Language Games

Nicholas Asher, Soumya Paul

Official URL

DOI : https://doi.org/10.1007/978-3-662-53826-5_1

Language Games

Nicholas Asher and Soumya Paul(✉)

Institut de Recherche en Informatique de Toulouse, Toulouse, France
nicholas.asher@irit.fr, soumya.paul@gmail.com

Abstract. In this paper we summarize concepts from earlier work and demonstrate how infinite sequential games can be used to model strategic conversations. Such a model allows one to reason about the structure and complexity of various kinds of winning goals that conversationalists might have. We show how to use tools from topology, set-theory and logic to express such goals. Our contribution in this paper is to offer a detailed examination of an example in which a player 'defeats himself' by going inconsistent, and to introduce a simple yet revealing way of talking about unawareness. We then demonstrate how we can use ideas from epistemic game theory to define various solution concepts and justify rationality assumptions underlying a conversation.

Keywords: Strategic reasoning · Conversations · Dialogues · Infinite games · Epistemic game theory

1 Introduction

A strategic conversation involves (at least) two people (agents) who have opposing interests concerning the outcome of the conversation. A debate between two political candidates is an instance. Each candidate has a certain number of points she wants to convey to the audience, and each wants to promote her own position and damage that of her opponent or opponents. In other words, each candidate wants to win. To achieve these goals each participant needs to plan for anticipated responses from the other. Debates are thus a sequence of exchange of messages at the end of which an agent may win, lose or draw. Similar strategic reasoning about what one says is a staple of board room or faculty meetings, bargaining sessions, etc.

It is therefore natural to model such conversations as games. Attempts to this end have been made in the past with the most notable of them being the use of signaling games [24] and the closely related persuasion games [15]. In a signaling game one player with a knowledge of the actual state sends a signal and the other player who has no knowledge of the state chooses an action, usually upon an interpretation of the received signal. The standard setup supposes that both players have common knowledge of each other's preference profiles as well as their own over a set of commonly known set of possible states, actions and signals.

The authors thank ERC grant 269427 for supporting this research.

DOI:10.1007/978-3-662-53826-5_1

However for modeling non-cooperative strategic contexts of sequential dynamic games, signaling games suffer from many drawbacks. Some of them can be summarised as follows (see [4] for a more comprehensive discussion):

- A game that models a non-cooperative setting, that is a setting where the preferences of the players are opposed, must be zero-sum. However, it has been shown [11] that under the zero-sum criterion, in equilibrium, the sending and receiving of any message has no effect on the receiver decision.
- In order to use games as part of a general theory of meaning, one has to make clear how to construct the game-context, which includes providing an interpretation of the game's ingredients (types, messages, actions). Franke [13] extended the setting of signaling games to that of *interpretation games* to address this issue. Such games encode a 'canonical context' for an utterance, in which relevant conversational implicatures may be drawn. The game structure is determined by the set of 'sender types'. Interpretation games model the interpretation of the messages and actions of a signaling game in a cooperative context for 'Gricean agents' quite well. But in the non-cooperative setting, things are much more intricate and problems remain (again see [4]).
- Signaling games are one-shot and fail to capture the dynamic nature of a strategic conversation. One can attempt to encode a sequence of moves of a particular player as a single message m sent by that player but then one runs into the problem of assigning correct utilities for m because such utilities depend again on the possible set of continuations of m.
- Finally, there is an inherent asymmetry associated with the setting of a signaling game - one player is informed of the state of the world but the other is not; one player sends a message but the other does not. Conversations (like debates), on the other hand, are symmetric - all participants should (and usually do) get equal opportunities to get their messages across.

Strategic conversations are thus special and have characteristics unique to them which have not been captured by previous game-theoretic models. Some of these important characteristics are as follows.

- Conversations are sequential and dynamic and inherently involve a 'turn-structure' which is important in determining the merit of a conversation to the participants. In other words, it is important to keep track of "who said what".
- A 'move' by a player in a linguistic game typically carries more semantic content than usually assumed in game theory. What a player says may have a set of 'implicatures', may be 'ambiguous', may be 'coherent/incoherent' or 'consistent/inconsistent' to what she had said earlier in the conversation. She may also 'acknowledge' other people's contributions or 'retract' her previous assertions. These features too have important consequences on the existence and complexity of winning strategies.
- Conversations typically have a 'Jury' who evaluates the conversation after it has ended and determines if one or more of the players have reached their goals – determines the winner. Players will spin the description of the game to

their advantage and so may not present an accurate view of what happened. The Jury can be a concrete or even a hypothetical entity who acts as a 'passive player' in the game. For example, in a courtroom situation there is a physical Jury who gives the verdict, whereas in a political debate the Jury is the audience or the citizenry in general. This means that the winning conditions of the players are affected by the Jury in that, they depend on what they believe that the Jury expects them to achieve.
- Epistemic elements thus naturally creep into such games. In particular, the players and the Jury have 'types'. In addition the players also have 'beliefs' about the types of the other players and that of the Jury. They strategize based on their beliefs and also update their beliefs after each turn.
- Lastly but most importantly, conversations do not have a 'set end'. When two people or a group of people engage in a conversation they do not know at the outset how many turns it will last or how many chances each player will get to speak (if at all). Sure in a 'conducted' conversation such as a political debate or a courtroom debate, there is usually a moderator whose job is to ensure that each player receives his or her fair chance to put their points across but even such a moderator does not know at the outset how the conversation will unfold and how many turns each player will receive. Players thus cannot strategize for a set horizon while starting a conversation. This rules out backward induction reasoning for both the players and we analysts.

With the above aspects in mind, [4] model conversations as infinite games over a countable 'vocabulary' V which they call Message Exchange games (ME games). In this paper, we first summarize the main results of [4] in a compact fashion but also add some new remarks concerning first order definability of conversational goals. We then add a more nuanced analysis of a particular dialogue excerpt (our example 4) and prove a theorem beyond the scope of [4], showing how unexpected moves can complicate the search for winning strategies. Finally in Sect. 3, we break new ground and add an epistemic layer to ME games.

Let's now turn to the basics of ME games. The intuitive idea behind an ME game is that a conversation proceeds in turns where in each turn one of the players 'speaks' or plays a string of letters from her own vocabulary. However, the player does not play just any sequence of arbitrary strings but sentences or sets of sentences that 'make sense'. To ensure this, the vocabulary V should have an exogenous semantics built-in. In order to achieve this, we exploit a semantic theory for discourse, SDRT [1]. SDRT develops a rich language to characterize the semantics and pragmatics of moves in dialogue. This means that we can exploit the notion of entailment associated with the language of SDRSs to track commitments of each player in an ME game. In particular, the language of SDRT features variables for dialogue moves that are characterized by contents that the move commits its speaker to. Crucially, some of this content involves predicates that denote rhetorical relations between moves—like the relation of *question answer pair* (qap), in which one move answers a prior move characterized by a question. The vocabulary V of an ME game thus contains a countable set of discourse constituent labels $\mathsf{DU} = \{\pi, \pi_1, \pi_2, \ldots\}$, and a finite set of discourse relation symbols $\mathcal{R} = \{\mathsf{R}, \mathsf{R}_1, \dots \mathsf{R}_n\}$, and formulas $\phi, \phi_1, \dots$ from some fixed language for describing elementary discourse move contents. V consists of formulas of the form $\pi\colon \phi$, where ϕ is a description of the content of the discourse unit labelled by π in a logical language like the language of higher order logic used, e.g., in Montague Grammar, and $\mathsf{R}(\pi, \pi_1)$, which says that π_1 stands in relation R to π. One such relation R is qap. Thus, each discourse relation symbolized in V comes with constraints as to when it can be coherently used in context and when it cannot.

2 Message Exchange Games

In this section we formally define Message Exchange games and state some of their properties and their use in modeling strategic conversations as explored at length in [4]. For simplicity, we shall develop the theory for the case of conversations that involve two participants, which we shall denote by Player 0 and Player 1. It will be straightforward to generalize it to the case where there are more than two players. Thus, in what follows, we shall let i range over the set of players $\{0, 1\}$. Furthermore, Player $-i$ will always denote Player $(1 - i)$, the opponent of Player i.

We first define the notion of a 'Jury'. As noted in Sect. 1, a Jury is any entity or a group of entities that evaluates a conversation and decides the winner. A Jury thus 'groups' instances of conversations as being winning for Player 0 or Player 1 or both.

For any set A let A^* be the set of all finite sequences over A and let A^ω be the set of all countably infinite sequences over A. Let $A^\infty = A^* \cup A^\omega$ and $A^+ = A^* \setminus \{\epsilon\}$. Now, let V be a vocabulary as defined at the end of Sect. 1 and let $V_i = V \times \{i\}$. This is to make explicit the 'turn-structure' of a conversation as alluded to in the introduction.

Definition 1. *A* Jury $\mathcal{J}$ *over* $(V_0 \cup V_1)^\omega$ *is a tuple* $\mathcal{J} = (\mathsf{win}_1, \mathsf{win}_2)$ *where* $\mathsf{win}_i \subseteq (V_0 \cup V_1)^\omega$ *is the* winning condition *or* winning set *for Player* i*.*

Given the definition of a Jury over $(V_0 \cup V_1)^\omega$ we define a Message Exchange game as:

Definition 2. *A Message Exchange game (ME game)* $\mathcal{G}$ *over* $(V_0 \cup V_1)^\omega$ *is a tuple* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ *where* $\mathcal{J}$ *is a Jury over* $(V_0 \cup V_1)^\omega$*.*

Formally the ME game $\mathcal{G}$ is played as follows. Player 0 starts the game by playing a non-empty sequence in V_0^+. The turn then moves to Player 1 who plays a non-empty sequence from V_1^+. The turn then goes back to Player 0 and so on. The game generates a play ρ_n after n (≥ 0) turns, where by convention, $\rho_0 = \epsilon$ (the empty move). A play can potentially go on forever generating an infinite play ρ_ω, or more simply ρ. Player i wins the play ρ iff $\rho \in \mathsf{win}_i$. $\mathcal{G}$ is zero-sum if $\mathsf{win}_i = (V_0 \cup V_1)^\omega \setminus \mathsf{win}_{-i}$ and is non zero-sum otherwise. Note that both player or neither player might win a non zero-sum ME game $\mathcal{G}$. The Jury of a zero-sum

ME game can be denoted simply as win where by convention $\mathsf{win} = \mathsf{win}_0$ and $\mathsf{win}_1 = (V_0 \cup V_1)^\omega \setminus \mathsf{win}$.

Plays are segmented into *rounds*—a move by Player 0 followed by a move by Player 1. A finite play of an ME game is (also) called a $\mathsf{history}$, and is denoted by ρ. Let Z be the set of all such histories, $Z \subseteq (V_0 \cup V_1)^*$, where $\epsilon \in Z$ is the empty history and where a history of the form $(V_0 \cup V_1)^+ V_0^+$ is a 0-history and one of the form $(V_0 \cup V_1)^+ V_1^+$ is a 1-history. We denote the set of i-histories by Z_i. By convention $\epsilon \in Z_1$. Thus $Z = Z_0 \cup Z_1$. For $\rho \in Z$, $\mathsf{turns}(\rho)$ denotes the total number of turns (by either player) in ρ. A $\mathsf{strategy}$ σ_i of Player i is thus a function from the set of $-i$-histories to V_i^+. That is, $\sigma_i : Z_{-i} \to V_i^+$. A play $\rho = x_0 x_1 \ldots$ of an ME game $\mathcal{G}$ is said to $\mathsf{conform}$ to a strategy σ_i of Player i if for every prefix ρ_j of ρ, $j = i(\mod 2)$ implies $\rho_{j+1} = \rho_j \sigma_i(\rho_j)$. A strategy σ_i is called $\mathsf{winning}$ for Player i if $\rho \in \mathsf{win}_i$ for every play ρ that conforms to σ_i.

Given how we have characterized the vocabulary $(V_0 \cup V_1)$, we can assumed a fixed meaning assignment function from EDUs to formulas the describe their contents. Then, a sequence of conversational moves can be represented as a graph (DU, E, ℓ), where DU is the set of vertices each representing a discourse unit, $E \subseteq \mathsf{DU} \times \mathsf{DU}$ a set of edges representing links between discourse units that are labeled by $\ell : E \to \mathcal{R}$ with discourse relations.[1]

Example 1. To illustrate this structure of conversations, consider the following example taken from [2] from a courtroom proceedings where a prosecutor is querying the defendant. We shall return to this example later on for a strategic analysis.

a. **Prosecutor**: *Do you have any bank accounts in Swiss banks, Mr. Bronston?*
b. **Bronston**: *No, sir.*
c. **Prosecutor**: *Have you ever?*
d. **Bronston**: *The company had an account there for about six months, in Zurich.*
e. **Prosecutor**: *Thank you Mr. Bronston.*

Example 2. We can view the conversation in Example 1 as an ME game as in Fig. 1. The figure shows a weakly connected graph, which represents a fully coherent conversation, with a set of discourse constituent labels $\mathsf{DU} = \{\pi_{\mathsf{bank}}, \pi_{\neg\mathsf{bank}}, \pi_{\mathsf{bank-elab}}, \pi_{\mathsf{company}}, \pi_{\mathsf{ack}}, \ldots\}$ and a set of relations $\mathcal{R} = \{\mathsf{qap}, \mathsf{q-followup}, \mathsf{ack}, \ldots\}$. The arrows depict the individual relation instances between the DUs.

ME game messages come with a conventionally associated meaning in virtue of the constraints enforced by the Jury; an agent who asserts a content of a message $\mathsf{commits}$ to that content, and it is in virtue of such commitments that other agents respond in kind. While SDRT has a rich language for describing

[1] We note that this is a simplification of SDRT which also countenances complex discourse units (CDUs) and another set of edges in the graph representation, linking CDUs to their simpler constituents. These edges represent parthood, not rhetorical relations. We will not, however, appeal to CDUs here.

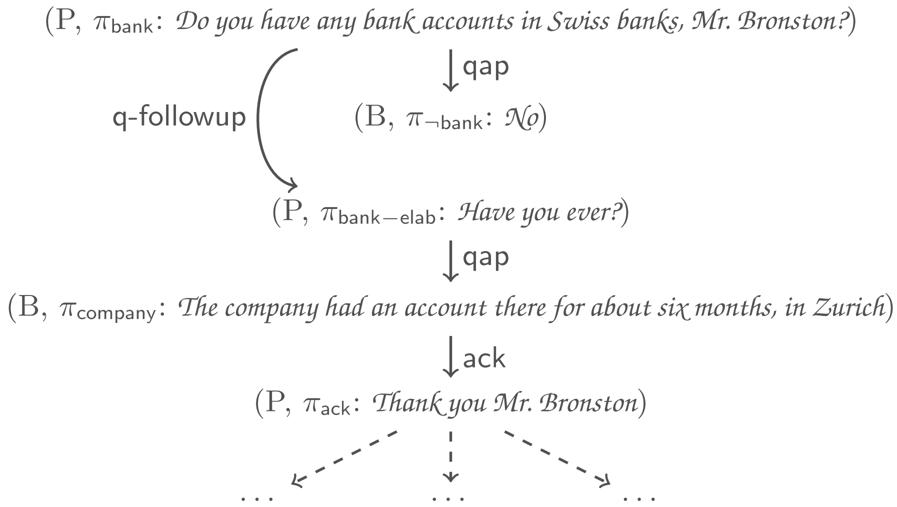

Fig. 1. An example ME game

dialogue moves, it is not explicit about how dialogue moves explicitly affect the commitments of the agents who make the moves or those who observe the moves. [25,26] link the semantics of the SDRT language with commitments explicitly (in two different ways). They augment the SDRT language with formulas that describe the commitments of dialogue participants, using a simple propositional modal syntax. Thus for any formula ϕ in the language of Montague Grammar that describes the content of a label $\pi \in \mathsf{DU}$, they add: $\neg\phi \mid \phi_1 \vee \phi_2 \mid \mathsf{C}_i\phi,\ i \in \{0,1\} \mid \mathsf{C}^*\phi$, with the derived operators $\wedge, \implies, \top, \bot$ are defined as usual, providing a propositional logic of commitments over the formulas that describe labels. Of particular interest are the commitment operators C_i and C^*. If ϕ is a formula for describing a content, $\mathsf{C}_i\phi$ is a formula that says that Player i commits to ϕ and $\mathsf{C}^*\phi$ denotes 'common commitment' of ϕ. Commitment is modeled as a Kripke modal operator via an alternativeness relation in a pointed model with a distinguished (actual) world w_0. This allows them to provide a semantics for discourse moves that links the making of a discourse move by an agent to her commitments: i's assertion of a discourse move ϕ, for instance, we will assume, entails a common commitment that i commits to ϕ, written $\mathsf{C}^*\mathsf{C}_i\phi$. They show how each discourse move ϕ defines an action, a change or update on the model's commitment structure; in the style of public announcement logic viz. [6,7]. For instance, if agent i asserts ϕ, then the commitment structure for the conversational participants is updated such so as to reflect the fact that $\mathsf{C}^*\mathsf{C}_i\phi$. Finally, they define an entailment relation $\models$ that ensures that $\phi \models \mathsf{C}^*\mathsf{C}_i\phi$. This semantics is useful because it allows us to move from sequences of discourse moves to sequences of updates on any model for the discourse language. See [25,26] for a detailed development and discussion.

ME games resemble infinite games that have been used in topology, set theory [19] and computer science [16] to study the descriptive complexity of different infinite sets. We can leverage some of the results from these areas to talk about the general 'shape' of conversations or to analyse the complexity of the winning conditions of the players. This has been extensively explored in [4]. We give a flavor of some of the applications here.

To do that we first need to define an appropriate topology on $(V_0 \cup V_1)^\omega$ which will allow us to characterize the descriptive complexity of the winning sets win_0 and win_1. We proceed as follows. We define the topology on $(V_0 \cup V_1)^\omega$ by defining the open sets to be sets of the form $A(V_0 \cup V_1)^\omega$ where $A \subseteq (V_0 \cup V_1)^*$. Such an open set will be often denoted as $\mathcal{O}(A)$. When A is a singleton set $\{x\}$ (say), we abuse notation and write $\mathcal{O}(\{x\})$ as $\mathcal{O}(x)$. The Borel sets are defined as the sigma-algebra generated by the open sets of this topology. The Borel sets can be arranged in a natural hierarchy called the Borel hierarchy which is defined as follows. Let Σ_1^0 be the set of all open sets. $\Pi_1^0 = \overline{\Sigma_1^0}$, the complement of the set of Σ_1^0 sets, is the set of all closed sets. Then for any $\alpha > 1$ where α is a successor ordinal, define Σ_α^0 to be the countable union of all $\Pi_{\alpha-1}^0$ sets and define Π_α^0 to be the complement of Σ_α^0. $\Delta_\alpha^0 = \Sigma_\alpha^0 \cap \Pi_\alpha^0$.

Definition 3 [19]**.** *A set A is called* complete *for a class Σ_α^0 (resp. Π_α^0) if $A \in \Sigma_\alpha^0 \setminus \Pi_\alpha^0$ (resp. $\Pi_\alpha^0 \setminus \Sigma_\alpha^0$) and $A \notin (\Sigma_\beta^0 \cup \Pi_\beta^0)$ for any $\beta < \alpha$.*

The Borel hierarchy represents the descriptive or structural complexity of the Borel sets. A set higher up in the hierarchy is structurally more complex than one that is lower down. Complete sets for a particular class of the hierarchy represent the structurally most complex sets of that class. We can use the Borel hierarchy and the notion of completeness to capture the complexity of winning conditions in conversations. For example, two typical sets in the fist level of the Borel hierarchy are defined as follows. Let $A \subseteq (V_0 \cup V_1)^+$, then

$$\mathsf{reach}(A) = \{\rho \in (V_0 \cup V_1)^\omega \mid \rho = xy\rho', y \in A\},\ \mathsf{safe}(A) = (V_0 \cup V_1)^\omega \setminus \mathsf{reach}(A)$$

A little thought convinces us that $\mathsf{reach}(A) \in \Sigma_1^0$ and $\mathsf{safe}(A) \in \Pi_1^0$. Let reachability be the class of sets of the form $\mathsf{reach}(A)$ and safety be the class of sets of the form $\mathsf{safe}(A)$.

Example 3. Returning to our example of Bronston and the Prosecutor, let us consider what goals the Jury expects each of them to achieve. The Jury will award its verdict in favor of the Prosecutor: (i) if he can eventually get Bronston to admit that (a) he had an account in Swiss banks, or (b) he never had an account in Swiss banks, or (ii) if Bronston avoids answering the Prosecutor forever. In the case of (i)a, Bronston is incriminated, (i)b, he is charged with perjury and (ii), he is charged with contempt of court. Bronston's goal is the complement of the above, that is to avoid either of the situations (i)a, (i)b and (ii). We thus see that the Jury winning condition for the Prosecutor is a Boolean combination of a reachability condition and the complement of a safety condition, which is in the first level of the Borel hierarchy.

Conversations, to be meaningful, must also satisfy certain natural constraints which the Jury might impose throughout the course of a play. Below we define some of these constraints and then go on to study the complexity of the sets satisfying them.

Let $\rho = x_0 x_1 x_2 \ldots$ be a play of an ME game $\mathcal{G}$ where $x_0 = \epsilon$ and $x_j \in V^+_{((j-1) \bmod 2)}$ is the sequence played by Player $((j-1) \bmod 2)$ in turn j. For every i define the function $\mathsf{du}_i : V_i^+ \to \wp(\mathsf{DU})$ such that $\mathsf{du}_i(x_j)$ gives the set of contributions (in terms of DUs) of Player i in the jth turn. By convention, $\mathsf{du}_i(x_j) = \emptyset$ for $x_j \in V^+_{-i}$.

Definition 4. *Let $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ be an ME game over $(V_0 \cup V_1)^\omega$. Let $\rho = x_0 x_1 x_2 \ldots$ be a play of $\mathcal{G}$. Then*

Consistency: ρ is consistent *for Player i if the set $\{\mathsf{du}_i(x_j)\}_{j>0}$ is consistent. Let CONS_i denote the set of consistent plays for Player i in $\mathcal{G}$.*

Coherence: Player i is coherent *on turn $j > 0$ of play ρ if for all $\pi \in \mathsf{du}_i(x_j)$ there exists $\pi' \in (\mathsf{du}_i(x_k) \cup \mathsf{du}_{-i}(x_{k-1}))$ where $k \leq j$ such that there exits $\mathsf{R} \in \mathcal{R}$ such that $(\pi' \mathsf{R} \pi \vee \pi \mathsf{R} \pi')$ holds. Let COH_i denote the set of all coherent plays for Player i in $\mathcal{G}$.*

Responsiveness: Player i is responsive *on turn $j > 0$ of play ρ if there exists $\pi \in \mathsf{du}_i(x_j)$ such that there exits $\pi' \in \mathsf{du}_{-i}(x_{j-1})$ such that $\pi' \mathsf{R} \pi$ for some $\mathsf{R} \in \mathcal{R}$. Let RES_i denote the set of responsive plays for Player i in $\mathcal{G}$. x_j (or abusing notation, π) will be sometimes called a* response move.

Rhetorical-cooperativity: Player i is rhetorically-cooperative *in ρ if she is both coherent and responsive in every turn of hers in ρ. ρ is rhetorically-cooperative if both the players are rhetorically-cooperative in ρ. Let RC_i denote the set of rhetorically-cooperative plays for Player i in $\mathcal{G}$ and let RC be the set of all rhetorically-cooperative plays.*

To define the constraints NEC and CNEC we need first the definition of an 'attack' and a 'response'. Thus

Definition 5. *Let $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ be an ME game over $(V_0 \cup V_1)^\omega$. Let $\rho = x_0 x_1 x_2 \ldots$ be a play of $\mathcal{G}$. Then*

Attack: $\mathsf{attack}(\pi', \pi)$ on Player $-i$ holds at turn j of Player i just in case $\pi \in \mathsf{du}_i(x_j)$, $\pi' \in \mathsf{du}_{-i}(x_k)$ for some $k \leq j$, there is an $\mathsf{R} \in \mathcal{R}$ such that $\pi' \mathsf{R} \pi$ and: (i) π' entails that $-i$ is committed to ϕ for some ϕ, (ii) ϕ entails that $\neg\phi$ holds. In such a case, we shall often abuse notation and denote it as $\mathsf{attack}(k, j)$. Furthermore, x_j or alternatively π shall be called an attack move. *An attack move is* relevant *if it is also a* response *move.* $\mathsf{attack}(k, j)$ *on $-i$ is* irrefutable *if there is no move $x_\ell \in V_{-i}$ in any turn $\ell > j$ such that $\mathsf{attack}(j, \ell)$ holds and $x_0 x_1 \ldots x_\ell$ is consistent for $-i$.*

Response: $\mathsf{response}(\pi', \pi)$ on Player $-i$ holds at turn j of Player i if there exits $\pi'' \in \mathsf{du}_i(x_\ell)$, $\pi' \in \mathsf{du}_{-i}(x_k)$ and $\pi \in \mathsf{du}_i(x_j)$ for some $\ell \leq k \leq j$, such that $\mathsf{attack}(\pi'', \pi')$ holds at turn k of Player $-i$, there exists $\mathsf{R} \in \mathcal{R}$ such that $\pi' \mathsf{R} \pi$ and π implies that (i) one of i's commitments ϕ attacked in π' is true or (ii) one of $-i$'s commitments in π' that entails that i was committed to $\neg\phi$ is false. We shall often denote this as $\mathsf{response}(k, j)$.

We can now define the constraints NEC and CNEC as follows.

Definition 6. *Let* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ *be an ME game over* $(V_0 \cup V_1)^\omega$*. Let* $\rho = x_0x_1x_2\dots$ *be a play of* $\mathcal{G}$*. Then*

NEC: NEC *holds for Player* i *in* ρ *on turn* j *if for all* $\ell, k,\ \ell \leq k < j$*, such that* $\mathsf{attack}(\ell, k)$*, there exists* $m,\ k < m \leq j$*, such that* $\mathsf{response}(k, m)$*.* NEC *holds for Player* i *for the entire play* ρ *if it holds for her in* ρ *for infinitely many turns. Let* NEC_i *denote the set of plays of* $\mathcal{G}$ *where* NEC *holds for player* i*.*

CNEC: CNEC *holds for Player* i *on turn* j *of* ρ *if there are fewer attacks on* i *with no response in* ρ_j *than for* $-i$*.* CNEC *holds for Player* i *over a* ρ *if in the limit there are more prefixes of* ρ *where* CNEC *holds for* i *than there are prefixes* ρ *where* CNEC *holds for* $-i$*. Let* CNEC_i *be the set of all plays of* $\mathcal{G}$ *where* CNEC *holds for* i*.*

For a zero-sum ME game $\mathcal{G}$, the structural complexities of most of the above constraints can be derived from another constraint which we call rhetorical decomposition sensitivity (RDS) which is defined as follows.

Definition 7. *Given a zero sum ME game* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathsf{win})$*,* win *is rhetorically decomposition sensitive* (RDS) *if for all* $\rho \in \mathsf{win}$ *and for all finite prefixes* ρ_j *of* ρ*,* $\rho_j \in Z_1$ *implies there exists* $x \in V_0^+$ *such that* $\mathcal{O}(\rho_j x) \cap \mathsf{win} = \emptyset$*.*

[4] show that if Player 0 has a winning strategy for an RDS winning condition win then win is a Π^0_2 complete set. Formally,

Proposition 1 [4]**.** *Let* $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathsf{win})$ *be a zero-sum ME game such that* win *is* RDS*. If Player 0 has a winning strategy in* $\mathcal{G}$ *then* win *is* Π^0_2 *complete for the Borel hierarchy.*

In the zero-sum setting, CONS_0, RES_0, COH_0, NEC_0 are all RDS and it is easy to observe that Player 0 has winning strategies in all these constraints (considered individually). Hence, as an immediate corollary to Proposition 1 we have

Corollary 1. CONS_0*,* RES_0*,* COH_0*,* NEC_0 *are* Π^0_2 *complete for the Borel hierarchy for a zero sum ME game.*

CNEC, on the other hand, is a structurally more complex constraint. This is not surprising because CNEC can be intuitively viewed as a limiting case of NEC. Indeed, this was formally shown in [4].

Proposition 2 [4]**.** CNEC_i *is* Π^0_3 *complete for the Borel hierarchy for a zero sum ME game.*

The above results have interesting consequences in terms of first-order definability. Note that certain infinite sequences over our vocabulary $(V_0 \cup V_1)$ can be coded up using first-order logic over discrete linear orders $(\mathbb{N}, <)$, where $\mathbb{N}$ is the set of non-negative natural numbers. Indeed, for every i and for every $a \in V_i$, let a^i_0 be a predicate such that given a sequence $x = x_0x_1\dots,\ x_j \in (V_0 \cup V_1)$

for all $j \geq 0$, $x \models a_0^i(j)$ iff $x_j = a$. Closing under finite Boolean operations and $\forall, \exists$, we obtain the logic $\mathsf{FO}(<)$. Now for any formula $\varphi \in \mathsf{FO}(<)$ and for any play ρ of an ME game $\mathcal{G}$, $\rho \models \varphi$ can be defined in the standard way. Thus every formula $\varphi \in \mathsf{FO}(<)$ gives a set of plays $\rho(\varphi)$ of $\mathcal{G}$ defined as: $\rho(\varphi) = \{\rho \in (V_0 \cup V_1)^\omega | \ \rho \models \varphi\}$. A set $A \subseteq (V_0 \cup V_1)^\omega$ is said to be $\mathsf{FO}(<)$ definable if there exists a $\mathsf{FO}(<)$ formula φ such that $A = \rho(\varphi)$. The following result is well-known.

Theorem 1 [21]**.** *$A \subseteq (V_0 \cup V_1)^\omega$ is $\mathsf{FO}(<)$ definable iff $A \in (\Sigma_2^0 \cup \Pi_2^0)$.*

Thus $\mathsf{FO}(<)$ cannot define sets that are higher than the second level of the Borel hierarchy in their structural complexity. Thus as a corollary of Proposition 2 and Corollary 1, we have

Corollary 2. *CONS_0, RES_0, COH_0, NEC_0 are $\mathsf{FO}(<)$ definable but not CNEC_i.*

This agrees with our intuition because as we observed, CNEC_i is a limit constraint and $\mathsf{FO}(<)$, being local [14], lacks the power to capture it. To define CNEC_i one has to go beyond $\mathsf{FO}(<)$ and look at more expressive logics. One such option is to augment $\mathsf{FO}(<)$ with a counting predicate cnt which ranges over $(\mathbb{N} \cup \{\infty\})$ [20]. Call this logic $\mathsf{FO}(<, \mathsf{cnt})$. One can write formulas of the type $\exists^\infty x \varphi(x)$ in $\mathsf{FO}(<, \mathsf{cnt})$ which says that "there are infinitely many x's such that $\varphi(x)$ holds." Note that it is straightforward to write a formula in $\mathsf{FO}(<, \mathsf{cnt})$ that describes CNEC_i. Another option is to consider the logic $\mathscr{L}_{\omega_1\omega}(FO, <)$ which is obtained by closing $\mathsf{FO}(<)$ under infinitary boolean connectives $\bigvee_j$ and $\bigwedge_j$. We can define a strict syntactic subclass of $\mathscr{L}_{\omega_1\omega}(FO, <)$, denoted $\mathscr{L}^*_{\omega_1\omega}(FO, <)$, where every formula is of the form $O_p O_q \ldots O_t \varphi_{pq\ldots t}$, where, for $k \in \{p, q, \ldots, t-1\}$, $O_k = \bigvee_k$ iff $O_{k+1} = \bigwedge_{k+1}$ and each $\varphi_{pq\ldots t}$ is an $(FO, <)$ formula, $p, q, \ldots, t \in \mathbb{N}$. That is, in every formula of $\mathscr{L}^*_{\omega_1\omega}(FO, <)$, the infinitary connectives are not nested and occur only in the beginning. We can then show that $\mathscr{L}^*_{\omega_1\omega}(FO, <)$ can express sets in any countable level of the Borel hierarchy. We do not go into further details here.

We now turn to strategic analyses of actual conversations. Consider this example, an excerpt from the 1988 Dan Quayle-Lloyd Bentsen Vice-Presidential debate which has exercised us now for several years, from the perspective of the theory of ME games developed above.

Example 4. Quayle (Q), a very junior and politically inexperienced Vice-Presidential candidate, was repeatedly questioned about his experience and his qualifications to be President. Till a point in the debate both of them were going neck to neck. But then to rebut doubts about his qualifications, Quayle compared his experience with that of the young John (Jack) Kennedy. To that, Bentsen (BN) made a discourse move that Quayle apparently did not anticipate. We give the relevant part of the debate below:

a. **Quayle**: *... the question you're asking is, "What kind of qualifications does Dan Quayle have to be president", [...] I have far more experience than many others that sought the office of vice president of this country. I have as much experience in the Congress as Jack Kennedy did when he sought the presidency.*

b. **Bensten**: *Senator, I served with Jack Kennedy. I knew Jack Kennedy. Jack Kennedy was a friend of mine. Senator, you're no Jack Kennedy.*
c. **Quayle**: *That was unfair, sir. Unfair.*
d. **Bensten**: *You brought up Kennedy, I didn't.*

Example (4) is an example of how a player can go inconsistent in a debate, which has disastrous consequences, if the Jury enforces consistency as a necessary component of any winning condition. But the analysis depends on the semantics of discourse relations. It would seem that Quayle was unaware that (Example 4b.) was a possible move for Bentsen in a strategy of countering his commitments (we shall talk more about unawareness shortly). However, note that Quayle's commitments in (Example 4a.) are not innocuous in the first place. He brings up as a comparison one of the most revered Presidents in contemporary American history; and while it is true that John F. Kennedy, like Quayle, was a relatively inexperienced junior senator when he ran for President in 1960, Quayle could have chosen many other figures for comparison—for instance, Richard Nixon's credentials prior to his taking the post of Vice-President in 1952 were also comparable to Quayle's. But by choosing JFK as a reference and by referring to him with his nickname 'Jack' used by his advisors and friends, Quayle made the suggestion or weak-implicature, that perhaps he would be comparable in other ways to JFK. It certainly put Quayle's experience or lack thereof in a favorable light.

Notice too that Quayle did not come out with a bald assertion of this implicature in (Example 4a.). He did not say

a'. *I have as much experience in the Congress and as much Presidential potential as Jack Kennedy did when he sought the presidency.*

He sensed this would be a dangerous move, opening him up to attack and perhaps even ridicule, either from his opponent or at least in the minds of the Jury. So instead, he couched his message in an implicit form.

Our intuition is that Quayle did not anticipate a direct attack on the implicature he was drawing out. Perhaps he was not even aware that he was making such an implicature, though our discussion of alternatives suggests that something like that implicature is there and the result of a choice of Quayle's comparison. In any case, Quayle had no real counter-move or strategy prepared, we feel.

So what happened with Quayle's response? (Example 4d.) in discourse theory terms is a 'commentary' on Bentsen's attack move. Commentaries carry with them a commitment by their speaker to the content they are commenting on. Now if the commentary's target is the *content* of what Bentsen said, then this is devastating for Quayle. By saying Quayle is no Kennedy, Bentsen is implicating something stronger, that Quayle is not of Presidential material. With commentary on the *content*, Quayle then commits to that content. In so doing he commits to his not being of Presidential stature when precisely his winning condition was to constantly come back to that commitment and reaffirm it. His commitments are now inconsistent, and inconsistency can be a game-losing property in a conversation. Moreover, this was an inconsistency involving an intrinsic property of Quayle's winning condition.

There is an alternative interpretation of the commentary move (Example 4d.) by Quayle. The commentary move is not about the content of Bensten's move but rather about the fact that Bensten made this move. This seems more plausible and it commits Quayle on the face of it only to the fact that Bentsen made a particular discourse move. But by not counter-attacking Bensten, Quayle sends a message that is terrible for him. First, he commits that the attack is coherent and responsive. Second, by not replying he concedes and commits to the proposition that the content of Bensten's move *and its implicatures* are not attackable. That is, Quayle implicates he has no means to refute the content of the attack. But this in turn implies that he implicitly must commit to their content. Hence, his non-reply makes his commitments look inconsistent.

Example 4 also lends itself to an analysis from the perspective of 'unawareness' of moves available to one player by the other player. What happens when Player 0 thinks that an ME game $\mathcal{G}$ is being played over a vocabulary $(V_0 \cup V_1)$ whereas Player 1 actually has moves available to him from a larger vocabulary $W_1 \supsetneq V_1$? That is $\mathcal{G} = ((V_0 \cup W_1)^\omega, \mathcal{J})$. To answer this question, we make use of the following result.

Proposition 3. *Let V and W be countable vocabularies such that $V \subsetneq W$. Then, a Σ_0^1 complete set in X^ω jumps to Δ_2^0 in Y^ω, and all other sets stay in the same level.*

To preserve the continuity of the text, we give the proof in the appendix. Proposition 3 thus implies that a winning set win which is Σ_1^0 in an ME game $\mathcal{G} = ((V_0 \cup V_1)^\omega, \mathcal{J})$ might be Δ_2^0 in an ME game $\mathcal{G}' = ((V_0 \cup W_1)^\omega, \mathcal{J})$ where $W_1 \supsetneq V_1$. win is hence more complex structurally in $\mathcal{G}'$. The result of this might be that even if Player 0 had a winning strategy σ_0 in $\mathcal{G}$, σ_0 might not be winning for her in $\mathcal{G}'$.

Coming back now to Example 4, Quayle believed that if he just made his comparison with John F. Kennedy, to whom he refers by his colloquial nickname used by friends and members of JFK's cabinet, no matter what the response Bentsen made, that is the responses of which he was aware in V_1 would hurt his chances. He had a simple goal, which we could characterize as a Σ_1^0 goal: mentioning this comparison. As such, he also had a simple winning strategy for achieving this goal. However, in the larger set of discourse moves, W_1 Bentsen had an attack that floored Quayle. In fact, we can easily show that Quayle had no winning strategy for keeping to his winning condition *over strings in* $(V_0 \cup V_1)^\omega$; given that his winning strategy depended on *his opponent*'s use of moves in V_1, all that Bentsen had to do to defeat Quayle was to use a coherent move in W_1 to upset Quayle's strategy. This is a simple-minded yet insightful analysis of the interesting and deep notion of unawareness which we wish to fully explore in our future work. To fully understand this phenomenon, one has to appeal to the theory of epistemic games, to which we now turn.

3 Imperfect Information and Epistemic Considerations

So far we have shown how to model strategic conversations as infinite sequential games and how to reason about the complexity of certain commonly used winning goals in such conversations in terms of both their topological and logical complexities. A couple of issues that we have not addressed are:

- Yes, a conversation at the outset can be potentially infinite. But still in real life, the Jury does end the game after a finite amount of time, after a finite number of turns. By doing so, how can it be sure that it has correctly determined the outcome of the conversation? In other words, how does the Jury, at any point in a conversation gauge how the players are faring and when does it decide to call it a day?
- How does the Jury determine the winning conditions win_0 and win_1? Surely, it does not come up with a arbitrary subset of $(V_0 \cup V_1)^\omega$ with an arbitrary Borel complexity.

To address the above questions [3] introduced the model of 'weighted ME games' or WME games. A WME game is similar to a ME game except that the Jury instead of specifying the winning sets win_i as subsets of $(V_0 \cup V_1)^\omega$, determines them on-the-fly. It does so by evaluating every move of each player by assigning a 'weight' or a 'score'. The cumulative weight of a conversation ρ is then the discounted sum of these individual weights. [3] also showed that given an $\epsilon > 0$ there exists a number n_ϵ such that the Jury can stop the game after n_ϵ turns and determine the winner, being sure that no player could have done more than ϵ better than what they had already done. We do not go into the details here but refer the interested reader to that paper.

In this section, we study the exact information structure implicit in the strategic reasoning in conversations by extending framework of ME games with epistemic notions. We use the well-established theory of type-structures, first introduced in [17] and widely studied since. We assume that each player $i \in (\{0,1\} \cup \{\mathcal{J}\})$ has a (possibly infinite) set of types T_i. With each type t_i of Player i is associated a (first-order) belief function $\beta_i(t_i)$ which assigns to t_i a probability distribution over the types of the other players. That is, $\beta_i : T_i \rightarrow \Delta(\prod_{j \neq i} T_j)$. $\beta_i(t_i)$ represents the 'beliefs' of type t_i of Player i about the types of the other players and the Jury. The higher-order beliefs can be defined in a standard way by iterating the functions β_i. We assume that each type t_i of each Player i starts the game with an initial belief $\beta_i(t_i) \in \Delta(\prod_{j \neq i} T_j)$, called the 'prior belief'. The players take turns in making their moves and after every move, all the players dynamically update their beliefs through Bayesian updates. The notions of 'optimal strategies', 'best-response', 'rationality', 'common belief in rationality' etc. can then be defined in the standard way (see [12]).

Having imposed the above epistemic structure on ME games, we can now reason about the 'rationality' of the players' strategies. In order to justify or predict the outcome of games, many different solution concepts viz., Nash equilibrium, iterated removal of dominated strategies, correlated equilibrium, rationalizability etc. have been proposed [5,10,22]. Most of them have also been characterized in terms of the exact belief structure and strategic behavior of the players (see [12] for an overview). We can borrow results from this rich literature to predict or justify outcomes in strategic conversations. The details of the above is on-going work and we leave it to an ensuing paper. However, let us apply the above concepts and analyze our original example of Bronston and the Prosecutor.

To illustrate the power of types, let us return to Example 1. One conversational goal of the Prosecutor in Example 1 is to get Bronston to commit to an answer eventually (and admit to an incriminating fact) or to continue to refuse to answer (in which case he will be charged with contempt of court). Under such a situation, the response (1d.) of Bronston is clearly a clever strategic move. Bronston's response (1d.) was a strategic move aimed to 'misdirect' the Jury $\mathcal{J}$. He believed that $\mathcal{J}$ was of a type that would be convinced by his ambiguous response and neither incriminate him nor charge him with perjury nor of contempt of court. His move was indeed rational, *given his belief* about the Jury type. It turns out that while the jury of a lower court $\mathcal{J}_1$ was not convinced of Bronston's arguments and charged him with perjury, a higher court $\mathcal{J}_2$ overturned the verdict and released him. Thus his belief agreed with $\mathcal{J}_2$ but not $\mathcal{J}_1$.

Powerful as the above techniques are, one has to exercise caution and define the moves, states and the types of the players carefully. Having too rich a type space can lead to inexistence results. For example, consider the following situation.

Example 5. Two philosophers Michael and Brian must occupy a panel discussion before an audience. They both have an extremely good opinion of themselves. Each philosopher's goal is to prove that he is better than the other by talking highly of himself. They exchange dialogues where in every turn a philosopher can boast of himself as long as he wants to but eventually has to stop and concede the turn to the other philosopher. The audience, unlike the philosophers, can become impatient and decide at any moment to stop the discussion, give its verdict and leave. It offers the win to the one who has spoken 'more' of himself.

Clearly, the above game does not have an equilibrium pair of strategies. To see this, suppose without loss of generality that Michael speaks first. He has to concede the turn to Brian after saying m_1 points in his own favour (say). Brian plays next and he says b_1 points in his own favour. Now suppose the audience decides to stop the conversation after k sentences have been uttered by both the players. We can always find a k such that neither Michael nor Brian has a winning strategy. Indeed, if $b_1 > m_1$ and $k = b_1 + m_1$ then Michael cannot win. However, if $k < 2p_1$ Brian cannot win. Thus, both Michael and Brian could have done better by having said a 'bit more' about themselves in their corresponding turns. Without equilibria, it is unclear what our speakers should do in such a situation. Such examples pose a challenge to a fundamental assumption amongst linguists and philosophers that conversation is a rational activity with optimal strategies for achieving speakers' goals.

Our example in fact follows from a general result by [18], which says that if the space of types is not a separable set then there always exists a game with no equilibrium. In the above game, associating the types of a player with possible subsets of her strategies, we see that the space of types is a set with a large cardinality ($> \aleph_1$) and hence we lose separability.

Conversationalists are aware implicitly of the dangers of such cases and debates have exogenous means of ensuring that there are optimal strategies for the speakers to follow. For instance, in debates there is usually a 'moderator' who ensures that all the participants get a fair chance to speak. She might interrupt a speaker and pass the turn on to another speaker. Note that this variant of our example game (Example 5) restores the presence of an equilibrium: each philosopher keeps speaking about himself till he is interrupted by the moderator - that is the best he can do anyway since he does not know in advance *when* he will be interrupted. More generally, we can restore separability (and hence the existence of equilibria) by limiting the set of types. One way is to require that each type (and hence each winning condition that players might countenance) be expressible in some language with a limited complexity. As long as the language is countable, separability can be restored for type spaces, and then by [18] any such game must have an equilibrium. Another way is to simply restrict the space of types to a strict subset of the entire space [8,9]. Thus not all possible subsets of the conversational space define rational or rationalizable conversational goals. In the case of our example (Example 5) this means that our philosophers should limit the set of types that they consider possible. For example, they might expect each turn to last for a maximum of 20 min (say) so that their belief closed set is restricted to types of players who speak for a maximum of 20 min in each turn. This ensures the presence of an equilibrium.

4 Conclusion

We believe that the work summarized and extended in this paper is the start of a novel yet powerful approach to study strategic conversations. We have but scratched the surface here and there are many directions into which we would like to delve deeper in the future. One such direction, as we already mentioned, is to work out the epistemic theory of ME games in full detail. That is our current work in progress. Another is that in the present work we have considered the Jury as a 'passive' entity - it simply evaluates the play and determines the winner. However, in real life situations, the Jury can be an 'active' member of the conversation itself. It can 'applaud' or 'criticize' moves of the players. Thus, the Jury can be seen as making these moves in the game. Based on what the players observe about the Jury, they may update or change their beliefs and vice-versa. Incorporating this into our ME games requires a modification of the current framework where the Jury is another player making moves from its own set of vocabulary. We plan to explore this in future work.

Finally, in addition to the Jury, debates usually also have a moderator whose job is to conduct the debate and assign turns to the players. The moderator may also actively 'pass comments' about the moves of the players. A fair moderator gives all the players equal opportunity to speak and put their points across. However, if the moderator is unfair, he may 'starve' a particular player by not letting her enough chance to speak, respond to attacks and so on. Exploring the effects the inclusion of a moderator in such conversations is another interesting topic which we leave for future work.

A Appendix

To prove Proposition 3 we shall refer to a result from [23].

Proposition 4 [23]**.** *If V is an infinite vocabulary, the subsets of V^ω of the form AV^ω, where A is a set of words of bounded length of V^* are clopen.*

We now prove Proposition 3.

Proof. First, we show that the set V^ω is closed but not open in the space W^ω. That is, $V^\omega \in (\Pi_1^0 \setminus \Sigma_1^0)$ in W^ω. Indeed, we have

$$V^\omega = \bigcap_{n \geq 0} V^n W^\omega$$

For every $n \geq 0$ we have that V^n is a set of words of bounded length of V^* and hence by Proposition 4 we have that $V^n W^\omega$ is clopen. Thus V^ω is closed. Also, V^ω is not open by the definition of open sets.

Now let $X \subset V^\omega$ be $(\Sigma_1^0 \setminus \Pi_1^0)$ in V^ω. By definition, we know that X is of the form AV^ω where $A \subset V^*$ Thus

$$X = AV^\omega = AW^\omega \cap V^\omega$$

Then since AW^ω is open (Σ_1^0) in W^ω and V^ω, as we just showed, is closed $(\Pi_1^0 \setminus \Sigma_1^0)$ in W^ω, their intersection is a Δ_2^0 set.

Next let $Y \subset X^\omega$ be $(\Pi_1^0 \setminus \Sigma_1^0)$ in V^ω. We show that Y is also closed in W^ω. Indeed, because the complement of Y in V^ω is of the form BV^ω for some $B \subset V^*$. Hence, the complement of Y in W^ω is

$$W^\omega \setminus Y = BW^\omega \cup W^*(W \setminus V)W^\omega$$

which is open.

References

1. Asher, N., Lascarides, A.: Logics of Conversation. Cambridge University Press, Cambridge (2003)
2. Asher, N., Lascarides, A.: Strategic conversation. Semant. Pragmat. **6**(2), 1–62 (2013). http://dx.doi.org/10.3765/sp.6.2
3. Asher, N., Paul, S.: Evaluating conversational success: weighted message exchange games. In: Hunter, J., Stone, M. (eds.) 20th workshop on the semantics and Pragmatics of Dialogue (SEMDIAL), New Jersey, USA, July 2016 (To appear)
4. Asher, N., Paul, S., Venant, A.: Message exchange games in strategic conversations. J. Philos. Log. (2016, in press)

5. Aumann, R.: Subjectivity and correlation in randomized strategies. J. Math. Econ. **1**, 67–96 (1974)
6. Baltag, A., Moss, L.S.: Logics for epistemic programs. Synthese **139**(2), 165–224 (2004)
7. Baltag, A., Moss, L.S., Solecki, S.: The logic of public announcements, common knowledge and private suspicions. Technical report SEN-R9922, