Building Reduced Model for MILP Optimization: Application to Demand Response of Residential Buildings
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To cite this version:
Camille Pajot, Nils Artiges, Benoît Delinchant, Yves Maréchal. Building Reduced Model for MILP Optimization: Application to Demand Response of Residential Buildings. Building Simulation Conference 2019, Sep 2019, Rome, Italy. hal-02364704

HAL Id: hal-02364704
https://hal.archives-ouvertes.fr/hal-02364704
Submitted on 15 Nov 2019

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Abstract
This paper addresses the topic of the electrical flexibility on the demand-side, by focusing on the residential buildings. Specifically, it aims to quickly formulate the optimal operation of building heat pumps, according to environmental or grid issues. Indeed the literature shows many examples of quantification of the flexibility impacts, but mostly relying on predefined strategies. To go from the evaluation of some strategies to the formulation of the optimal planning, we are introducing a methodology based on the automatic generation of models dedicated to mixed-integer linear programming (MILP) optimization. Finally, the method was applied to a new residential building during a month of winter.

Introduction
Flexibility context on the electrical system
According to the Intergovernmental Panel on Climate Change (2018), climate-related risks to health, livelihoods, food security, water supply, human security, and economic growth are projected to increase with global warming. Limiting global warming to 1.5°C requires the energy system to undergo a rapid transition. One solution to reduce the CO₂ emitted by the energy system is to increase the share of renewable energies into the energy production mix. As a massive integration of variable renewable energies in the power system could lead to stability issues, flexibility becomes key to the energy transition (IRENA (2018)). In order to increase the flexibility means on the electrical system, the consumption site has been involved through the concept of Demand-Side Management (Meyabadi and Deihimi (2017)).

According to the International Energy Agency (2018), the global buildings sector accounts for more than 55% of global electricity demand, so that they represent a massive and diffuse electricity consumption. Besides electrical appliances, the electricity can be converted into heat in order to cover the thermal needs. This possibility of conversion from electricity to heat is usually called Power-to-Heat (P2H) and allows the use of the flexibility on a thermal load for electrical grid purposes (Bloess et al. (2018)). As buildings can store heat into their own envelope thanks to thermal inertia, P2H can be applied to buildings equipped with heat pumps, electric heaters or even electric boilers.

Approaches for DSM modeling
In order to quantify the impacts of a DSM (Demand Side Management) strategy, load forecast models are required. In the particular case of using heat pumps flexibility, the load to be predicted corresponds to the thermal needs of the building. The literature mostly shows two approaches for heat load forecasting:

1. Data-driven models
2. Physical models

In the first case, the load predictions rely on historical data, on which various machine learning methods (from linear regression to neural networks) are applied for the future load forecast (Yildiz et al. (2017), Amasyali and El-Gohary (2018)). In order to be successful, these methods require a large amount of data. Indeed, in order to provide efficient hours-ahead building load forecasts, Ke et al. (2016) used 15-minutes building load data from May 2012 to April 2014. Similarly, Bacher et al. (2013) used measurements over a two-year period with a 10-minute time step. When as many data are not available, an alternative to these "black box" models is to exploit the laws of physics for thermal transfers, also called "white box" or "grey box" (Harish and Kumar (2016)). Besides solving a data issue, using physical laws can lead to greater reliability of the load forecast. A large range of physical models can be found in the literature, from very-detailed to low-level models (Reinhart and Davila (2016)). Very-detailed models are generally based on the thermal zone concept, which considers parts of the building with a homogeneous temperature as a single "thermal zone" and decomposed buildings elements such as walls with a finite volume method (Peuportier and Blanc (1990)). Software with detailed thermal models, including all the thermal zones and the energy systems are used for regulation purposes, such as guarantee-
ing the respect of the maximal energy consumption of a building during the design phase (Allegrini et al. (2015)). In the opposite, thermal models such as Resistance-Capacitance (RC) networks applied to an entire building can significantly reduce the simulation time of the thermal load forecasting model. Therefore, when scaling up from building to the district, RC networks tend to be preferred (Elci et al. (2018)). Another alternative is available to gain time during the design of the study case: model generation tools. By using standard languages, these tools have the main advantage to provide reusable models. For instance, Remmen et al. (2018) provide TEASER (Tool for Energy Analysis and Simulation for Efficient Retrofit) for the creation of building models in the Modelica language.

