Conversation and Games

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Abstract. In this paper we summarize concepts from earlier work and demonstrate how infinite sequential games can be used to model strate-gic conversations. Such a model allows one to reason about the structure and complexity of various kinds of winning goals that conversationalists might have. We show how to use tools from topology, set-theory and logic to express such goals. We then show how to tie down the notion of a winning condition to specific discourse moves using techniques from Mean Payoff games and discounting. We argue, however, that this still requires another addition from epistemic game theory to define appro-priate solution and rationality underlying a conversation.

Keywords: Strategic reasoning · Conversations · Dialogues · Infinite games · Epistemic game theory

1 Introduction

Conversations have a natural analysis as games. They involve typically at least two agents, each with their own interests and goals. These goals may be compatible, or they may conflict; but in either case, one agents' successfully achieving her conversational goals will typically depend upon her taking her interlocutor's goals and interests into account. In cooperative conversations where agents' goals are completely aligned, conversational partners may still need to coordinate actions, even linguistic actions. A strategic or non-cooperative conversation involves (at least) two people (agents) who have opposing interests concerning the outcome of the conversation. A debate between two political candidates is an instance. Each candidate has a certain number of points to convey to the audience, and each wants to promote her own position and damage her opponent's or opponents'. To achieve these goals, each participant typically needs to plan for anticipated responses from the other.

This paper surveys some results from what we feel is an exciting new application of games to language. The core of formal results are summarized from [4,6]; the part on weighted and discounted games draws from [3] but also introduces new material; the last section points to work in progress.

Various game-theoretic models for cooperative conversation have been proposed, most notably the model of signalling games [22]. Another closely related

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model is that persuasion games [15]. In a signalling game one player with a knowledge of the actual state sends a signal and the other player who has no knowledge of the state chooses an action, usually upon an interpretation of the received signal. The standard setup supposes that both players have common knowledge of each other's preference profiles as well as their own over a set of commonly known set of possible states, actions and signals. However for modeling non-cooperative strategic contexts of sequential dynamic games, signalling games suffer from many drawbacks. We summarise below the difficulties we see (see [6] for a more comprehensive discussion):

- A game that models a non-cooperative setting, that is a setting where the preferences of the players are opposed, must be zero-sum. However, it has been shown [11] that in a zero-sum criterion, in equilibrium, the sending and receiving of any message has no effect on the receiver's decision. Signaling games typically assign a game a finite horizon; backward inductions arguments threaten to conclude that communication should not occur in such situations.
- In order to use games as part of a general theory of meaning, one has to make clear how to construct the game-context, which includes providing an interpretation of the game's ingredients (types, messages, actions). Franke's extension of signalling, games, *interpretation games*, addresses this issue [13]. Such games encode a 'canonical context' for an utterance, in which relevant conversational implicatures may be drawn. The game structure is determined by the set of 'sender types'. Interpretation games model the interpretation of the messages and actions of a signaling game in a co-operative context for 'Gricean agents' quite well. But in the non-cooperative setting, things get very intricate and problems remain.
- Signalling games are one-shot and fail to capture the dynamic nature of a strategic conversation. One can attempt to encode a finite sequence of moves of a particular player as a single message m sent by that player but then one runs into the problem of assigning correct utilities for m because such utilities depend again on the possible set of continuations of m.
- Finally, there is an inherent asymmetry associated with the setting of a signalling game - one player is informed of the state of the world but the other is not; one player sends a message but the other does not. Conversations (like debates), on the other hand, are symmetric - all participants should (and usually do) get equal opportunities to get their messages across.

Strategic conversations are thus special and have characteristics unique to them which, to our knowledge, have not been captured in other frameworks. Here is a short list of these characteristics:

- Conversations are sequential and dynamic and inherently involve a 'turn-structure' which is important in determining the merit of a conversation to the participants. In other words, it is important to keep track of "who said what".
- A 'move' by a player in a linguistic game typically carries more semantic content than usually assumed in game theory. What a player says may have a

set of 'implicatures', may be 'ambiguous', may be 'coherent/incoherent' or 'consistent/inconsistent' with regards to what she had said earlier in the conversation. She may also 'acknowledge' other people's contributions or 'retract' her previous assertions. These features too have important consequences on the existence and complexity of winning strategies.

– Conversations typically have a 'Jury' who evaluates the conversation and determines if one or more of the players have reached their goals. In other words a Jury determines the winners in a conversation, if there is a winner. Players will spin the description of the game to their advantage and so may not present an accurate view of what happened. The Jury can be a concrete or even a hypothetical entity who acts as a 'passive player' in the game. For example, in a courtroom situation there is a physical Jury who gives the verdict, whereas in a political debate the Jury is the audience or the citizenry in general. This means that the winning conditions of the players are affected by the Jury in that, they depend on what they believe that the Jury expects them to achieve.
– Conversations do not have a 'set end'. When two or more people engage in a conversation they do not know at the outset how many turns it will last or how many chances each player will get to speak (if at all). In a more scripted conversation like a political debate or a courtroom debate, there may be a moderator whose job is to ensure that each player receives his or her fair chance to put their points across; but even such a moderator may not know at the outset how the conversation will unfold and how many turns each player will receive. Players thus cannot strategize for a set horizon while starting a conversation. This rules out backward induction reasoning for both the players and analysts of conversation.
– Finally, epistemic elements are a natural component of such games. The players and the Jury have 'types', and players have 'beliefs' about the types of the other players and the Jury. They strategize based on their beliefs and also update their beliefs after each turn.

The first four considerations led [6] to model conversations as infinite games over a countable 'vocabulary' V. They call such games *Message Exchange games* (ME games). The intuitive idea behind an ME game is that a conversation proceeds in turns where in each turn one of the players 'speaks' or plays a string of letters from her own "vocabulary". The two vocabularies are distinguished in order to keep track of who said what, which is crucial to the analysis of a conversation. We will assume that both players use the same expressions in a set V to communicate, but that when 0 uses a symbol $v \in V$, she is actually playing $(v, 0)$, which allows us to see that it was 0 that played v at a certain point in the sequence; and when 1 plays v, he's actually playing $(v, 1)$.

