destabilizing role. Second, we have reported that the magnetic field curvature tends to further destabilize the mode in combination with the temperature gradient, suggesting a destabilization through an interchange mechanism³⁸. We have shown that there exists a threshold in temperature gradient below which the magnetic curvature is stabilizing. We have identified a response of trapped particles to the tearing mode which results in a kinetic damping of the mode. The threshold in temperature gradient appears when this kinetic damping is balanced by the interchange destabilization due to the combination of the temperature gradient and the magnetic field curvature. These findings open the way to a deeper understanding of the tearing instability in tokamaks in the presence of kinetic effects and shed light on the dependence of its growth rate on a set of tokamak control parameters.

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Appendix A: Exact linear solution of the gyro-kinetic equation

The starting point to obtain the linear response of electrons is the Vlasov equation in conservative form using a Hamilto-

nian formalism

$$\frac{\partial F}{\partial t} - [\mathcal{H}, F] = 0 \quad (\text{A1})$$

Here $[X, Y]$ represents the Poisson brackets between X and Y , i.e. $[X, Y] = \partial_{\mathbf{x}} X \partial_{\mathbf{p}} Y - \partial_{\mathbf{p}} X \partial_{\mathbf{x}} Y$, where $\mathbf{x}$ and $\mathbf{p}$ are the position and momenta, forming a set of canonical variables satisfying Hamilton's equations.

The calculation is performed in the so-called δF -approximation. For this purpose we decompose the distribution function and the Hamiltonian into equilibrium and perturbed parts, respectively,

$$F = F_{\text{eq}} + \delta F \quad (\text{A2})$$

$$\mathcal{H} = \mathcal{H}_{\text{eq}} + \delta \mathcal{H} \quad (\text{A3})$$

and assume the gyrokinetic ordering $|\delta F|/F_{\text{eq}} \approx \rho_*$ and $|\delta \mathcal{H}|/|\mathcal{H}_{\text{eq}}| \approx \rho_*$. Vlasov equation (A1) for electrons is then linearized and written for δF as

$$\frac{\partial \delta F}{\partial t} - [\mathcal{H}_{\text{eq}}, \delta F] - [\delta \mathcal{H}, F_{\text{eq}}] = 0 \quad (\text{A4})$$

The equilibrium Hamiltonian is given by

$$\mathcal{H}_{\text{eq}} = \frac{|\mathbf{p} - eZ_e \mathbf{A}_{\text{eq}}|^2}{2m_e} + eZ_e \phi_{\text{eq}} \quad (\text{A5})$$

where $\mathbf{A}_{\text{eq}}$ and ϕ_{eq} are respectively the equilibrium vector and electrostatic potentials, $\mathbf{p} = m_e \mathbf{v} + eZ_e \mathbf{A}$ is the electron momentum and Z_e the atomic number of the species, which is $Z_e = -1$ in this particular case where only electrons are considered. The perturbed Hamiltonian is written in terms of the perturbed vector and electrostatic potentials, $A_{\parallel}$ and ϕ , respectively, as

$$\delta \mathcal{H} = eZ_e (\phi - v_{\parallel, \text{eq}} A_{\parallel}) \quad (\text{A6})$$

where $v_{\parallel, \text{eq}}$ is the projection of the electron velocity onto the unperturbed magnetic field lines. This formulation can lead to simple insightful equations of motion when expressed in canonical variables $(\mathbf{x}, \mathbf{p})$. However, the periodicity of particle motion in a tokamak suggests the use of a different set of canonically conjugated variables, called action-angle variables and represented by $\mathbf{J} = (J_1, J_2, J_3)$ and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$, respectively. These angles describe the gyro-motion (α_1), the motion in the poloidal direction (α_2) and the motion in the toroidal direction (α_3). The equations of motion using these coordinates can be written as

$$\dot{\boldsymbol{\alpha}} = \frac{\partial \mathcal{H}_{\text{eq}}}{\partial \mathbf{J}} = \boldsymbol{\Omega}_{\text{eq}}(\mathbf{J}) \quad \dot{\mathbf{J}} = -\frac{\partial \mathcal{H}_{\text{eq}}}{\partial \boldsymbol{\alpha}} \quad (\text{A7})$$

where the angles are supposed to vary linearly in time giving the characteristic frequencies of motion at equilibrium $\boldsymbol{\Omega} =$

$(\Omega_1, \Omega_2, \Omega_3)$. For completeness, these frequencies are derived in detail in Appendix C.

We define an equilibrium quantity X_{eq} as the average of the quantity X over the angles α , i.e.

$$X_{\text{eq}} \equiv \int d^3\alpha X \quad (\text{A8})$$

which means that the only dependence of X_{eq} is on the actions $\mathbf{J}$, i.e. $X_{\text{eq}} = X_{\text{eq}}(\mathbf{J})$. The equilibrium distribution function and the equilibrium Hamiltonian are therefore functions of the motion invariants and the perturbed distribution function and Hamiltonian are functions of $(\alpha, \mathbf{J}, t)$. Following this definition, the equations of motion A7 introduce the actions as motion invariants, i.e. $\dot{\mathbf{J}} = \mathbf{0}$. Therefore, without any loss of generality, any equilibrium quantity can be defined as a function of the motion invariants associated to the three directions of periodicity.

The periodicity of the perturbed quantities with respect to the angles α allows us to perform a Fourier expansion

$$\left\{ \begin{array}{c} \delta\mathcal{H} \\ \delta F \end{array} \right\}(\alpha, \mathbf{J}, t) = \sum_{\mathbf{n}\omega} \left\{ \begin{array}{c} \delta\mathcal{H}_{\mathbf{n}\omega} \\ \delta F_{\mathbf{n}\omega} \end{array} \right\}(\mathbf{J}) \exp[i(\mathbf{n} \cdot \alpha - \omega t)] \quad (\text{A9})$$

where $\mathbf{n} = (n_1, n_2, n_3)$ are the wave numbers associated to the angles $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and ω is the frequency of the tearing mode. Plugging Eq. (A9) into the linearized Vlasov equation (A4) and using the equations of motion (A7), an exact linear solution can be found

$$\delta F_{\mathbf{n},\omega} = -\frac{\mathbf{n} \cdot \partial_{\mathbf{J}} F_{\text{eq}}}{\omega - \mathbf{n} \cdot \Omega} \delta \mathcal{H}_{\mathbf{n},\omega} \quad (\text{A10})$$

In the adiabatic limit, the first motion invariant is proportional to the magnetic moment $\mu = m_e v_{\perp} / (2B)$, i.e. $J_1 = m_e \mu / (eZ_e)$. The second action, J_2 , is written in terms of the toroidal flux of the magnetic field Φ as $J_2 = eZ_e \Phi + \frac{1}{2\pi} \oint m_e v_{\parallel} d\ell$, where ℓ is the coordinate along the magnetic field line and $v_{\parallel}$ is the component of the guiding-centre velocity parallel to the magnetic field. Finally, the third action J_3 is the toroidal canonical momentum $J_3 \equiv P_{\varphi} = -eZ_e \psi + m_e R v_{\varphi}$, with ψ the poloidal flux of the magnetic field, R is the major radius and v_{φ} is the projection of the electron velocity onto the toroidal direction. For highly passing electrons, the second action is closer to a radial function than J_3 is. Therefore, performing the change of variable $\mathbf{J} \rightarrow \mathbf{I}$, where $\mathbf{I} = (J_1, J_2, \mathcal{H}_{\text{eq}})$ makes it possible to introduce radial gradients of the equilibrium. This change of variable introduces the equilibrium Hamiltonian, i.e. the energy in the absence of any perturbation, which is more convenient for physical analysis. The numerator of Eq. (A10) can therefore be written as follows

$$\mathbf{n} \cdot \partial_{\mathbf{J}} F_{\text{eq}} = \mathbf{n} \cdot \frac{\partial \mathbf{I}}{\partial \mathbf{J}} \cdot \frac{\partial F_{\text{eq}}}{\partial \mathbf{I}} = n_1 \frac{\partial F_{\text{eq}}}{\partial J_1} + \mathbf{n} \cdot \Omega \frac{\partial F_{\text{eq}}}{\partial \mathcal{H}_{\text{eq}}} + n_2 \frac{\partial F_{\text{eq}}}{\partial J_2} \quad (\text{A11})$$

We can now consider that $\Omega_1 \gg \omega, \Omega_2, \Omega_3$. This ordering yields the exact linear response to be used in the following

$$\delta F_{\mathbf{n},\omega} = \frac{\partial F_{\text{eq}}}{\partial \mathcal{H}_{\text{eq}}} \delta \mathcal{H}_{\mathbf{n},\omega} + \frac{1}{B} \frac{\partial F_{\text{eq}}}{\partial \mu} \delta \mathcal{H}_{\mathbf{n}',\omega} - \frac{\omega \partial_{\mathcal{H}_{\text{eq}}} F_{\text{eq}} + n_2 \partial_{J_2} F_{\text{eq}}}{\omega - \mathbf{n}^* \cdot \Omega} \delta \mathcal{H}_{\mathbf{n}^*,\omega} \quad (\text{A12})$$

where $\mathbf{n}' = (n_1 \neq 0, n_2, n_3)$ and $\mathbf{n}^* = (n_1 = 0, n_2, n_3)$.

Appendix B: Perturbed parallel current

The perturbed parallel current is calculated as

$$j_{\parallel} = eZ_e \int d^3\mathbf{p} (v_{\parallel,\text{eq}} \delta F + \delta v_{\parallel} F_{\text{eq}}) \quad (\text{B1})$$

where the perturbed parallel velocity is expressed in terms of the parallel vector potential

$$\delta v_{\parallel} = -\frac{eZ_e}{m_e} A_{\parallel} \quad (\text{B2})$$

Therefore, the perturbed parallel current reads

$$j_{\parallel} = eZ_e \int d^3\mathbf{p} \left(v_{\parallel,\text{eq}} \delta F - \frac{eZ_e}{m_e} A_{\parallel} F_{\text{eq}} \right) \quad (\text{B3})$$

Passing from $(\mathbf{x}, \mathbf{p})$ to action-angle variables $(\alpha, \mathbf{J})$ at this point requires to pass from a three-dimensional to a six-dimensional space. For this purpose we use the δ -Dirac function such that $j_{\parallel}$ for electrons is calculated as a 6D integration.

$$j_{\parallel}(\mathbf{x}, t) = eZ_e \int d^3\mathbf{p} d^3\mathbf{x}' \delta(\mathbf{x}' - \mathbf{x}) \left(v_{\parallel,\text{eq}} \delta F - \frac{eZ_e}{m_e} A_{\parallel} F_{\text{eq}} \right) \quad (\text{B4})$$

where $\mathbf{x}'$ is a spatial position for the integration. We can now make the canonical change of variable $(\mathbf{x}, \mathbf{p}) \rightarrow (\alpha, \mathbf{J})$ which

gives

$$j_{\parallel}(\mathbf{x}, t) = eZ_e \int d\tau^* d\alpha_1 \delta(\mathbf{x}' - \mathbf{x}) \left(v_{\parallel, \text{eq}} \delta F - \frac{eZ_e}{m_e} A_{\parallel} F_{\text{eq}} \right) \quad (\text{B5})$$

where we have separated the integration variables into gyrophase $d\alpha_1 \equiv d\varphi_c$ and $d\tau^* = d\alpha_2 d\alpha_3 dJ_1 dJ_2 dJ_3$. We make use of Parseval's identity for Fourier series,

$$\int \frac{d\alpha_1}{2\pi} \delta(\mathbf{x}' - \mathbf{x}) \delta F = \sum_{n_1} [\delta(\mathbf{x}' - \mathbf{x})]_{n_1} \delta F_{n_1}^{\dagger} \quad (\text{B6})$$

where the $\dagger$ designates the complex conjugate of the perturbed quantity. The n_1 mode of the Dirac function can be

$$j_{\parallel}(\mathbf{x}, t) = eZ_e \int d\tau^* \sum_{n_1} (-i)^{n_1} \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot (\mathbf{x}_G - \mathbf{x})} J_{n_1}(k_{\perp} \rho_c) \left(v_{\parallel, \text{eq}} \delta F_{n_1}^{\dagger} - \frac{eZ_e}{m_e} A_{\parallel n_1}^{\dagger} F_{\text{eq}} \right) \quad (\text{B8})$$

Here the perturbed quantities are functions of $(\mathbf{x}_G, \mathcal{H}_{\text{eq}}, P_{\varphi})$. Note that through this Hamiltonian formalism one can obtain the parallel current associated to the particle motion, in terms of particle coordinates $\mathbf{x}$, using physical quantities describing the guiding-center motion, which depend on guiding-center coordinates $\mathbf{x}_G$. Similarly, we calculate the n_1 -mode of each component of δF and we plug it into the expression of the parallel current in Eq. (B8). Anticipating the fact that we will later use the deeply passing particle approximations, we pass to an integration in guiding center coordinates $d\tau^* = d\alpha_2 d\alpha_3 dJ_1 dJ_2 dJ_3 \rightarrow \mathcal{J} d\mathbf{x}_G dv_{\parallel, \text{eq}} dv_{\perp}$, where $\mathcal{J}$ is the Jacobian of the transformation. We now make use again of the ordering $\Omega_1 \gg \omega, \Omega_2, \Omega_3$, which implies that the third term in the right-hand side of Eq. (4) is negligible except for $n_1 = 0$. Therefore, we keep only the $n_1 = 0$ terms, which strictly speaking is the exactly the same as performing a gyro-average. Using the Bessel function property $\sum_{n=-\infty}^{+\infty} J_n^2(x) = 1$ ³⁴ and the inverse Fourier transform

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \delta \mathcal{H}^{\dagger}(\mathbf{k}, t) e^{-i\mathbf{k} \cdot \mathbf{x}} = \delta \mathcal{H}(\mathbf{x}, t) \quad (\text{B9})$$

we obtain the following general expression for the parallel current in the gyro-kinetic approach using the magnetic limit, i.e. neglecting the electrostatic potential

$$\begin{aligned} j_{\parallel}(\mathbf{x}, t) = & -e^2 \int \mathcal{J} dv_{\parallel, \text{eq}} d\mu \left\{ v_{\parallel, \text{eq}}^2 \partial_{\mathcal{H}_{\text{eq}}} F_{\text{eq}} A_{\parallel}(\mathbf{x}, t) \right. \\ & + v_{\parallel, \text{eq}}^2 \frac{1}{B} \frac{\partial F_{\text{eq}}}{\partial \mu} (1 - J_0^2) A_{\parallel}(\mathbf{x}, t) \\ & + \sum_{n_2, n_3} v_{\parallel, \text{eq}}^2 \frac{\omega \partial_{\mathcal{H}_{\text{eq}}} F_{\text{eq}} + n_2 \partial_{J_2} F_{\text{eq}}}{\omega - n_2 \Omega_2 - n_3 \Omega_3} J_0^2 A_{\parallel n_2, n_3, \omega}(\mathbf{x}) \\ & \times e^{i(n_2 \alpha_2 + n_3 \alpha_3 - \omega t)} \\ & \left. + \frac{1}{m_e} F_{\text{eq}} A_{\parallel}(\mathbf{x}, t) \right\} \quad (\text{B10}) \end{aligned}$$

calculated using the definition of the Dirac distribution

$$\begin{aligned} [\delta(\mathbf{x}' - \mathbf{x})]_{n_1} &= \int \frac{d\alpha_1}{2\pi} \delta(\mathbf{x}' - \mathbf{x}) e^{-in_1 \alpha_1} \\ &= \int \frac{d\alpha_1}{2\pi} \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}' - i\mathbf{k} \cdot \mathbf{x}} e^{-in_1 \alpha_1} \quad (\text{B7}) \end{aligned}$$

Therefore, the parallel current density in Eq. (B5) can be expressed using the Bessel function of the first kind, defined as $\int d\alpha_1/(2\pi) e^{i(k_{\perp} \rho_c \cos \alpha_1 - n_1 \alpha_1)} = (-i)^{n_1} J_{n_1}(k_{\perp} \rho_c)$

where $J_0 \equiv J_0(k_{\perp} \rho_c)$, J_0^2 represents the gyro-average operator applied twice and the squared electron charge $Z_e^2 = 1$ has been removed for the sake of clarity. We choose the equilibrium distribution function

$$F_{\text{eq}} = \frac{n_{\text{eq}}}{(2\pi T_{\text{eq}}/m_e)^{3/2}} e^{-\frac{\mathcal{H}_{\text{eq}} - \mathcal{U}_{\text{eq}}}{T_{\text{eq}}}} \quad (\text{B11})$$

where n_{eq} and T_{eq} are the equilibrium density and temperature, respectively, and $\mathcal{U}_{\text{eq}}$ is a function which depends on the motion invariants. For our purpose, this function will be odd in $v_{\parallel}$, in order to generate an equilibrium parallel current. After integration, the first and last terms of the right-hand side in expression (B10) cancel out. For electrons, one can take the small orbit limit and approximate the Bessel function as $J_0^2(k_{\perp} \rho_c) \approx 1$ for $k_{\perp} \rho_c \ll 1$. Therefore, the second term with the $\partial F_{\text{eq}}/\partial \mu$ vanishes. Finally, $j_{\parallel}(\mathbf{x}, t)$ is written as

$$j_{\parallel}(\mathbf{x}, t) = \frac{e^2}{T_{\text{eq}}} \sum_{n_2, n_3} \left\langle v_{\parallel, \text{eq}}^2 \frac{\omega - n_2 T_{\text{eq}} \partial_{J_2} \log F_{\text{eq}}}{\omega - n_2 \Omega_2 - n_3 \Omega_3} \right\rangle A_{\parallel n_2, n_3, \omega}(\mathbf{x}) e^{i(n_2 \alpha_2 + n_3 \alpha_3 - \omega t)} \quad (\text{B12})$$

where the notation $\langle \dots \rangle$ has been used for simplicity to represent an average over gyro-centre equilibrium velocity space weighted by the equilibrium distribution function

$$\langle \dots \rangle = \int \mathcal{J} dv_{\parallel, \text{eq}} d\mu \dots F_{\text{eq}} \quad (\text{B13})$$

If we restrict our analysis to deeply passing electrons, we can write $\alpha_2 = \theta$ and $\alpha_3 = \varphi$, where θ and φ are the poloidal and toroidal angles, respectively. Consequently $n_2 = m$ and $n_3 = n$, with m and n the poloidal and toroidal mode numbers, respectively. The frequencies Ω_2 and Ω_3 are calculated in detail in Appendix C. For deeply passing particles, these frequencies can be approximated as

$$\Omega_2 \approx \frac{v_{\parallel, \text{eq}}}{q R_0} \quad (\text{B14})$$

$$\Omega_3 \approx \frac{2q}{r} \frac{m_e v_{\parallel, \text{eq}}^2 + \mu B_0}{e Z_e B_0 R_0} + \frac{v_{\parallel, \text{eq}}}{R_0} \quad (\text{B15})$$

$$j_{\parallel}(\mathbf{x}, t) = \rho_{\star} e n_0 v_{\text{th}} \frac{2}{\hat{T}_{\text{eq}}} \sum_{m, n, \omega} \left\langle \hat{v}_{\parallel}^2 \frac{\hat{\omega} - \hat{\omega}_{\star g}}{\hat{\omega} - \hat{k}_{\parallel} \hat{v}_{\parallel} - \hat{\omega}_D} \right\rangle \delta \hat{A}_{\parallel m, n, \omega}(\mathbf{x}) e^{i(m\theta + n\varphi - \omega t)} \quad (\text{B16})$$

with $\hat{\cdot}$ representing normalized quantities. The parallel wave vector $k_{\parallel}$ is given by $k_{\parallel} = (m/q + n)/R_0$ and the magnetic drift frequency ω_D is given by $\omega_D = 2qn/r (m_e v_{\parallel, \text{eq}}^2 + \mu B_0) / (e Z_e B_0 R_0)$. Notice that $k_{\parallel}$ vanishes on the rational surface defined as $q = -m/n$. In the remainder of this paper, the eq subscript for the parallel velocity will be dropped for the sake of simplicity. Note that $\rho_{\star}$ gives the typical ordering between the equilibrium and the perturbed distribution function and $e n_0 v_{\text{th}}$ is the normalization for the current. In this expression we have introduced the generalized diamagnetic frequency $\omega_{\star g} = m T_{\text{eq}} \partial_{J_2} \log F_{\text{eq}}$, which includes all the spatial dependence of the equilibrium. For thermal passing particles, at lowest order in $\rho_{\star}$, one can write $J_2 \approx e Z_e \Phi$. Therefore, the derivative with respect to J_2 can be reduced to a derivative with respect to the radial position.