Once the consumption models created, the strategies of demand-side management can be applied and evaluated. The performance of a strategy can be quantified by introducing evaluation indicators, such as the CO$_2$ emissions, the operative temperature or peak shaving. However, in the case of simulation models, each strategy should be pre-defined, then tested and evaluated afterward, so that a simulation-based approach cannot provide new solutions. To go further in the formulation of DSM scenarios according to its purposes, optimization is thus needed. For this reason, this paper aims to develop a methodology to define optimal operation strategies of a building heat pump.

**Paper structure**

First, the methodology implemented to quickly generate optimization models for DSM on buildings heating loads will be presented. Then, the methods will be applied to a residential building, aiming either to minimize its electric peak power or its CO$_2$ emissions. In the results section, the optimization results obtained on the study case will be presented. Finally, after a discussion section, a conclusion will be drawn.

**Methods**

**Quick generation of optimization models**

The paper aims to provide a method to quickly determine optimal heat pumps operation strategies. Optimization allows finding the best DSM strategy according to criteria, instead of testing several predefined strategies in simulation and evaluating the impact afterward. For this purpose, a first requirement is heating loads models suited for optimization. Then, the generation of these models has to be fast and replicable. To do so, we rely on the OMEGAlpes open source software$^1$, which allows a quick design of optimization problems. The models are generated into a Mixed-Integer Linear Programming (MILP) formulation in order to quickly provide a solution with a large number of variables. Thus, the thermal load model has to respect this linear formulation. This is a key point that we are addressing in this paper, and especially for non-linearities occurring in radiative exchanges.

**Thermal load forecast**

The district scale can be very different from the building scale by the number of stakeholders involved in the energy decisions, and thus by the availability of relevant data. Indeed, consumption data are much easier to get for study cases at a building scale, than for an entire district. This poor data access when scaling up to the district scale needs to be taken into consideration. Thus, data-driven load forecast methods can be difficult to apply to an entire district. Moreover, a thermal load is much more correlated to data such as the external temperature for low-insulated buildings than for the new high-efficiency ones. Therefore, this paper only focuses on a physical modeling approach.

However, as they require a large amount of specific data on each building, very-detailed models are out of scope too. Besides not being available, a very large amount of data could lead to computational issues during the resolution of the optimization problem. Therefore, reduced models are wished for the optimization model of the heating load prediction, as soon the level of modeling is effective enough to describe buildings responses to DSM events.

Low-level RC networks could fit the requirements since their utilization in order to simulate the impact of DSM strategies on building heating loads was validated in a previous study (Pajot et al. (2018)). Moreover, the model has to be linear in order to fit with the linear formulation of the optimization problem (MILP). As there are many variants of these models, this study focus on the RC model used for regulatory studies in both the French and Swiss contexts (RT2012 and SIA 2044) and drawn in Figure 1. The model described in Figure 1 is related to SIA 2044 and was extracted from the framework City Energy Analyst from ETH Zurich (Fonseca et al. (2016)).

Figure 1: Resistance-Capacitance network model

Six nodes of temperature are linked by five thermal resistances and one capacitance. The three nodes on the left of the thermal model are related to external temperatures, while the right part describes internal behaviours.

Then, the top represents the ventilation, through an

$^1$https://gricad-gitlab.univ-grenoble-alpes.fr/omegalpes
heat flow rates (Φ_{vent}) between the ambient air (T_{int}) and the injected air (θ_{ea}), according to:

\[ Φ_{vent} = H_{EA}(T_{int} - θ_{ea}) \]  

(1)

The bottom part represents the heat exchanges at the building mass. The temperature at its internal surfaces is expressed by \( θ_m \), while \( θ_{em} \) stands for its external surfaces. Thus, the transmission losses with outside (Φ_{trans}) can be expressed by the expression (2). Moreover, an internal heat capacity (C_m) is connected to the node (m), since the main source of heat storage in a building corresponds to its mass.

Finally, the light surfaces of the building, such as the windows, are integrated into the node (c). This node reflects an average behavior between these surfaces, the building mass and the ambient air. The transmission losses occurring at light surfaces are thus considered by Φ_{trans, c} as expressed in (2).

\[
\begin{align*}
Φ_{trans} &= H_{EC}(θ_c - θ_{ec}) \\
Φ_{trans, c} &= H_{EM}(θ_m - θ_{em})
\end{align*}
\]  

(2)

However, these losses can be compensated by heat gains, from solar radiation, but also occupancy, appliances and lighting. All the gains from inside or outside can be split between the three internal nodes of temperature (T_{int}, θ_c and θ_m) and are respectively called Φ_a, Φ_c and Φ_m, as shown Figure 1. Splitting the internal gain between the nodes was realized with coefficients whose values were extracted from the Swiss norm SIA 2044, according to the calculations found in Fonseca et al. (2016). Therefore, the heat flow rates from lightning (Φ_{int}), occupancy (Φ_{ip}) and appliances (Φ_{ia}) are distributed according to (3) to form the internal gains.

\[
\begin{align*}
Φ_{int, a} &= (1 - f_{ria})Φ_{al} + (1 - f_{rip})Φ_{ip} + (1 - f_{ria})Φ_{ia} \\
Φ_{int, m} &= f_{ria}Φ_{al} + f_{rip}Φ_{ip} + f_{ria}Φ_{ia}
\end{align*}
\]  

(3)

The external gains (Φ_a) can be split into the net solar radiation to the building (I_{sol}) and heat flow rates the re-irradiated to the sky (I_{rad}).

\[ Φ_a = I_{sol} - I_{rad} \]  

(7)

Both the incident and the re-irradiated heat flow rates can be divided between the walls, the windows and the roof (8 and 14). The incident solar gains to the building depend on the average value of the solar radiation (8).

\[ I_{sol} = I_{sol}^w(γ_{win} + γ_{wall} + γ_{roof}) \]  

(8)

Where:

\[
\begin{align*}
γ_{win} &= A_{win} * (1 - F_F) * F_{sh_{win}} \\
γ_{wall} &= A_{wall} * R_{SE} * a_{wall} * U_{wall} \\
γ_{roof} &= A_{roof} * R_{SE} * a_{roof} * U_{roof}
\end{align*}
\]  

(9)

Despite benefiting from solar radiation, buildings are continuously exposed to the sky, so that radiative exchanges occur between buildings elements (windows, walls and roof) and the sky. For each element x, this re-irradiated heat flow to the sky, which is non-linear regarding surface temperature (θ), can be expressed as (10) and is usually found under the form (11) in building physics applications.

\[ I_{rad, x} = ε_x * σ * A_x^e * (T_{sky}^4 - θ_x^4) \]  

(10)

\[ I_{rad, x} = \frac{ε_x σ(T_{sky}^2 + θ_x^2)(T_{sky} + θ_x) * A_x^e * (T_{sky} - θ_x)}{h_{rad, x}} \]  

(11)

Where \( ε_x \) is the emissivity of the element x, \( σ \) the Stefan-Boltzmann constant, \( T_{sky} \) the temperature of the sky, \( θ_x \) the temperature defined previously, \( h_{rad, x} \) is an external radiative heat transfer coefficient for the element x and \( A_x^e \) is its effective solar collecting area (12) according to the norm ISO 13790 (2008):

\[ A_x^e = F_{f,x} * R_{SE} * U_x * A_x \]  

(12)

Where \( R_{SE} \) is the external surface heat resistance of the opaque part, \( F_{f,x} \) is the form factor of the element \( x \) (0.5 for vertical surfaces and 1 for horizontal surfaces), \( U_x \) is the thermal transmittance of the element \( x \) and \( A_x \) is its surface.