However, a conversationalist does not play just any sequence of arbitrary strings but sentences or sets of sentences that 'make sense'. To ensure this, the vocabulary V should have a built-in, exogenous semantics. [6] identify V with the language of a semantic theory for discourse, SDRT [1]. SDRT's language characterizes the semantics and pragmatics of moves in dialogue. This means that we can exploit the notion of entailment associated with the language of SDRSs to track commitments of each player in an ME game. In particular, the language of SDRT features variables for dialogue moves that are characterized by contents that the move commits its speaker to. Crucially, some of this content involves predicates that denote rhetorical relations between moves—like the relation of *question answer pair* (qap), in which one move answers a prior move characterized by a question. The vocabulary V of an ME game thus contains a countable set of discourse constituent labels $\mathsf{DU} = \{\pi, \pi_1, \pi_2, \ldots\}$, and a finite set of discourse relation symbols $\mathcal{R} = \{\mathsf{R}, \mathsf{R}_1, \ldots \mathsf{R}_n\}$, and formulas $\phi, \phi_1, \ldots$ from some fixed language for describing elementary discourse move contents. V consists of formulas of the form $\pi\colon \phi$, where ϕ is a description of the content of the discourse unit labelled by π in a logical language like the language of higher order logic used, e.g., in Montague Grammar, and $\mathsf{R}(\pi, \pi_1)$, which says that π_1 stands in relation R to π. One such relation R is qap. Thus, each discourse relation represented in V comes with constraints as to when it can be coherently used in a context and when it cannot.

2 Message Exchange Games

We now formally define Message Exchange games, state some of their properties and show how they model strategic conversations, as explored in [6]. For simplicity, we restrict our description to conversations with two participants, whom we denote by Player 0 and Player 1. It is straightforward to generalize ME games to the case where there are more than two players. Thus, in what follows, we let i range over the set of players $\{0, 1\}$. Furthermore, Player $-i$ will always denote Player $(1 - i)$, the opponent of Player i.

We first define the notion of a 'Jury'. As noted in Sect. 1, a Jury is an entity or a group of entities that evaluates a conversation and decides the winner. A Jury thus 'groups' instances of conversations as being winning for Player 0 or Player 1 or both.

For any set A let A^* be the set of all finite sequences over A and let A^ω be the set of all countably infinite sequences over A. Let $A^\infty = A^* \cup A^\omega$ and $A^+ = A^* \backslash \{\epsilon\}$. Now, let V be a vocabulary as defined at the end of Sect. 1 and let $V_i = V \times \{i\}$.

Definition 1. *A* Jury $\mathcal{J}$ *over* $(V_0 \cup V_1)^\omega$ *is a tuple* $\mathcal{J} = (\mathsf{win}_1, \mathsf{win}_2)$ *where* $\mathsf{win}_i \subseteq (V_0 \cup V_1)^\omega$ *is the* winning condition *or* winning set *for Player* i*.*

Given the definition of a Jury over $(V_0 \cup V_1)^\omega$ we define a Message Excahge game game as:

Definition 2. *A Message Exchange game (ME game)* $\mathcal{G}$ *over* $(V_0 \cup V_1)^\omega$ *is a tuple* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ *where* $\mathcal{J}$ *is a Jury over* $(V_0 \cup V_1)^\omega$*.*

Formally an ME game $\mathcal{G}$ is played as follows. Player 0 starts the game by playing a non-empty sequence in V_0^+. The turn then moves to Player 1 who plays a non-empty sequence from V_1^+. The turn then goes back to Player 0 and so on. The game generates a play ρ_n after n (≥ 0) turns, where by convention, $\rho_0 = \epsilon$ (the empty move). A play can potentially go on forever generating an infinite play ρ_ω, or more simply ρ. Player i wins the play ρ iff $\rho \in \mathsf{win}_i$. $\mathcal{G}$ is $\mathsf{zero\text{-}sum}$ if $\mathsf{win}_i = (V_0 \cup V_1)^\omega \backslash \mathsf{win}_{-i}$ and is $\mathsf{non\ zero\text{-}sum}$ otherwise. Note that both player or neither player might win a non zero-sum ME game $\mathcal{G}$. The Jury of a zero-sum ME game can be denoted simply as win where by convention $\mathsf{win} = \mathsf{win}_0$ and $\mathsf{win}_1 = (V_0 \cup V_1)^\omega \backslash \mathsf{win}$.

The basic structure of an ME game means that plays are segmented into *rounds*—a move by Player 0 followed by a move by Player 1. A finite play of an ME game is (also) called a $\mathsf{history}$, and is denoted by ρ. Let Z be the set of all such histories, $Z \subseteq (V_0 \cup V_1)^*$, where $\epsilon \in Z$ is the empty history and where a history of the form $(V_0 \cup V_1)^+ V_0^+$ is a 0-history and one of the form $(V_0 \cup V_1)^+ V_1^+$ is a 1-history. We denote the set of i-histories by Z_i. Thus $Z = Z_0 \cup Z_1$. For $\rho \in Z$, $\mathsf{turns}(\rho)$ denotes the total number of turns (by either player) in ρ. A $\mathsf{strategy}$ σ_i of Player i is thus a function from the set of $-i$-histories to V_i^+. That is, $\sigma_i : Z_{-i} \to V_i^+$. A play $\rho = x_0 x_1 \ldots$ of an ME game $\mathcal{G}$ is said to $\mathsf{conform}$ to a strategy σ_i of Player i if for every prefix ρ_j of ρ, $j = i (\mod 2)$ implies $\rho_{j+1} = \rho_j \sigma_i(\rho_j)$. A strategy σ_i is called $\mathsf{winning}$ for Player i if $\rho \in \mathsf{win}_i$ for every play ρ that conforms to σ_i.