Appendix C: Equilibrium frequencies of motion

The motion of a charged particle in toroidal geometry can be divided into a parallel and a drift motion (at lower order),

where q is the safety factor, which represents the helicity of the magnetic field lines, and r is the minor radius. B_0 and R_0 are respectively, the modulus of the magnetic field and the major radius, measured at the magnetic axis. The perturbed parallel current will be written in normalized units. For this purpose we normalize the velocities to the thermal velocity of electrons v_{th} , the distances to R_0 , the frequencies to the transit frequency $\omega_t = v_{\text{th}}/R_0$, the equilibrium distribution function to n_0/v_{th}^3 , with n_0 some normalizing density, the parallel vector potential to $B_0 R_0 \rho_{\star}^2$ and the temperature to a normalizing temperature defined as $T_0 = m_e v_{\text{th}}^2/2$. The parallel current can then be rewritten as

$\mathbf{v} = v_{\parallel} \mathbf{b} + \mathbf{v}_g$, in which we omit the $\mathbf{E} \times \mathbf{B}$ drift. Note that this term can be added, however one needs to be careful that it is independent of the toroidal motion so that the toroidal momentum can still be a motion invariant. The time evolution of the motion can be given by,

$$\dot{\psi} = \mathbf{v} \cdot \nabla \psi \quad (\text{C1})$$

$$\dot{\theta} = v_{\parallel} \mathbf{b} \cdot \nabla \theta + \mathbf{v}_g \cdot \nabla \theta \quad (\text{C2})$$

$$\dot{\varphi} = v_{\parallel} q \mathbf{b} \cdot \nabla \theta + \mathbf{v}_g \cdot \nabla \varphi \quad (\text{C3})$$

where ψ is the poloidal flux, θ and φ are the poloidal and toroidal coordinates, respectively. The magnetic drift velocity $\mathbf{v}_g$ is given by,

$$\mathbf{v}_g = \frac{m v_{\parallel}^2 + \mu B_0}{e B_0} \left(\mathbf{b}_0 \times \frac{\nabla B_0}{B_0} \right) \quad (\text{C4})$$

The energy is defined as

$$E = \frac{1}{2} m v_{\parallel}^2 + \mu B(r, \theta) \quad (\text{C5})$$

so that equation (C2) is expressed at lowest order in $\rho_{\star}$, the Larmor radius normalized to the tokamak major radius, as

follows

$$\dot{\theta} = \epsilon_{\parallel} \sqrt{2/m} [E - \mu B_0(\theta)]^{1/2} \frac{\mathbf{B}_0 \cdot \nabla \theta}{B_0} \quad (\text{C6})$$

$\epsilon_{\parallel}$ designates the sign of the parallel velocity. We define the helical angle as $\zeta = \varphi - q\theta$. The time derivative of ζ gives,

$$\dot{\zeta} = \dot{\varphi} - q(\bar{\psi})\dot{\theta} \quad (\text{C7})$$

where $\bar{\psi}$ is the flux surface at the resonance layer. Using eq. (C2) and the fact that $q(\psi) - q(\bar{\psi}) = (dq/dr)\delta\psi$, we get,

$$\dot{\zeta} = \frac{dq}{d\psi} \delta\psi \dot{\theta} + \mathbf{v}_g \cdot \nabla \zeta \quad (\text{C8})$$

It follows that the three angular motions are found to be the gyromotion, the poloidal motion and the toroidal motion (also called, respectively, the bounce and precessional motions, for trapped particles). Their respective frequencies are written as,

$$\Omega_1 = \Omega_b \oint \frac{d\theta}{2\pi} \frac{1}{\dot{\theta}} \dot{\gamma} \approx \Omega_b \oint \frac{d\theta}{2\pi} \frac{1}{\dot{\theta}} \frac{eB_0}{m} \quad (\text{C9})$$

$$\Omega_2 = \Omega_b \approx 2\pi \left(\oint \frac{d\theta}{\dot{\theta}} \right)^{-1} \quad (\text{C10})$$

$$\Omega_3 = \Omega_b \oint \frac{d\theta}{2\pi} \frac{1}{\dot{\theta}} \dot{\varphi} \approx \Omega_b \oint \frac{d\theta}{2\pi} \frac{1}{\dot{\theta}} \mathbf{v}_g \cdot [-q'(\bar{\psi})\theta \nabla \psi + \nabla((\varphi - q(\bar{\psi})\theta))] + q(\bar{\psi})\Omega_b \oint \frac{d\theta}{2\pi} \quad (\text{C11})$$

Note that for passing particles, $\oint = \int_{-\pi}^{+\pi}$ whereas for trapped particles oscillating on $[-\theta_0, \theta_0]$, $\oint = \int_{-\theta_0}^{+\theta_0}$.

1. The poloidal frequency Ω_2

At the lowest order, the parallel velocity dominates the drift velocity, and the bounce frequency can be rewritten as,

$$\Omega_2^{-1} = \oint \frac{d\theta}{2\pi} \frac{1}{\mathbf{b} \cdot \nabla \theta} \frac{1}{v_{\parallel}} \quad (\text{C12})$$

We consider a simple circular equilibrium, with a large aspect ratio where the equilibrium magnetic field is given by

$$\mathbf{B}_0 = \nabla \psi \times \nabla \zeta \quad (\text{C13})$$

In this geometry, the magnitude of the equilibrium magnetic field can be expressed to the first order as $B_{(0)} = B_0(1 - \epsilon \cos \theta)$. Since $\mathbf{b} \cdot \nabla \theta \approx 1/qR$, we can write

$$\Omega_2^{-1} = \oint \frac{d\theta}{2\pi} \frac{qR}{v_{\parallel}} \quad (\text{C14})$$

Taking into account that the major radius is expressed as

$$R = R_0(1 + \epsilon \cos \theta) \quad (\text{C15})$$

and that the parallel velocity can be approximated in terms of the energy and radial position to the first order in ϵ as

$$v_{\parallel} = \epsilon_{\parallel} \sqrt{2(E - \mu B_{(0)})/m} = \epsilon_{\parallel} \sqrt{\frac{2E}{m}} \sqrt{1 - \frac{\mu B_{(0)}}{E}} \quad (\text{C16})$$

one can write Eq. (C14) as

$$\Omega_2^{-1} = \epsilon_{\parallel} q R_0 \sqrt{\frac{m}{2E}} \bar{\Omega}_b^{-1} \quad (\text{C17})$$

where $\epsilon_{\parallel} = \text{sign}(v_{\parallel})$ and $\bar{\Omega}_b^{-1}$ is the integral given by

$$\bar{\Omega}_b^{-1} = \oint \frac{d\theta}{2\pi} \frac{1}{\sqrt{1 - \lambda(1 - \epsilon \cos \theta)}} \quad (\text{C18})$$

with $\lambda = \mu B_0/E$. Keeping the first order effects in ϵ , we define a parameter κ which differentiates the magnetically trapping and passing particles

$$\kappa^2 = \frac{2\epsilon\lambda}{1 - (1 - \epsilon)\lambda} \quad (\text{C19})$$

and we rewrite Eq. (C18) as

$$\bar{\Omega}_b^{-1} = \frac{\kappa}{\sqrt{2\epsilon\lambda}} \oint \frac{d\theta}{2\pi} \frac{1}{\sqrt{1 - \kappa^2 \sin^2(\theta/2)}} \quad (\text{C20})$$

The integral (C20) is to be solved separately for passing and trapped particles by integrating over the appropriate boundaries

$$\text{for passing particles}(\kappa < 1) : \theta_1 = -\pi, \theta_2 = +\pi \quad (\text{C21})$$

$$\text{for trapped particles}(\kappa > 1) : \theta_1 = -\theta_0, \theta_2 = +\theta_0 \quad (\text{C22})$$

Consequently, an appropriate change of variables is made for treating each class of particles. For passing particles, we make the change of variable $u = \theta/2$, and for trapped particles $\sin u = \kappa \sin(\theta/2)$ such that $0 \leq u \leq \pi/2$ and $0 \leq \theta \leq \theta_0$. The integral (C20) is then expressed in terms of the elliptic integral of the first kind $\mathbb{K}(m)$ as defined in Ref. 34,

$$\mathbb{K}(m) = \int_0^{\pi/2} d\theta \frac{1}{\sqrt{1 - m \sin^2 \theta}} \quad (\text{C23})$$

It finally comes that

$$\bar{\Omega}_b^{-1} = \frac{\kappa}{\sqrt{2\epsilon\lambda}} \frac{2}{\pi} \begin{cases} \mathbb{K}(\kappa^2) & \text{for passing particles} \\ \frac{1}{\kappa} \mathbb{K}(1/\kappa^2) & \text{for trapped particles} \end{cases} \quad (\text{C24})$$

Finally, the poloidal frequency Ω_2 is calculated for passing and trapped particles, locally, in circular geometry to the leading order in ϵ and it is expressed as

$$\Omega_2 = \epsilon_{\parallel} \frac{q}{R_0} \sqrt{\frac{2E}{m}} \frac{\sqrt{2\epsilon\lambda}}{\kappa} \frac{\pi}{2} \begin{cases} \mathbb{K}^{-1}(\kappa^2) & , \kappa < 1 \\ \kappa \mathbb{K}^{-1}(1/\kappa^2) & , \kappa > 1 \end{cases} \quad (\text{C25})$$

2. The toroidal frequency Ω_3

The precessional frequency is given to the first order in ϵ by

$$\Omega_3 = \bar{\Omega}_b \oint \frac{d\theta}{2\pi} \frac{\mu B_{(0)} + mv_{\parallel}^2}{\sqrt{1 - \lambda(1 - \epsilon \cos \theta)}} \frac{\mathbf{B}_{(0)} \times \nabla B_{(0)}}{eB_{(0)}^3} \cdot \left[-\frac{dq}{d\psi} s\theta \nabla \psi + \nabla \zeta \right] + q(\bar{\psi}) \bar{\Omega}_b \oint \frac{d\theta}{2\pi} \quad (\text{C26})$$

In order to evaluate the integral in Eq. (C26), we explicit the projection of the curvature term onto $\nabla \psi$ and $\nabla \zeta$ using Eq. (C13)

$$(\mathbf{B}_0 \times \nabla B_0) \cdot \nabla \psi = -\nabla B_0 \cdot (\nabla \psi \times \nabla B_0) = -\nabla B_0 \cdot [\nabla \psi \times (\nabla \psi \times \nabla \zeta)] \quad (\text{C27})$$

$$(\mathbf{B}_0 \times \nabla B_0) \cdot \nabla \zeta = \nabla B_0 \cdot (\nabla \zeta \times \nabla B_0) = -\nabla B_0 \cdot [\nabla \zeta \times (\nabla \psi \times \nabla \zeta)] = -|\nabla \zeta|^2 |\nabla \psi|^2 (\partial B_0 / \partial \zeta) \quad (\text{C28})$$

Writing $\mu B_0 + mv_{\parallel}^2 = 2E - \mu B_0$, the expression of the toroidal frequency Ω_3 becomes,

$$\Omega_3 = \bar{\Omega}_b \frac{EB_0}{eR_0 B_0^3} |\nabla \zeta|^2 |\nabla \psi| \oint \frac{d\theta (2 - \lambda)(s\theta \sin \theta + \cos \theta)}{2\pi \sqrt{1 - \lambda(1 - \epsilon \cos \theta)}} + \delta_{\text{passing}} q(r) \Omega_2 \quad (\text{C29})$$

Since $d\psi/dr = B_0 r/q$ and $|\nabla \zeta| \approx q/r$, it follows that Eq. (C29) is written

$$\Omega_3 = \frac{q}{r} \frac{E}{eB_0 R_0} \bar{\Omega}_d + \delta_{\text{passing}} q(r) \Omega_2 \quad (\text{C30})$$

Where $\bar{\Omega}_d$ is given in terms of Ω_2 given by Eq. (C25)

$$\bar{\Omega}_d = \bar{\Omega}_b (2 - \lambda) \Omega_{\kappa} \quad (\text{C31})$$

the frequency $\bar{\Omega}_d$ can be expressed as

$$\bar{\Omega}_d = \begin{cases} \frac{4\epsilon + \kappa^2(1-2\epsilon)}{2\epsilon + (1-\epsilon)\kappa^2} \left[1 + \frac{2}{\kappa^2} \left(\frac{\mathbb{E}(\kappa^2)}{\mathbb{K}(\kappa^2)} - 1 \right) + \frac{4s}{\kappa^2} \left(\frac{\mathbb{E}(\kappa^2)}{\mathbb{K}(\kappa^2)} - \frac{\pi}{2} \frac{\sqrt{1-\kappa^2}}{\mathbb{K}(\kappa^2)} \right) \right] & \text{for passing particles } (\kappa < 1) \\ \frac{4\epsilon + \kappa^2(1-2\epsilon)}{2\epsilon + (1-\epsilon)\kappa^2} \left[2 \frac{\mathbb{K}(1/\kappa^2)}{\mathbb{E}(1/\kappa^2)} - 1 + 4s \left(\frac{\mathbb{K}(1/\kappa^2)}{\mathbb{E}(1/\kappa^2)} + \frac{1}{\kappa^2} - 1 \right) \right] & \text{for trapped particles } (\kappa > 1) \end{cases} \quad (\text{C36})$$

Appendix D: Impact of the magnetic field curvature on the tearing instability through mode-mode coupling

In all the previous sections, we have considered a single harmonic for the perturbation, thus neglecting possible couplings

With $2 - \lambda = \frac{4\epsilon + \kappa^2(1-2\epsilon)}{2\epsilon + (1-\epsilon)\kappa^2}$ and Ω_{κ} is the integral given by

$$\Omega_{\kappa} = \int_{\theta_1}^{\theta_2} \frac{d\theta}{2\pi} \frac{(s\theta \sin \theta + \cos \theta)}{\sqrt{1 - \lambda(1 - \epsilon \cos \theta)}} \quad (\text{C32})$$

The integral (C32) is also solved separately for passing and trapped particles by integrating over the appropriate boundaries given in Eqs. (C21) and (C22). For the $\cos \theta$ integral, we notice that $\cos \theta = 1 - \sin^2 \frac{\theta}{2}$, so the convenient change of variable would be $u = \theta/2$ for passing particles and $\sin u = \sin \theta/2$ for trapped particles. As for the $\sin \theta$ integral, we solve it integrating by parts as we notice that

$$\frac{d}{d\theta} \left(\sqrt{1 - \kappa^2 \sin^2 \frac{\theta}{2}} \right) = -\frac{\kappa^2}{2} \frac{\sin \theta}{2\sqrt{1 - \kappa^2 \sin^2 \frac{\theta}{2}}} \quad (\text{C33})$$

Therefore,

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{2\pi} \frac{\theta \sin \theta}{\sqrt{1 - \kappa^2 \sin^2 \frac{\theta}{2}}} = -\frac{4}{\kappa^2} \left[\frac{\theta}{2\pi} \sqrt{1 - \kappa^2 \sin^2 \left(\frac{\theta}{2} \right)} \right]_{\theta_1}^{\theta_2} + \frac{4}{\kappa^2} \int_{\theta_1}^{\theta_2} \frac{d\theta}{2\pi} \sqrt{1 - \kappa^2 \sin^2 \left(\frac{\theta}{2} \right)} \quad (\text{C34})$$

The term between brackets is equal to zero for trapped particles, as $\kappa^2 \sin^2(\theta/2) = 1$ for $\theta = \pm\theta_0$. However, for passing particles, $\theta_1 = -\pi$ and $\theta_2 = +\pi$, it does not vanish and it gives $\sqrt{1 - \kappa^2}$. Using the standard definition of the complete elliptic integrals of the first kind given by (C23) and that of the second kind given by³⁴

$$\mathbb{E}(m) = \int_0^{\pi/2} d\theta \sqrt{1 - m \sin^2 \theta} \quad (\text{C35})$$

between the different harmonics. However multiple rational surfaces may be present within the domain and so allows double tearing modes and toroidal coupling of modes, a process which is known to be stabilising⁶. We have neglected also the electrostatic potential and treated the instability in a purely

magnetic approach. Of course, although they can be insightful, these approximations make our model less accurate as we neglect some physical mechanisms that are important in understanding the role of magnetic curvature. For this reason, we investigate in this section the effect of curvature through toroidal coupling of main resonating mode, which we consider here as the principal harmonic, and one of its side bands. The electrostatic potential is considered here. However, Poisson equation is not solved because we use the ideal MHD condition outside of the resonant surface to relate $\tilde{\phi}$ to $\tilde{A}_{\parallel}$. We start with the electromagnetic drift kinetic equation that describes the electron dynamics. The perturbed distribution function δF is decomposed into adiabatic and non adiabatic parts

$$\delta F = \frac{e}{T_{\text{eq}}} F_{\text{eq}} \phi + g \quad (\text{D1})$$

where F_{eq} is the background distribution, considered as a Maxwellian, and g is the non adiabatic distribution function, satisfying the equation

$$(\omega - k_{\parallel} v_{\parallel} - \omega_D) g = -(\omega - \omega_{*}) \frac{e F_{\text{eq}}}{T_{\text{eq}}} (\phi - v_{\parallel} A_{\parallel}) \quad (\text{D2})$$

where ϕ and $A_{\parallel}$ are the perturbed electrostatic potential and the parallel component of the magnetic vector potential, respectively. In this equation, the magnetic drift frequency is expressed as a differential operator

$$\omega_D = -i \mathbf{v}_D \cdot \nabla = \frac{v_{\parallel}^2 + \mu B}{e B_0 R_0} \left(\sin \theta \frac{\partial}{\partial r} + \cos \theta \frac{\partial}{r \partial \theta} \right) \quad (\text{D3})$$

r and θ are respectively the radial coordinate and the poloidal angle. We consider the toroidal coupling of modes m and $m+1$, where m is the poloidal wave number. Perturbations are then expressed in the form

$$Q = Q_m e^{im\theta} + Q_{m+1} e^{i(m+1)\theta} \quad (\text{D4})$$

Assuming $\partial_r g_m = \partial_r g_{m+1} = 0$ and separating the main oscillating component from the side band, the Vlasov equation is rewritten as

$$\mathcal{M} \begin{pmatrix} g_m \\ g_{m+1} \end{pmatrix} = S \begin{pmatrix} \phi_m - v_{\parallel} A_{\parallel m} \\ \phi_{m+1} - v_{\parallel} A_{\parallel m+1} \end{pmatrix} \quad (\text{D5})$$

where the matrix $\mathcal{M}$ is the square matrix

$$\mathcal{M} = \begin{pmatrix} \omega - k_{\parallel m} v_{\parallel} & \frac{i(m+1)}{2r} v_D \\ \frac{i m}{2r} v_D & \omega - k_{\parallel m+1} v_{\parallel} \end{pmatrix} \quad (\text{D6})$$

$S = -(\omega - \omega_{*n}) e F_{\text{eq}} / T_{\text{eq}}$ and $v_D = (v_{\parallel}^2 + \mu B) / e B_0 R_0$, B_0 is the equilibrium magnetic field. The parallel wave vector $k_{\parallel}$ takes a different form depending on the considered poloidal mode such that $k_{\parallel m} = (m/q_m + n)/R$ is evaluated at the resonant surface r_m and $k_{\parallel m+1} = ((m+1)/q_{m+1} + n)/R$ at the resonant surface r_{m+1} . The solution of Eq. (D5) can be obtained by inverting the matrix $\mathcal{M}$,

$$\mathcal{M}^{-1} = \frac{1}{|\mathcal{M}|} \begin{pmatrix} \omega - k_{\parallel m+1} v_{\parallel} & -\frac{i(m+1)}{2r} v_D \\ -\frac{i m}{2r} v_D & \omega - k_{\parallel m} v_{\parallel} \end{pmatrix} \quad (\text{D7})$$

where $|\mathcal{M}| = \det(\mathcal{M}) = (\omega - k_{\parallel m} v_{\parallel})(\omega - k_{\parallel m+1} v_{\parallel}) + v_D^2 m(m+1)/4r^2$ is the determinant of matrix $\mathcal{M}$. The dispersion relation of the tearing mode is obtained by integrating Ampère's law for each mode. In the constant- ψ approximation, this reads in normalized units

$$\delta_e^2 \Delta'_m A_{\parallel m} = 2\pi \int_{\mathbb{R}} dx \int dv_{\parallel} v_{\perp} dv_{\perp} v_{\parallel} g_m \quad (\text{D8})$$

$$\delta_e^2 \Delta'_{m+1} A_{\parallel m+1} = 2\pi \int_{\mathbb{R}} dx \int dv_{\parallel} v_{\perp} dv_{\perp} v_{\parallel} g_{m+1} \quad (\text{D9})$$

In matrix form this system of equations can be written as

$$\begin{pmatrix} \Delta'_m & 0 \\ 0 & \Delta'_{m+1} \end{pmatrix} \begin{pmatrix} A_{\parallel m} \\ A_{\parallel m+1} \end{pmatrix} = \mathcal{I} \begin{pmatrix} A_{\parallel m} \\ A_{\parallel m+1} \end{pmatrix} \quad (\text{D10})$$

where $\mathcal{I}$ is a matrix that will be calculated in the following. The perturbed distribution functions associated to modes m and $m+1$, respectively at the resonant surfaces r_m and r_{m+1} are expressed in normalized units as

$$g_m = -\frac{(\omega - k_{\parallel m+1} v_{\parallel})(\phi_m - 2v_{\parallel} A_{\parallel m}) - \frac{i m+1}{2r} v_D (\phi_{m+1} - 2v_{\parallel} A_{\parallel m+1})}{(\omega - k_{\parallel m} v_{\parallel})(\omega - k_{\parallel m+1} v_{\parallel}) + \frac{m(m+1)}{4r^2} v_D^2} (\omega - \omega_{*n}) \frac{1}{T_{\text{eq}}} \frac{n_e}{(\pi T_{\text{eq}})^{3/2}} e^{-\frac{v_{\parallel}^2 + v_{\perp}^2}{T_{\text{eq}}}} \quad (\text{D11})$$

$$g_{m+1} = -\frac{-\frac{i m}{2r} v_D (\phi_m - 2v_{\parallel} A_{\parallel m}) + (\omega - k_{\parallel m} v_{\parallel})(\phi_{m+1} - 2v_{\parallel} A_{\parallel m+1})}{(\omega - k_{\parallel m} v_{\parallel})(\omega - k_{\parallel m+1} v_{\parallel}) + \frac{m(m+1)}{4r^2} v_D^2} (\omega - \omega_{*n}) \frac{1}{T_{\text{eq}}} \frac{n_e}{(\pi T_{\text{eq}})^{3/2}} e^{-\frac{v_{\parallel}^2 + v_{\perp}^2}{T_{\text{eq}}}} \quad (\text{D12})$$

Therefore, the response of mode m is given by the combination of the modes m and $m+1$ of the potential. The same

can be said about the mode $m+1$. The integrals in Eqs.

(D8) (first line of matrix $\mathcal{I}$) and (D9) (second line of matrix $\mathcal{I}$) are respectively calculated around the resonant surfaces r_m and r_{m+1} .

The resonant condition $\omega - k_{\parallel}v_{\parallel}$ at one of the resonant surfaces can be simplified when evaluated at the other sur-

face. In fact, away from the resonant surface r_m , the parallel streaming motion is much faster than the frequency of the mode, however close to that surface, the resonance is satisfied and one can write $\omega - k_{\parallel m}v_{\parallel} \approx 0$. This allows to write

$$(\omega - k_{\parallel m+1}v_{\parallel})|_{r_m} = \omega - \left(\frac{m+1}{q_{m+1}} + n\right) \bigg|_{r_m} v_{\parallel} = \omega - k_{\parallel m}v_{\parallel} - \frac{v_{\parallel}}{q_{m+1}} \approx -\frac{v_{\parallel}}{q_{m+1}} \quad (\text{D13})$$

$$(\omega - k_{\parallel m}v_{\parallel})|_{r_{m+1}} = \omega - \left(\frac{m}{q_m} + n\right) \bigg|_{r_{m+1}} v_{\parallel} = \omega - k_{\parallel m+1}v_{\parallel} - \frac{v_{\parallel}}{q_m} \approx \frac{v_{\parallel}}{q_m} \quad (\text{D14})$$

in normalized units. Therefore, we have implicitly normalized the frequencies to the transit frequency $\omega_t = v_{th}/R_0$.

We assume that the magnetic drift is constant on each of the resonant surfaces (the local approximation). Inside the resonant layer, we consider that the corresponding electrostatic potential is equal to zero. And outside of it, the ideal

MHD condition applies i.e. the parallel electric field vanishes at that position. Therefore, on the resonance surface r_m , $E_{\parallel m+1} = 0$, i.e. $\phi_{m+1} = 2\omega A_{\parallel m+1} k_{\parallel m+1}^{-1}$ and on $r_m + 1$; $E_{\parallel m} = 0$, i.e. $\phi_m = 2\omega A_{\parallel m} k_{\parallel m}^{-1}$. On these grounds, we calculate each term of matrix $\mathcal{I}$ of Eq. (D10)

$$\mathcal{I}_{m,m} \approx -\frac{1}{\delta_e^2} \frac{4\pi}{T_{eq}} \int dv_{\parallel} v_{\perp} dv_{\perp} \frac{v_{\parallel}^3}{q_{m+1}} (\omega - \omega_{*}) \frac{n_e}{(\pi T_{eq})^{3/2}} e^{-\frac{v_{\perp}^2 + v_{\parallel}^2}{T_{eq}}} \frac{q_{m+1}}{v_{\parallel}} \int dx \frac{1}{k'_{\parallel m} x v_{\parallel} - \omega + \frac{m(m+1)v_D^2 q_{m+1}}{4r_m^2 v_{\parallel}}} \quad (\text{D15})$$

$$\begin{aligned} \mathcal{I}_{m,m+1} = & -i \frac{1}{\delta_e^2} \frac{4\pi}{T_{eq}} \int dv_{\parallel} v_{\perp} dv_{\perp} (\omega - \omega_{*}) \frac{n_e}{(\pi T_{eq})^{3/2}} e^{-\frac{v_{\perp}^2 + v_{\parallel}^2}{T_{eq}}} \frac{m+1}{q_{m+1}} \frac{1}{2r_m} v_D \\ & \left\{ \int dx \frac{\omega}{k'_{\parallel m+1} k'_{\parallel m} v_{\parallel} x^2 - \omega k'_{\parallel m+1} x + \frac{m(m+1)v_D^2 q_{m+1}}{4r_m^2 v_{\parallel}} k'_{\parallel m+1} x} + v_{\parallel} \int dx \frac{1}{k'_{\parallel m} x v_{\parallel} - \omega + \frac{m(m+1)v_D^2 q_{m+1}}{4r_m^2 v_{\parallel}}} \right\} \quad (\text{D16}) \end{aligned}$$

After integration over x and $v_{\perp}$ and using the definition of the complex logarithm, the matrix elements $\mathcal{I}_{m,m}$ and $\mathcal{I}_{m,m+1}$ become $\mathcal{I}_{m,m} =$

$$-i \frac{1}{\delta_e^2} \frac{4\sqrt{\pi} n_{eq}}{|k'_{\parallel m}| \sqrt{T_{eq}}} \left[\omega - \omega_{*n} \left(1 + \left(2T_{eq} - \frac{3}{2} \right) \eta_e \right) \right] \text{ and } \mathcal{I}_{m,m+1} = 0.$$

Similar calculations for $\mathcal{I}_{m+1,m}$ and $\mathcal{I}_{m+1,m+1}$ lead to the matrix $\mathcal{I}$

$$\mathcal{I} = -i \frac{1}{\delta_e^2} \frac{4\sqrt{\pi} n_{eq}}{\sqrt{T_{eq}}} \left[\omega - \omega_{*n} \left(1 + \left(2T_{eq} - \frac{3}{2} \right) \eta_e \right) \right] \begin{pmatrix} \frac{1}{|k'_{\parallel m}|} & 0 \\ 0 & \frac{1}{|k'_{\parallel m+1}|} \end{pmatrix} \quad (\text{D17})$$

with vanishing non-diagonal terms. Looking for non trivial solutions of Eq. (D10) is equivalent to solving the equation

$$\left(\Delta'_m + \frac{\Gamma}{|k'_{\parallel m}|} \right) \left(\Delta'_{m+1} + \frac{\Gamma}{|k'_{\parallel m+1}|} \right) = 0 \quad (\text{D18})$$

where Γ reads

$$\Gamma = i \frac{1}{\delta_e^2} \frac{4\sqrt{\pi} n_{eq}}{\sqrt{T_{eq}}} \left[\omega - \omega_{*n} \left(1 + \left(2T_{eq} - \frac{3}{2} \right) \eta_e \right) \right] \quad (\text{D19})$$

The solutions for the modes m and $m+1$ are therefore decoupled from each other. This indicates that there is no effect of the curvature of the field on the stability of the tearing mode in the absence of particle trapping.

Appendix E: Magnetic Hamiltonian for deeply trapped particles

Without any loss of generality, the Hamiltonian can be decomposed into equilibrium and perturbed components, namely

$$\mathcal{H} = \mathcal{H}_{eq} + \delta\mathcal{H}_{A_{\parallel}} \quad (\text{E1})$$

where the equilibrium Hamiltonian is simply

$$\mathcal{H}_{eq} = \frac{1}{2}mv_{\parallel}^2 + \mu B \quad (\text{E2})$$

and the perturbed Hamiltonian is, for the case of electrons,

$$\delta\mathcal{H}_{A_{\parallel}} = ev_{\parallel}A_{\parallel} \quad (\text{E3})$$

The parallel component of the perturbed vector potential is written as a single helicity perturbation in real space

$$A_{\parallel} = A_{\parallel} e^{i(m\theta + n\varphi)} \quad (\text{E4})$$

where $A_{\parallel}$ is the amplitude of the perturbation and the time dependence has been omitted for the sake of clarity. However, the perturbations introduced here exist in real space and do not have the same expression in the space of canonically conjugated variables that are suitable for the description of particle trajectories. Therefore, the hamiltonian needs to be written in terms of the action-angle variables of the deeply trapped particles. For this purpose we use the relations

$$\theta = \theta_0 \sin \alpha_2 \quad (\text{E5})$$

$$\varphi = \alpha_3 + q\theta_0 \sin \alpha_2 \quad (\text{E6})$$

where θ_0 is the bounce poloidal angle. Note that the previous expressions, although strictly speaking valid for deeply trapped particles, can be applied for poloidal angles up to $\theta_0 \approx \pi/2$ and therefore apply also for trapped particles (not barely trapped particles though). Moreover, the parallel velocity can be expressed in terms of the angle α_2 ,

$$v_{\parallel} = v_{\parallel 0} \cos \alpha_2 \quad (\text{E7})$$

where $v_{\parallel 0}$ is the parallel velocity on the mid-plane.

Introducing the expressions E5 and E6 that provide θ and φ as functions of α_2 and α_3 , the parallel component of the vector potential is then expressed as

$$A_{\parallel} = A_{\parallel} e^{i(m+nq)\theta_0 \sin \alpha_2} e^{in\alpha_3} \quad (\text{E8})$$

Using the Jacobi-Anger expansion

$$e^{ix \cos \alpha} = \sum_{p=-\infty}^{\infty} i^p J_p(x) e^{ip\alpha} \quad (\text{E9})$$

where $J_p(x)$ is the Bessel function of the first kind of order p , we can write the perturbed Hamiltonian as

$$\delta\mathcal{H}_{A_{\parallel}} = ev_{\parallel 0} A_{\parallel} \frac{1}{2} e^{in\alpha_3} (e^{i\alpha_2} + e^{-i\alpha_2}) \sum_{p=-\infty}^{\infty} J_p((m+nq)\theta_0) e^{ip\alpha_2} \quad (\text{E10})$$

The multiplication by $v_{\parallel}$ introduces therefore a shift in the p summation index

$$\delta\mathcal{H}_{A_{\parallel}} = ev_{\parallel 0} A_{\parallel} \sum_{n_2=-\infty}^{\infty} \frac{n_2 J_{n_2}((m+nq)\theta_0)}{(m+nq)\theta_0} e^{in_2\alpha_2} e^{in\alpha_3} \quad (\text{E11})$$

where we have used the property

$$J_{p+1}(x) + J_{p-1}(x) = \frac{2p}{x} J_p(x) \quad (\text{E12})$$

Finally, the n_2, n_3, ω component of the perturbed hamiltonian is found to be

$$\delta\mathcal{H}_{A_{\parallel}, n_2, n_3, \omega} = ev_{\parallel 0} A_{\parallel} \frac{n_2 J_{n_2}((m+nq)\theta_0)}{(m+nq)\theta_0} \quad (\text{E13})$$

which reduces to $\delta\mathcal{H}_{A_{\parallel}, n_2=0, n_3, \omega} = 0$.

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