In buildings physic literature, numerous simplifications are realized for the expression of the external radiation heat transfer coefficients (h_{rad, x}) from empirical values (around 5W/m²K) to more complex calculation depending on the wind speed (Evangelisti et al. 2017). The Standard UNI EN ISO 6946 recommends to express h_{rad, x} as follows (13).

\[ h_{rad, x} = 4 * ε_x * σ * \left( \frac{T_{sky}^t + θ_x^t - 1}{2} \right)^3 \]  

(13)
Therefore the re-irradiated heat flow can be expressed by a polynomial of $\theta_c^{-1}$ and $T_{\text{sky}}$ (14).

$$
\begin{align*}
I_{\text{rad}} &= p(\theta_c^{t-1}, T_{\text{sky}})(k_{\text{win}} + k_{\text{wall}} + k_{\text{roof}}) \\
p(x, y) &= (y - x)(x + y)^3
\end{align*}
$$

(14)

Where:

$$
\begin{align*}
k_{\text{win}} &= F_{f,\text{win}} \cdot R_{\text{SE}} \cdot U_{\text{win}} \cdot A_{\text{win}} \cdot \epsilon_{\text{win}} \cdot \sigma/2 \\
k_{\text{wall}} &= F_{f,\text{wall}} \cdot R_{\text{SE}} \cdot U_{\text{wall}} \cdot A_{\text{wall}} \cdot \epsilon_{\text{wall}} \cdot \sigma/2 \\
k_{\text{roof}} &= F_{f,\text{roof}} \cdot R_{\text{SE}} \cdot U_{\text{roof}} \cdot A_{\text{roof}} \cdot \epsilon_{\text{roof}} \cdot \sigma/2
\end{align*}
$$

(15)

In the case of Dynamic Thermal Simulation (DTS), the resolution of the previous equations (or variants) is realized once per time step. However, in order to be integrated into an optimization approach, specific requirements need to be considered.

**Optimization requirements**

In opposite to DTS models, the optimization model has to integrate all the time steps simultaneously to find the optimal operation of the heating system. Thus, no iterative process can be incorporated into the MILP formulation of the study case, which represents a big difference with the simulation models. Besides considering the time-dependency when it is the case, all the equations should be acausal. Thus, all the dynamic variables calculated at each iteration of the simulation become as many decisions variables as the number of time steps and differential equations can be written as difference equations. This way, all equations from the simulation model can be converted into optimization ones and expressed as optimization constraints.

Moreover, the constraints expressed in a MILP model can only be described as linear expressions of the decision variables. However, the Stefan-Boltzmann law expressed in (14) is non-linear and cannot be integrated as such in the MILP model. Indeed, the $\theta_c$ value is calculated at each iteration during the simulation process and therefore is a decision variable in the optimization model. In order to be considered, this heat flow rate has to be linearized. In the case of a MILP formulation, one solution is to introduce new constraints with binary variables. Nevertheless, this method could lead to computational issues when the expression to be linearized is time-dependent. More traditional approaches include using Taylor development (16) to express the function around a point of interest $(x_i)$ or several ones through a piecewise linear function.

$$
f(x) \approx \sum_{i=0}^{n} \frac{f^{(i)}(x_i)}{i!}(x - x_i)^i
$$

(16)

Since the temperature of the sky ($T_{\text{sky}}$) only depends on weather data and is entirely known out of the optimization, the expression of the re-irradiated heat flow has to be linearized only with respect to $\theta_c$.

The range of variation of $\theta_c$ is relatively small as the temperature of the surfaces is quite close to the mean ambient temperature of a temperature-regulated building. For this reason, an estimation of the mean temperature of the thermal zone ($T_{\text{mean}}$) is chosen as the point of interest in the linearization, so that $I_{\text{rad}}$ can be expressed as (17).

$$
I_{\text{rad}}(\theta_c^{-1}) \approx \sum_{i=0}^{n} f^{(i)}(i) \frac{T_{\text{mean}}}{i!}(\theta_c^{-1} - T_{\text{mean}})^i
$$

(17)

A first approach consist in assuming $\theta_c$ to be constant, i.e. considering the Taylor formula (17) at the order 0. Then, a second step is realized with a development at the first order. The two methods are illustrated in Figure 2 and will be developed in the next subsection.

![Figure 2: Illustration of the linearization method](image)

**Linearization results**

As explained previously, the MILP formulation of the optimization problem requires all equations to be linear in order to be set as constraints. As $I_{\text{rad}}$ can be expressed as a fourth-degree polynomial of $\theta_c$ (14), two linearization methods were presented:

1. The first linearization considered a fixed value of $\theta_c$ equals to $T_{\text{mean}}$ (17 at order 0).
2. The second method assumed a linear variation of $I_{\text{rad}}$ depending on $\theta_c$ (17 at order 1).

Dynamic results of these linearizations are shown in Figure 3 on a 48-hour period, with a time step of 10 minutes. For reference, the real calculation of $I_{\text{rad}}$ is represented by a blue line, while the orange and green bullets respectively stand for the first and second method.

First, it can be noticed that results obtained by the first method change per stages. This can be explained by the time steps of the data. As the external temperature is hourly predicted, the sky temperature is calculated with an hourly time step. As the value of $\theta_c^{-1}$ in the method is set to $T_{\text{mean}} \forall t$, the estimation of $I_{\text{rad}}$ only fluctuate according to $T_{\text{sky}}^t$. In the other hand, the orange curve representing the second linearization method fits all variations with a good approximation.
The mean absolute error (18) obtained during a month of winter with $T_{\text{mean}}$ set to $T_{\text{set}} = 19^\circ\text{C}$ is 0.89% for the first method and 0.0013% for the second method.

$$\text{Error} = \frac{1}{\text{period length}} \sum_{\text{period}} \left| \frac{\hat{I}_{\text{rad}} - I_{\text{rad}}}{|I_{\text{rad}}|} \right|$$

In these cases, the approximation of the mean value of $\theta_c$ ($19^\circ\text{C}$) is very close to the real mean temperature ($18.9^\circ\text{C}$). However, this method was applied to a thermally controlled building, so that the mean temperature of the surfaces is not supposed to vary much. In cases of non-controlled buildings or during load shedding strategies, higher variations of temperature may occur on the surfaces. Therefore, the prediction was realized again with an estimation of $T_{\text{mean}}$ with variations of +/- 1$^\circ\text{C}$ and +/- 4$^\circ\text{C}$, with the same real mean temperature. The results are shown in Table 1.

<table>
<thead>
<tr>
<th>$T_{\text{mean}}$</th>
<th>Error - lin. 1</th>
<th>Error - lin. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>15$^\circ\text{C}$</td>
<td>15 %</td>
<td>0.30 %</td>
</tr>
<tr>
<td>18$^\circ\text{C}$</td>
<td>3.3 %</td>
<td>0.014 %</td>
</tr>
<tr>
<td>19$^\circ\text{C}$</td>
<td>0.89 %</td>
<td>0.0013 %</td>
</tr>
<tr>
<td>20$^\circ\text{C}$</td>
<td>4.8 %</td>
<td>0.028 %</td>
</tr>
<tr>
<td>23$^\circ\text{C}$</td>
<td>17 %</td>
<td>0.35 %</td>
</tr>
</tbody>
</table>

2. Minimizing the peak consumption power

For an eco-district, lowering its environmental impact is very important, so that minimizing the CO$_2$ emissions related to the heating consumption of the buildings may be more and more investigated. However, the CO$_2$ emissions generated for the production of a kilowatt-hour is time-dependent, according to the production mix needed to match the consumption. Shifting electrical loads to low-pollution periods can easily be realized in case of a building thermally fed by heat pumps and will be explored in this paper.

The CO$_2$ emissions are calculated from the consumption of the heat pump, according to the hourly emission of the French electrical system (19).

$$CO_2\text{em.} = \frac{1}{COP_{HP}} \sum_{\text{January}} c_{o_2} \Phi_{hc}^t$$

The CO$_2$ emissions of the French electrical system ($c_{o_2}$) are taken during the year 2017, while the power needs ($\Phi_{hc}^t$) result from the optimal strategy. In this case, the heat provided to the heat pumps comes from groundwater whose temperature can be considered as constant, so that COP$_{HP}$ too.

Then, a more local point of view was adopted with the minimization of the peak consumption power. Peak shaving strategies can be crucial for distribution system operators, as they can avoid congestion on the power lines and reduce the risk of instability on the entire power system. For this reason, the second objective studied in this paper is the minimization of the peak consumption power required for the building heating needs.

From an electric point of view, the time of the year when the grid is the most at risk in France is the winter. For this reason, both of these objectives will be applied for the month of January. During this period, the dynamic was modeled with a time step of 10 minutes. Moreover, to ensure simplicity in the formulation, this paper focuses on the operation of the heat pump of a single building of the block.

As an optimal operation of the heat pump could affect the thermal comfort of the building occupants, a specific constraint has been defined. In order to express the thermal comfort, this paper relies on the operative temperature calculated as the mean value between the temperatures representing the radiative and the convective heat flows for the occupants, according to the approximation defined by ASHRAE (2013). With the definition of $\theta_c$, the norm calculates $T_{\text{operative}}$ as follows (20):

$$T_{\text{operative}} = 0.69 \times \theta_c + 0.31 \times T_{\text{int}}$$

The constraint ensures that the operative temperature, stays between +/- 1$^\circ\text{C}$ around the temperature set-point.

Then, all the external and internal flows are required in order to build the thermal model. Internal flows.
are usually estimated thanks to occupancy prediction (see Figure 4), while the external heat flow rates rely on weather data.

![Figure 4: Internal heat flow rates per area](image)

In this study, representative weather files for the location were found online (EnergyPlus (2018)) and extracted for the month of January.

Data relative to the building were obtained from mandatory studies for the construction of new residential buildings. An occupancy schedule was realized and provides us the internal gains of the building per surface (Figure 4).

Standard data about the envelope of the building were also available and the most important parameters (U-values, areas, emissivity: \( \epsilon \) and absorptivity: \( \alpha \)) are summarized in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>U [W/(m² K)]</th>
<th>A [m²]</th>
<th>( \epsilon )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Windows</td>
<td>1.1</td>
<td>520</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Walls</td>
<td>0.18</td>
<td>1990</td>
<td>0.9</td>
<td>0.6</td>
</tr>
<tr>
<td>Roof</td>
<td>0.12</td>
<td>263</td>
<td>0.9</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Once the model built, three optimization problems were launched: a reference scenario, the minimization of the CO\(_2\) emissions and the minimization of the maximal peak power. For comparison, the operation for reference scenario corresponds to the heating supply providing the least variation of the operative temperature around its set-point.

Results

In this section, the results for optimal operation of the heat pump of a new residential building during January are detailed. First two mono-objective approaches are investigated in order to study both the environmental and financial objectives. Then, the results from a multi-objectives study are presented to find some trade-offs between these points of view.

Mono-objective optimizations

As explained before, a reference scenario was defined as the heat pump operation needed to lower the gap between the operative temperature and its set-point. Then, two optimal scenarios respectively minimizing the consumption peak and the CO\(_2\) emissions have been studied through a MILP formulation.

With 93745 variables (80353 continuous and 13392 binary) and 271859 non-zeros, this optimization problem was solved within 32 seconds with the Gurobi solver on an Intel bicore i5 2.4 GHz CPU. The results obtained from the three operation strategies can be found in Table 3, in terms of electrical peak power (\( P_{\text{elec peak}} \)), CO\(_2\) emissions (CO\(_2\) em.), electrical consumption (Elec cons.) and mean operative temperature (\( T_{\text{mean}}^{\text{op}} \)). For comparison to other buildings, it can be added that the considered building includes an area of 3436m\(^2\) heated by a heat pump with a coefficient of performance equals to 4.

![Figure 5: Pareto diagram for trade-off between CO\(_2\) emissions reduction and electrical peak shaving](image)

As we can see, the reference scenario is the worst from three points of views (the emissions of CO\(_2\), the electrical peak power and electrical consumption). Even from the operative temperature perspective, 0.1°C more on average are obtained by minimizing the peak power.

For this first objective, the heat pump operation allows dividing the peak power by 11 (91% of diminution), while reducing both the CO\(_2\) emissions and the energy consumption, from 43%. In order to reach a reduction of the CO\(_2\) emissions from 53% with the second optimization, the power peak decreases from 67% and the energy consumption is lowered with 52%.

In both cases, the energy consumption decreases as much as CO\(_2\) emissions, which suggests that reducing the energy consumption could lead to a similar reduction of the CO\(_2\) emissions.

Multi-objectives optimization

In order to find trade-offs between the two points of views, study cases with weighted objectives have been realized and are shown on a Pareto diagram (Figure 5). Thus, several possibilities can be found allowing stakeholders to choose the better compromise according to them, between peak shaving and the desire of minimizing the CO\(_2\) emissions. These trade-off scenarios can be found in the bottom-left area of the Figure 5, where increasing the peak allows to consumed more during low-CO\(_2\) periods and vice-versa.

<table>
<thead>
<tr>
<th></th>
<th>Ref.</th>
<th>Obj. peak</th>
<th>Obj. CO(_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{\text{elec peak}} )</td>
<td>50.0 kW</td>
<td>4.41 kW</td>
<td>16.7 kW</td>
</tr>
<tr>
<td>CO(_2) em.</td>
<td>173 kg</td>
<td>98.6 kg</td>
<td>82.0 kg</td>
</tr>
<tr>
<td>Elec cons.</td>
<td>12 MWh</td>
<td>6.9 MWh</td>
<td>5.8 MWh</td>
</tr>
<tr>
<td>( T_{\text{mean}}^{\text{op}} )</td>
<td>19.0°C</td>
<td>19.1°C</td>
<td>18.7°C</td>
</tr>
</tbody>
</table>
Finally, a specific trade-off providing an electrical peak power of 5.70 kW, while emitting 82.7 kg of CO₂ was selected (see Figure 5). The heat pump operation is drawn Figure 6 in line with the dynamic CO₂ emissions from the French electrical system (during 2017) and the operative temperature of the building, which must remain between 18°C and 20°C.

**Perspectives**

As explained previously, this work was based on a low-level RC-model, with one capacitance. As these elements represent the thermal storage capacity of the building, further work is needed to confirm the accuracy of the results. To do so, this approach could be confronted with the results obtained with a highly-detailed simulation model, by applying the optimal strategies obtained and evaluate the same indicators (power peak, CO₂ emissions, electrical consumption and mean operative temperature).

Another perspective is to extend this methodology to the entire block of buildings. This step only requires the envelope data from the buildings (U-values, areas, emissivity, and absorptivity of the walls and windows), as well as the internal gains.

**Conclusion and perspectives**

**Conclusion**

In this paper, two main subjects have been developed. A generic methodology for quick generation of MILP optimization models for DSM on buildings heating loads at the district scale, suitable for any building type. A low-level RC model dedicated for thermal dynamic simulation was converted into a MILP optimization model, by applying two linearization methods of the heat flow rate Re-irradiated to the sky ($I_{rad}$). Providing the best estimation, the second method, based on a Taylor development at the first order, was kept for an illustration on the study case. The application of the optimization models on a new residential building, in order to find the best operation of the heat pump according to two criteria: the CO₂ emissions and the peak power. First, the two objectives have been studied separately and compared to a reference scenario. Then, combining the two point of views was investigated by weighting the objectives. These results were shown in a Pareto diagram and a trade-off was selected for further studies. Finally, dynamic results of the heat pump operation were presented for a 48-hour period.

**Figure 6: Heat pump operation for the selected trade-off according to CO₂ rate and operative temperature**

It can be noticed that the heat pump is operating preferably during the low-CO₂ emissions periods while being constrained by the operative temperature range to maintain. By considering a trade-off instead of the solution of minimizing the CO₂ emissions, the power peaks during low-CO₂ periods are limited to 5.70 kW, instead of 16.7kW.

**Nomenclature**

- $A_x$: Area of the element x
- $A_{sol}^x$: Effective solar collecting area of x
- $C_m$: Internal heat capacity
- $CO_2$: CO₂ emissions of a French electrical kWh at time t (during 2017)
- $CO_2_{em}$: Total CO₂ emissions
- $COP_{HP}$: Heat pump coefficient of performance
- $ε_x$: Emissivity of the element x
- $F_F$: Frame area faction coefficient
- $F_{f,x}$: Form factor of the element x
- $f_{cin}^x$: SIA 2044 internal coefficient for the node c/m
- $f_{ra}^a$: SIA 2044 coefficient for appliances
- $f_{rl}^c$: SIA 2044 coefficient for lightning
- $f_{rp}^a$: SIA 2044 coefficient for occupancy
- $f_{sa}^c$: SIA 2044 external coefficient for the node c/m
- $h_{rad,x}$: External radiative heat transfer coefficient for the element x
- $H_{XY}$: Thermal transmission coefficient between the nodes X and Y
- $I_{rad}$: Estimation of the heat flow rate re-irradiated to the sky
- $I_{rad,x}$: Heat flow rate re-irradiated to the sky from the element x
- $I_{sol}$: Net solar radiation to the building
- $I_{sol}$: Average value of the net solar radiation to the building
- $Φ_a$: Heat flow rate at the node a
- $Φ_{ext}^a$: External heat flow rate at the node a
- $Φ_{int}^a$: Internal heat flow rate at the node a
- $Φ_c$: Heat flow rate at the node c
- $Φ_m$: Heat flow rate at the node m
\( \Phi_{\text{ext}} \) \( \frac{c}{m} \) External heat flow rate at the node
\( \Phi_{\text{int}} \) \( \frac{c}{m} \) Internal heat flow rate at the node
\( \Phi_{\text{h,c}} \) Building heating/cooling power
\( \Phi_{\text{h,ce}} \) Convective part of heating/cooling
\( \Phi_{\text{h,r}} \) Radiative part of heating/cooling
\( \Phi_{\text{ia}} \) Internal gains from appliances
\( \Phi_{\text{il}} \) Internal gains from lightning
\( \Phi_{\text{ip}} \) Internal gains from occupancy
\( \Phi_{\text{trans}} \) Heat flow rate due to transmission through the envelope at the node \( k \)
\( \Phi_{\text{ex}} \) External heat gains
\( \Phi_{\text{vent}} \) Heat flow rate due to ventilation
\( \text{RSE} \) Thermal resistance of external surfaces according to ISO 6946
\( \sigma \) Stefan-Boltzmann constant
\( \theta_{\text{ea}} \) Temperature at the node \( e_a \) [°C]
\( \theta_{\text{ec}} \) Temperature at the node \( e_c \) [°C]
\( \theta_{\text{em}} \) Temperature at the node \( e_m \) [°C]
\( \theta_x \) Temperature at the node \( x \) [°C]
\( \theta_m \) Temperature at the thermal mass [°C]
\( T_{\text{int}} \) Temperature of the ambient air [°C]
\( T_{\text{mean}} \) Estimation of the mean temperature of the surfaces [°C]
\( T_{\text{operative}} \) Operative temperature of the zone
\( T_{\text{sky}} \) Temperature of the sky [K]
\( U_{\text{x}} \) Thermal transmittance of \( x \)

References


Intergovernmental Panel on Climate Change (2018). Summary - Global Warming of 1.5°C.