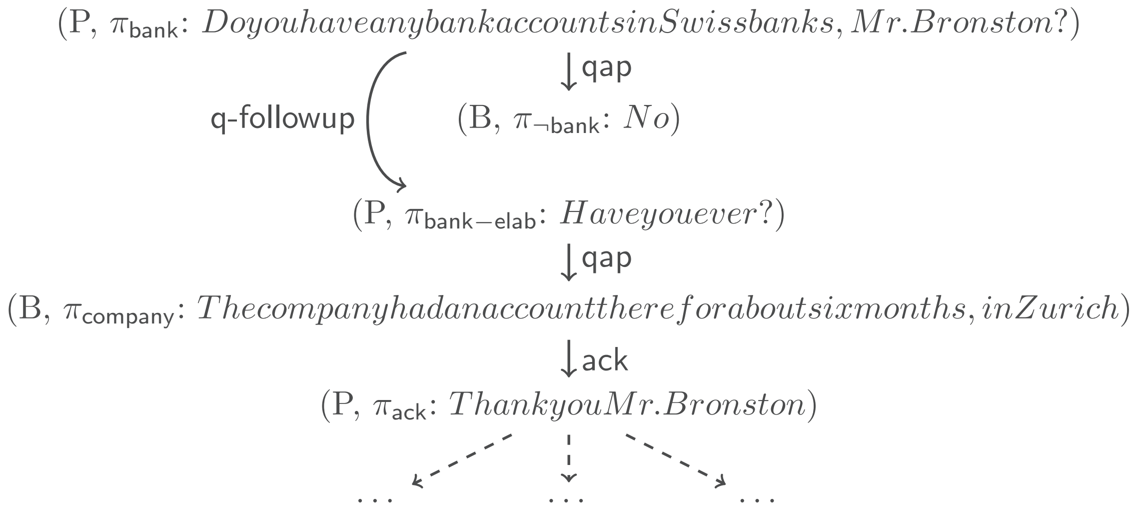
Given how we have characterized the vocabulary $(V_0 \cup V_1)$, we have a fixed meaning assignment function from EDUs to formulas describing their contents. Then, a sequence of conversational moves can be represented as a graph (DU, E, ℓ), where DU is the set of vertices each representing a discourse unit, $E \subseteq \mathsf{DU} \times \mathsf{DU}$ a set of edges representing links between discourse units that are labeled by $\ell : E \to \mathcal{R}$ with discourse relations.[1]

Example 1. To illustrate this structure of conversations, consider the following example taken from [2] from a courtroom proceedings where a prosecutor is querying the defendant. We shall return to this example later on for a strategic analysis.

a. **Prosecutor:** *Do you have any bank accounts in Swiss banks, Mr. Bronston?*
b. **Bronston:** *No, sir.*
c. **Prosecutor:** *Have you ever?*
d. **Bronston:** *The company had an account there for about six months, in Zurich.*
e. **Prosecutor:** *Thank you Mr. Bronston.*

Example 2. We can view the conversation in Eg. 1 as a play of an ME game as follows.

[1] We note that this is a simplification of SDRT which also countenances complex discourse units (CDUs) and another set of edges in the graph representation, linking CDUs to their simpler constituents. These edges represent parthood, not rhetorical relations. We will not, however, appeal to CDUs here.



The picture shows a weakly connected graph with a set of discourse constituent labels

$$\mathsf{DU} = \{\pi_{\mathsf{bank}}, \pi_{\neg\mathsf{bank}}, \pi_{\mathsf{bank-elab}}, \pi_{\mathsf{company}}, \pi_{\mathsf{ack}}, \ldots\}$$

and a set of relations

$$\mathcal{R} = \{\mathsf{qap}, \mathsf{q-followup}, \mathsf{ack}, \ldots\}$$

The arrows depict the individual relation instances between the DUs. A weakly connected graph represents a fully coherent conversation, in which each player's contribution is coherently linked with a preceding one. The graph also reveals that each player responds to a contribution of the other; this is a property that [6] call *responsiveness* (*vide infra*).

ME game messages come with a conventionally associated meaning in virtue of the constraints enforced by the Jury; an agent who asserts a content of a message commits to that content, and it is in virtue of such commitments that other agents respond in kind. While SDRT has a rich language for describing dialogue moves, earlier work did not make explicit how dialogue moves explicitly affect the commitments of the agents who make the moves or those who observe the moves. [24,25] link the semantics of the SDRT language with commitments explicitly (in two different ways). They augment the SDRT language with formulas that describe the commitments of dialogue participants, using a simple propositional modal syntax. Thus for any formula ϕ in the language of dynamic semantics that describes the content of a label $\pi \in \mathsf{DU}$, they add:

$$\neg\phi \mid \phi_1 \vee \phi_2 \mid \mathsf{C}_i\phi,\ i \in \{0,1\} \mid \mathsf{C}^*\phi$$

with the derived operators $\wedge, \implies, \top, \bot$ are defined as usual, providing a propositional logic of commitments over the formulas that describe labels. Of particular interest are the commitment operators C_i and C^*. If ϕ is a formula for describing a content, $\mathsf{C}_i\phi$ is a formula that says that Player i commits to ϕ and $\mathsf{C}^*\phi$ denotes 'common commitment' of ϕ. Commitment is modelled as a Kripke modal operator via an alternativeness relation in a pointed model with a distinguished (actual) world w_0. This allows them to provide a semantics for discourse moves that links the making of a discourse move by an agent to her commitments: i's assertion of a discourse move ϕ, for instance, we will assume, entails a common commitment that i commits to ϕ, written $\mathsf{C}^*\mathsf{C}_i\phi$. They show how each discourse move ϕ defines an action, a change or update on the model's commitment structure; in the style of public announcement logic viz. [8,9]. For instance, if agent i asserts ϕ, then the commitment structure for the conversational participants is updated such so as to reflect the fact that $\mathsf{C}^*\mathsf{C}_i\phi$. Finally, they define an entailment relation $\models$ that ensures that $\phi \models \mathsf{C}^*\mathsf{C}_i\phi$. This semantics is useful because it allows us to move from sequences of discourse moves to sequences of updates on any model for the discourse language. See [24,25] for a detailed development and discussion.

ME games resemble infinite games like Banach Mazur or Gale-Stewart games that have been used in topology, set theory [18] and computer science [16]. We can leverage some of the results from these areas to talk about the general 'shape' of conversations or to analyse the complexity of the winning conditions of the players in ME games. For instance, [23] shows that ME games, like Banach Mazur games or Gale-Stewart games, are determined. Other features have been extensively explored in [6]. We give a flavor of some of the applications here.

To do that we first need to define an appropriate topology on $(V_0 \cup V_1)^\omega$ which will allow us to characterize the descriptive complexity of the winning sets win_0 and win_1. We proceed as follows. We define the topology on $(V_0 \cup V_1)^\omega$ by defining the open sets to be sets of the form $A(V_0 \cup V_1)^\omega$ where $A \subseteq (V_0 \cup V_1)^*$. Such an open set will be often denoted as $\mathcal{O}(A)$. When A is a singleton set $\{x\}$ (say), we abuse notation and write $\mathcal{O}(\{x\})$ as $\mathcal{O}(x)$. The Borel sets are defined as the sigma-algebra generated by the open sets of this topology. The Borel sets can be arranged in a natural hierarchy called the Borel hierarchy which is defined as follows. Let Σ_1^0 be the set of all open sets. $\Pi_1^0 = \overline{\Sigma_1^0}$, the complement of the set of Σ_1^0 sets, is the set of all closed sets. Then for any $\alpha > 1$ where α is a successor ordinal, define Σ_α^0 to be the countable union of all $\Pi_{\alpha-1}^0$ sets and define Π_α^0 to be the complement of Σ_α^0. $\Delta_\alpha^0 = \Sigma_\alpha^0 \cap \Pi_\alpha^0$.

Definition 3 [18]. *A set A is called* complete *for a class Σ_α^0 (resp. Π_α^0) if $A \in \Sigma_\alpha^0 \setminus \Pi_\alpha^0$ (resp. $\Pi_\alpha^0 \setminus \Sigma_\alpha^0$) and $A \notin (\Sigma_\beta^0 \cup \Pi_\beta^0)$ for any $\beta < \alpha$.*

The Borel hierarchy represents the descriptive or structural complexity of the Borel sets. A set higher up in the hierarchy is structurally more complex than one that is lower down. Complete sets for a particular class of the hierarchy represent the structurally most complex sets of that class. We can use the Borel hierarchy and the notion of completeness to capture the complexity of winning conditions in conversations. For example, two typical sets in the fist level of the Borel hierarchy are defined as follows. Let $A \subseteq (V_0 \cup V_1)^+$, then

$$\mathsf{reach}(A) = \{\rho \in (V_0 \cup V_1)^\omega | \rho = xy\rho', y \in A\}$$

and

$$\mathsf{safe}(A) = (V_0 \cup V_1)^\omega \setminus \mathsf{reach}(A)$$

A little thought shows that $\mathsf{reach}(A) \in \Sigma_1^0$ and $\mathsf{safe}(A) \in \Pi_1^0$. Let reachability be the class of sets of the form $\mathsf{reach}(A)$ and safety be the class of sets of the form $\mathsf{safe}(A)$.

Example 3. Returning to our example of Bronston and the Prosecutor, let us consider what goals the Jury expects each of them to achieve. The Jury will award its verdict in favor of the Prosecutor: (i) if he can eventually get Bronston to admit that (a) he had an account in Swiss banks, or (b) he never had an account in Swiss banks, or (ii) if Bronston avoids answering the Prosecutor forever. In the case of (i)a, Bronston is incriminated, (i)b, he is charged with perjury and (ii), he is charged with contempt of court. Bronston's goal is the complement of the above, that is to avoid either of the situations (i)a, (i)b and (ii). We thus see that the Jury winning condition for the Prosecutor is a boolean combination of a reachability condition and the complement of a safety condition, which is in the first level of the Borel hierarchy.

Conversations typically must also satisfy certain natural constraints which the Jury might impose throughout the course of a play. Here are some constraints defined in [6]. We will then study the complexity of the sets satisfying them.

Let $\rho = x_0x_1x_2\dots$ be a play of an ME game $\mathcal{G}$ where $x_0 = \epsilon$ and $x_j \in V^+_{((j-1) \bmod 2)}$ is the sequence played by Player $((j-1) \bmod 2)$ in turn j. For every i define the function $\mathsf{du}_i : V_i^+ \to \wp(\mathsf{DU})$ such that $\mathsf{du}_i(x_j)$ gives the set of contributions (in terms of DUs) of Player i in the jth turn. By convention, $\mathsf{du}_i(x_j) = \emptyset$ for $x_j \in V^+_{-i}$.

Definition 4. *Let $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ be an ME game over $(V_0 \cup V_1)^\omega$. Let $\rho = x_0x_1x_2\dots$ be a play of $\mathcal{G}$. Then*

Consistency: ρ is consistent *for Player i if the set $\{\mathsf{du}_i(x_j)\}_{j>0}$ is consistent. Let* CONS_i *denote the set of consistent plays for Player i in $\mathcal{G}$.*

Coherence: Player i is coherent *on turn $j > 0$ of play ρ if for all $\pi \in \mathsf{du}_i(x_j)$ there exists $\pi' \in (\mathsf{du}_i(x_k) \cup \mathsf{du}_{-i}(x_{k-1}))$ where $k \leq j$ such that there exits $\mathsf{R} \in \mathcal{R}$ such that $(\pi' \mathsf{R} \pi \vee \pi \mathsf{R} \pi')$ holds. Let* COH_i *denote the set of all coherent plays for Player i in $\mathcal{G}$.*

Responsiveness: Player i is responsive *on turn $j > 0$ of play ρ if there exists $\pi \in \mathsf{du}_j(x_j)$ such that there exits $\pi' \in \mathsf{du}_{-i}(x_{j-1})$ such that $\pi' \mathsf{R} \pi$ for some $\mathsf{R} \in \mathcal{R}$. Let* RES_i *denote the set of responsive plays for Player i in $\mathcal{G}$. x_j (or abusing notation, π) will be sometimes called a* response move*.*

Rhetorical-cooperativity: Player i is rhetorically-cooperative *in ρ if she is both coherent and responsive in every turn of hers in ρ. ρ is rhetorically-cooperative if both the players are rhetorically-cooperative in ρ. Let* RC_i *denote the set of rhetorically-cooperative plays for Player i in $\mathcal{G}$ and let* RC *be the set of all rhetorically-cooperative plays.*

To define two more constraints, NEC and CNEC, we need definitions of an 'attack' and a 'response'.

Definition 5. *Let $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ be an ME game over $(V_0 \cup V_1)^\omega$. Let $\rho = x_0x_1x_2\dots$ be a play of $\mathcal{G}$. Then*

Attack: $\mathsf{attack}(\pi', \pi)$ *on Player* $-i$ *holds at turn* j *of Player* i *just in case* $\pi \in \mathsf{du}_i(x_j)$, $\pi' \in \mathsf{du}_{-i}(x_k)$ *for some* $k \leq j$, *there is an* $\mathsf{R} \in \mathcal{R}$ *such that* $\pi' \mathsf{R} \pi$ *and: (i)* π' *entails that* $-i$ *is committed to* ϕ *for some* ϕ, *(ii)* ϕ *entails that* $\neg\phi$ *holds. In such a case, we shall often abuse notation and denote it as* $\mathsf{attack}(k, j)$. *Furthermore,* x_j *or alternatively* π *shall be called an* attack move. *An attack move is* relevant *if it is also a* response *move.* $\mathsf{attack}(k, j)$ *on* $-i$ *is* irrefutable *if there is no move* $x_\ell \in V_{-i}$ *in any turn* $\ell > j$ *such that* $\mathsf{attack}(j, \ell)$ *holds and* $x_0 x_1 \ldots x_\ell$ *is consistent for* $-i$.

Response: $\mathsf{response}(\pi', \pi)$ *on Player* $-i$ *holds at turn* j *of Player* i *if there exits* $\pi'' \in \mathsf{du}_i(x_\ell)$, $\pi' \in \mathsf{du}_{-i}(x_k)$ *and* $\pi \in \mathsf{du}_i(x_j)$ *for some* $\ell \leq k \leq j$, *such that* $\mathsf{attack}(\pi'', \pi')$ *holds at turn* k *of Player* $-i$, *there exists* $\mathsf{R} \in \mathcal{R}$ *such that* $\pi' \mathsf{R} \pi$ *and* π *implies that (i) one of* i*'s commitments* ϕ *attacked in* π' *is true or (ii) one of* $-i$*'s commitments in* π' *that entails that* i *was committed to* $\neg\phi$ *is false. We shall often denote this as* $\mathsf{response}(k, j)$.

Definition 6. *Let* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ *be an ME game over* $(V_0 \cup V_1)^\omega$. *Let* $\rho = x_0 x_1 x_2 \ldots$ *be a play of* $\mathcal{G}$. *Then*

NEC*:* NEC *holds for Player* i *in* ρ *on turn* j *if for all* $\ell, k,\ \ell \leq k < j$, *such that* $\mathsf{attack}(\ell, k)$, *there exists* $m,\ k < m \leq j$, *such that* $\mathsf{response}(k, m)$. NEC *holds for Player* i *for the entire play* ρ *if it holds for her in* ρ *for infinitely many turns. Let* NEC_i *denote the set of plays of* $\mathcal{G}$ *where* NEC *holds for player* i.

CNEC*:* CNEC *holds for Player* i *on turn* j *of* ρ *if there are fewer attacks on* i *with no response in* ρ_j *than for* $-i$. CNEC *holds for Player* i *over a* ρ *if in the limit there are more prefixes of* ρ *where* CNEC *holds for* i *than there are prefixes* ρ *where* CNEC *holds for* $-i$. *Let* CNEC_i *be the set of all plays of* $\mathcal{G}$ *where* CNEC *holds for* i.

For a zero-sum ME game $\mathcal{G}$, the structural complexities of most of the above constraints can be derived from the constraint of rhetorical decomposition sensitivity (RDS), which is a crucial feature of many conversational goals and is defined as follows.

Definition 7. *Given a zero sum ME game* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathsf{win})$, win *is rhetorically decomposition sensitive* (RDS) *if for all* $\rho \in \mathsf{win}$ *and for all finite prefixes* ρ_j *of* ρ, $\rho_j \in Z_1$ *implies there exists* $x \in V_0^+$ *such that* $\mathcal{O}(\rho_j x) \cap \mathsf{win} = \emptyset$.

[6] show that if Player 0 has a winning strategy for an RDS winning condition win then win is a Π_2^0 complete set. Formally,

Proposition 1 [6]. *Let* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathsf{win})$ *be a zero-sum ME game such that* win *is* RDS. *If Player 0 has a winning strategy in* $\mathcal{G}$ *then* win *is* Π_2^0 *complete for the Borel hierarchy.*

In the zero-sum setting, CONS_0, RES_0, COH_0, NEC_0 are all RDS and it is easy to observe that Player 0 has winning strategies in all these constraints (considered individually). Hence, as an immediate corollary to Proposition 1 we have

Corollary 1. CONS_0, RES_0, COH_0, NEC_0 *are* Π^0_2 *complete for the Borel hierarchy for a zero sum ME game.*

CNEC, on the other hand, is a structurally more complex constraint. This is not surprising because CNEC can be intuitively viewed as a limiting case of NEC. Indeed, this was formally shown in [6].

Proposition 2 [6]**.** CNEC_i *is* Π^0_3 *complete for the Borel hierarchy for a zero sum ME game.*

The above results have interesting consequences in terms of first-order definability. Note that certain infinite sequences over our vocabulary $(V_0 \cup V_1)$ can be coded up using first-order logic over discrete linear orders $(\mathbb{N}, <)$, where $\mathbb{N}$ is the set of non-negative natural numbers. Indeed, for every i and for every $a \in V_i$, let a^i_0 be a predicate such that given a sequence $x = x_0 x_1 \ldots,\ x_j \in (V_0 \cup V_1)$ for all $j \geq 0$, $x \models a^i_0(j)$ iff $x_j = a$. Closing under finite boolean operations and $\forall, \exists$, we obtain the logic $\mathsf{FO}(<)$. Now for any formula $\varphi \in \mathsf{FO}(<)$ and for any play ρ of an ME game $\mathcal{G}$, $\rho \models \varphi$ can be defined in the standard way. Thus every formula $\varphi \in \mathsf{FO}(<)$ gives a set of plays $\rho(\varphi)$ of $\mathcal{G}$ defined as:

$$\rho(\varphi) = \{\rho \in (V_0 \cup V_1)^\omega \mid \rho \models \varphi\}$$

A set $A \subseteq (V_0 \cup V_1)^\omega$ is said to be $\mathsf{FO}(<)$ definable if there exists a $\mathsf{FO}(<)$ formula φ such that $A = \rho(\varphi)$. The following result is well-known.

Theorem 1 [20]**.** $A \subseteq (V_0 \cup V_1)^\omega$ *is* $\mathsf{FO}(<)$ *definable if and only if* $A \in (\Sigma^0_2 \cup \Pi^0_2)$.

Thus $\mathsf{FO}(<)$ cannot define sets that are higher than the second level of the Borel hierarchy in their structural complexity. Thus as a corollary of Proposition 2 and Corollary 1, we have

Corollary 2. CONS_0, RES_0, COH_0, NEC_0 *are all* $\mathsf{FO}(<)$ *definable but* CNEC_i *is not.*

This agrees with our intuition because as we observed, CNEC_i is a limit constraint and $\mathsf{FO}(<)$, being local [14], lacks the power to capture it. To define CNEC_i one has to go beyond $\mathsf{FO}(<)$ and look at more expressive logics. One such option is to augment $\mathsf{FO}(<)$ with a counting predicate cnt which ranges over $(\mathbb{N} \cup \{\infty\})$ [19]. Call this logic $\mathsf{FO}(<, \mathsf{cnt})$. One can write formulas of the type $\exists^\infty x \varphi(x)$ in $\mathsf{FO}(<, \mathsf{cnt})$ which says that "there are infinitely many x's such that $\varphi(x)$ holds." Note that it is straightforward to write a formula in $\mathsf{FO}(<, \mathsf{cnt})$ that describes CNEC_i. Another option is to consider the logic $\mathscr{L}_{\omega_1\omega}(FO, <)$ which is obtained by closing $\mathsf{FO}(<)$ under infinitary boolean connectives $\bigvee_j$ and $\bigwedge_j$. We can define a strict syntactic subclass of $\mathscr{L}_{\omega_1\omega}(FO, <)$, denoted $\mathscr{L}^*_{\omega_1\omega}(FO, <)$, where every formula is of the form $O_p O_q \ldots O_t \varphi_{pq\ldots t}$, where, for $k \in \{p, q, \ldots, t-1\}$, $O_k = \bigvee_k$ iff $O_{k+1} = \bigwedge_{k+1}$ and each $\varphi_{pq\ldots t}$ is an $(FO, <)$ formula, $p, q, \ldots, t \in \mathbb{N}$. That is, in every formula of $\mathscr{L}^*_{\omega_1\omega}(FO, <)$, the infinitary connnectives are not nested and occur only in the beginning. We can then show that $\mathscr{L}^*_{\omega_1\omega}(FO, <)$ can express sets in any countable level of the Borel hierarchy.

3 Weighted Message Exchange Games

So far we have reviewed how the framework of Message Exchange games models strategic conversations as infinite sequential games and how we can use it to analyze the complexity of certain intuitive, winning goals in such conversations in terms of both their topological and logical complexities. Nevertheless, there are two issues with ME games that still need to be addressed.

– Let's suppose that a conversation at the outset can be potentially infinite. But still in real life, the Jury ends the game after a finite number of turns. By doing so, how can it be sure that it has correctly determined the outcome of the conversation? In other words, how does the Jury, at any point in a conversation gauge how the players are faring and how can it reliably (or even rationally) choose a winner in a finite time?
– How does the Jury determine the winning conditions win_0 and win_1? Surely, it does not come up with a arbitrary subset of $(V_0 \cup V_1)^\omega$ with an arbitrary Borel complexity.

To address the above questions, [3] introduced the model of *weighted ME games* or WME games. A WME game is an ME game where the Jury specifies the winning sets win_i as subsets of $(V_0 \cup V_1)^\omega$ by evaluating each move of every player. It does this by assigning a 'weight' or a 'score' to the moves. The cummulative weight of a conversation ρ is then the discounted sum of these individual weights.

More formally, let $\mathbb{Z}$ be the set of all integers and $\mathbb{Z}_+$ be the set of non-negative integers. For any $n \in \mathbb{Z}_+$ let $[n] = [0, n-1] \cap \mathbb{Z}_+ = \{0, 1, \ldots, n-1\}$. A weight function is a function $w : (Z_0 \times V_1^+ \cup Z_1 \times V_0^+) \to \{0,1,2\} \times \{0,1,2\}$. Intuitively, given a history $\rho \in Z$, w assigns a tuple of integers $(a_0, a_1) = w(\rho, x)$ to the next legal move x of the play ρ. A weight of 0 is intended to denote a 'bad' move, 1 a 'neutral' or 'average' move, and 2 is intended to denote a 'good' or 'strong' move. An example of a 'strong' move is an attack CDU whereas an example of a 'bad' move can be an incoherent CDU, as defined in Sect. 2. Note that the weight function, w depends on the current history of the game in that, given two different histories $\rho_1, \rho_2 \in Z$, it might be the case that $w(\rho_1, x) \neq w(\rho_2, x)$ for the same continuing move x. For notational simplicity, in what follows, given a play $\rho = x_0 x_1 \ldots$, we shall denote by $w_i^j(\rho)$, the weight assigned by w to Player i in the jth turn of ρ ($j \geq 1$). That is, if $w(\rho_{j-1}, x_j) = (a_0, a_1)$, then $w_0^j(\rho) = a_0$ and $w_1^j(\rho) = a_1$.

A discounting factor is a real $\lambda \in (0,1)$. For every play ρ of an ME game $\mathcal{G}$, the Jury, using some discounting factor λ, computes the discounted-weight of ρ for each player i, which is denoted by $w_i(\rho)$ and is defined as:

Definition 8. *Let ρ be a play of $\mathcal{G}$ and let λ be a discounting factor. Then the discounted-weight of ρ for Player i is given by*

$$w_i(\rho) = \sum_{j \geq 1} \lambda^{j-1} w_i^j(\rho)$$

We can now consider the Jury simply as a tuple (w, λ) where w is a weight function and λ is a discounting factor.[2] And formally define WME games as:

Definition 9. *A Weighted Message Exchange game (WME game) is a tuple* $\mathcal{G} = ((V_0 \cup V_1)^\omega, (w, \lambda))$.

We can now use w and λ to implicitly determine the winning sets win_i of the players and turn $\mathcal{G}$ into either a zero-sum or a non zero-sum game.

Definition 10. *Let* $\mathcal{G} = ((V_0 \cup V_1)^\omega, (w, \lambda))$ *be WME game. Then*

i. **Zero-sum:** $\mathsf{win} = \{\rho \in (V_0 \cup V_1)^\omega | w_0(\rho) \geq w_1(\rho)\}$.
ii. **Non-zero sum:** *Fix constants* $\nu_i \in \mathbb{R}$ *called 'thresholds'. Then,*

$$\mathsf{win}_i = \{\rho \in (V_0 \cup V_1)^\omega | w_i(\rho) \geq \nu_i\}.$$

For this exposition, we concentrate on the zero-sum setting. Winning strategies are then defined as in Sect. 2. We can also define the notions of best-response and ϵ-best-response strategies for a given $\epsilon > 0$. This leads to the definition of a Nash-equilibrium and an ϵ-Nash-equilibrium. It can also be shown that ϵ-Nash-equilibia always exist in WME games (see [3] for more details). It was also shown in [3] that given an $\epsilon > 0$ there exists $n_\epsilon \in \mathbb{Z}_+$ such that after n_ϵ turns neither player can gain more than just a 'small amount' than what they have already gained so far. More formally,

Proposition 3 [3]. *Let* $\mathcal{G} = ((V_0 \cup V_1)^\omega, (w, \lambda))$ *be a WME game. Then given* $\epsilon > 0$ *we have for Player* i *and any play* ρ *of* $\mathcal{G}$

$$\sum_{j=1}^{n_\epsilon} \lambda^{j-1} w_i^j(\rho) - \epsilon \leq w_i(\rho) \leq \sum_{j=1}^{n_\epsilon} \lambda^{j-1} w_i^j(\rho) + \epsilon$$

where $n_\epsilon \leq \frac{\ln[\frac{\epsilon}{2}(1-\lambda)]}{\ln \lambda}$.

Thus if the Jury stops the conversation ρ after n_ϵ turns it is guaranteed that no player could have gained more than ϵ from what they have already gained so far. Thus, it may already be able to come to a conclusion after n_ϵ turns of the game - if Player i has already gained much more than 2ϵ than Player $-i$, then i may be declared the winner.

We have thus answered both the questions posed at the beginning of the section.

Let's now consider an application of WME games to the segment of a real-life debate.

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[2] Note that [3] considers the discounting as a function of the history rather than a constant factor which, arguably, better reflects real-life situations. We stick to a constant discounting factor here for the simplicity of presentation. The main concepts remain the same.

Example 4. Consider the following excerpt from the 1988 Dan Quayle-Lloyd Bentsen Vice-Presidential debate that has exercised us now for several years. Quayle (Q), a very junior and politically inexperienced Vice-Presidential candidate, was repeatedly questioned about his experience and his qualifications to be President. Till a point in the dbate both of them were going neck to neck. But then to rebut doubts about his qualifications, Quayle compared his experience with that of the young John (Jack) Kennedy. To that, Bentsen (BN) made a discourse move that Quayle apparently did not anticipate. We give the relevant part of the debate below where for the simplicity of the ensuing analysis we have labeled each CDU:

a. **Quayle:** *... the question you're asking is, "What kind of qualifications does Dan Quayle have to be president,"*
b. **Quayle:** *... I have far more experience than many others that sought the office of vice president of this country. I have as much experience in the Congress as Jack Kennedy did when he sought the presidency.*
c. **Bensten:** *Senator, I served with Jack Kennedy. I knew Jack Kennedy. Jack Kennedy was a friend of mine. Senator, you're no Jack Kennedy.*
d. **Quayle:** *That was unfair, sir. Unfair.*
e. **Bensten:** *You brought up Kennedy, I didn't.*

Let us analyze the above exchange from the perspective of a WME game. Without loss of generality suppose Quayle is Player 0 and Bensten is Player 1. Let us denote by ρ all the conversation that took place before the above exchange. Since both of them were neck-neck till then we can assume that both had gained a weight of c (say) that far. Next, Quayle makes moves (a) and (b) which might be considered an average move at that point (the audience applauds but is skeptical). So we can assign $w(\rho, \langle a\rangle\langle b\rangle) = (1,1)$ - Bensten neither gains nor loses from this move of Quayle. Bensten then makes the brilliant move (b) which does serious damage to Quayle. The audience bursts with applause. Hence, we set $w(\rho\langle a\rangle\langle b\rangle, \langle c\rangle) = (0,2)$. Quayle is unable to retaliate to (b) and makes another rather timid move (c) which has even a negative impact to his cause on the audience. The audience is still basking in Bensten's previous move and we set $w(\rho\langle a\rangle\langle b\rangle\langle c\rangle, \langle d\rangle) = (1,1)$. Bensten goes ahead and cements his position further by making another attack move (d) on Quayle. We hence set $w(\rho\langle a\rangle\langle b\rangle\langle c\rangle\langle d\rangle, \langle e\rangle) = (0,2)$.

Now suppose the Jury (in this case the audience) is using a discount factor λ. The discounted-weights to Quale and Bensten are respectively:

$$w_Q(\rho\langle a\rangle\langle b\rangle\langle c\rangle\langle d\rangle\langle e\rangle) = 1 + \lambda^2$$

and

$$w_{BN}(\rho\langle a\rangle\langle b\rangle\langle c\rangle\langle d\rangle\langle e\rangle) = 1 + 2\lambda + \lambda^2 + 2\lambda^3$$

We thus see that $w_{BN}(\rho\langle a\rangle\langle b\rangle\langle c\rangle\langle d\rangle\langle e\rangle) > w_Q(\rho\langle a\rangle\langle b\rangle\langle c\rangle\langle d\rangle\langle e\rangle)$ for any value of $\lambda \in (0,1)$. Not just that, even if after the above initial slump, Quayle plays in such a way that every move he makes is a brilliant move and every move

Bensten makes is a disaster, Quayle still cannot recover and gain more than Bensten eventually for values of λ as high as 0.8! Discounting thus reiterates the fact that it is always beneficial to makes ones best moves earlier on in a debate. This also 'colours' the weighting function of the Jury in ones favour.

In passing, we would like to remark that Quayle never recovered from one disastrous move in that debate and lost handily as is rightly predicted by our model.

4 Imperfect Information and Epistemic Considerations

WME games address certain open questions in the theory of ME games, as we have shown in the previous section. But they give rise to other questions as well.

- How does the Jury determine a weighting scheme?
- If the Jury is identified simply with a weighting function and a discount factor, and players know these parameters, they can determine when the Jury will end the game. So don't WME games fall prey to troublesome backwards induction arguments that ME games were designed to avoid?

Concerning the first question, we've shown that the predictions of WME games hold for a wide range of weighting schemes, but indeed it is clear that different Juries will have different weighting schemes. Consider how a partisan audience say of a political candidate c reacts to his discourse moves and how a audience hostile to c's views reacts. The U.S. Presidential primary debates and general debates show that these reactions can vary widely. In particular, Juries may be biased and only "hear what they want to hear," even to the extent that they ignore inconsistencies or incoherences on the part of their preferred player. Concrete Juries adopt the weighting schemes they do, in virtue of their beliefs and desires. Thus, weighting schemes may vary quite widely, and a conversational participant should be as well informed as she can be about the Jury she wants to sway.

The second question needs a negative response. [3] simply assumes that the Jury's characteristics are unknown to the conversational participants. But this is not really realistic, especially in virtue of our response to the first question above. So in this section, we study the exact information structure implicit in the strategic reasoning in conversations by extending framework of ME games with epistemic notions. We use the well-established theory of type-structures, first introduced in [17] and widely studied since. We assume that each player $i \in (\{0,1\} \cup \{\mathcal{J}\})$ has a (possibly infinite) set of types T_i. With each type t_i of Player i is associated a (first-order) belief function $\beta_i(t_i)$ which assigns to t_i a probability distribution over the types of the other players. That is, $\beta_i : T_i \rightarrow \Delta(\prod_{j \neq i} T_j)$. $\beta_i(t_i)$ represents the 'beliefs' of type t_i of Player i about the types of the other players and the Jury. The higher-order beliefs can be defined in a standard way by iterating the functions β_i. We assume that each type t_i of each Player i starts the game with an initial belief $\beta_i(t_i) \in \Delta(\prod_{j \neq i} T_j)$, called the 'prior belief'. The players take turns in making their moves and after every move, all the players dynamically update their beliefs through Bayesian updates. The notions of 'optimal strategies', 'best-response', 'rationality', 'common belief in rationality' etc. can then be defined in the standard way (see [12]).

Having imposed the above epistemic structure on ME games, we can now reason about the 'rationality' of the players' strategies. In order to justify or predict the outcome of games, many different solution concepts viz., Nash equilibrium, iterated removal of dominated strategies, correlated equilibrium, rationalizability etc. have been proposed [7, 10, 21]. Most of them have also been characterized in terms of the exact belief structure and strategic behavior of the players (see [12] for an overview). We can borrow results from this rich literature to predict or justify outcomes in strategic conversations. The details of the above is ongoing work and we leave it to an ensuing paper. However, let us apply the above concepts and analyze our original example of Bronston and the Prosecutor.

To illustrate the power of types, let us return to Eg. 1. One conversational goal of the Prosecutor in Eg. 1 is to get Bronston to commit to an answer eventually (and admit to an incriminating fact) or to continue to refuse to answer (in which case he will be charged with contempt of court). Under such a situation, the response 1d of Bronston is clearly a clever strategic move. Bronston's response (1d) was a strategic move aimed to 'misdirect' the Jury $\mathcal{J}$. He believed that $\mathcal{J}$ was of a type that would be convinced by his ambiguous response and neither incriminate him nor charge him with perjury nor of contempt of court. His move was indeed rational, *given his belief* about the Jury type. It turns out that while the jury of a lower court $\mathcal{J}_1$ was not convinced of Bronston's arguments and charged him with perjury, a higher court $\mathcal{J}_2$ overturned the verdict and released him. Thus his belief agreed with $\mathcal{J}_2$ but not $\mathcal{J}_1$.

We now return briefly to the information players have about the Jury in WME games. Intuitively, each Player i is uncertain about: (i) the type of the other player $(1-i)$, (ii) the strategy that $(1-i)$ is employing and (iii) the type of the Jury which is the discounting factor λ and the weight function w. We thus assume that at every history ρ of an ME/WME game $\mathcal{G}$ each type $t_i \in \mathcal{T}_i$ of Player i has beliefs on:

1. the set of types $T_{(1-i)}$ of Player $(1-i)$,
2. the set of strategies $S_{(1-i)}$ of Player $(1-i)$,
3. the weight function w.
4. the discounting factor λ.

Although going into the details of each one of these points would take too long for this exposition, we can show that each of these factors can be modelled precisely preserving our intuitions. This supplies us with a needed answer to our second question. Indeed, the Jury ends the game after a finite number of turns n (say), and from its viewpoint, the game is finite. But note that the players are uncertain about the exact value of n and hold beliefs about it. Hence, from their viewpoint, although the game ends after finitely many turns, they do not know the exact number of turns. Thus, intuitively a rational player is one who strategizes for a wide range of possibilities for the value of n [this will be elaborated presently]. For her, the game is 'potentially infinite'. And hence, we as analysts, model the situation as an infinite game as well. In [5], we argued that an infinitary approach was needed to handle both technical issues having to do with Backwards Induction arguments as well as to capture the intuition that a conversationalist, to be sure of succeeding in convincing a Jury of a particular position, should be prepared to argue for her position for "as long as it takes" and to answer every possible objection by an opponent. Since the list of possible objections is most likely infinite, the analyst must provide an infinitary game-theoretic framework. These points still hold once we add an epistemic layer to WME games.

5 Conclusion

In this paper we have summarized concepts from earlier work and have demonstrated how infinite sequential games paired with the notion of a Jury, ME games, can be used to model strategic conversations. Such a model allows one to reason about the structure and complexity of various kinds of winning goals that conversationalists might have. We have shown how to use tools from topology, set-theory and logic to express such goals. We then discussed a problem with pure ME games: how can an actual Jury reliably determine a winner or winners in a conversation after only finitely many rounds. We addressed this issue by moving to Weighted ME (WME) games. We showed how to apply elements of WME games to a snippet of a historic moment in American political debates. However, WME games, we also showed, don't furnish a completely satisfactory analysis, because though the Jury can reliably determine a winner or winners of a conversation after a finite moment, this information crucially cannot be common knowledge of the participants without re-introducing the damaging backwards induction arguments that ME games were originally designed to solve. We then demonstrated how we can use ideas from epistemic game theory would in principle solve this problem.

Thus, what we have put forward in this paper is a framework for an epistemic, game-theoretic approach to conversation. As far as we know, this approach is utterly different from any other model proposed for the study of linguistic conversation, though it may have other applications as well. There are many directions into which we would like to delve deeper in the future. One such direction, as we already mentioned, is to work out the epistemic theory of ME games in full detail. That is our current work in progress. Another direction has to do with a more detailed investigation of the Jury, or possible Juries. So far we have considered the Jury as a 'passive' entity; it simply evaluates the play and determines the winner. In real life situations, however, the Jury actively participate in the conversation itself, albeit typically in a limited way. It can applaud or boo moves of the players. Thus, the Jury can be seen as making these moves in the game. Based on what the players observe about the Jury, they may update or change their beliefs and vice-versa. Incorporating this into our ME games requires a modification of the current framework where the Jury is another player making moves from its own set of vocabulary. We plan to explore this in future work.

Finally, in addition to the Jury, debates usually also have a moderator whose job is to conduct the debate and assign turns to the players. The moderator may also actively 'pass comments' about the moves of the players. A fair moderator gives all the players equal opportunity to speak and put their points across. However, if the moderator is unfair, he may 'starve' a particular player by not letting her enough chance to speak, respond to attacks and so on. Exploring the effects a biased moderator can have on conversations is another interesting, future topic of research.

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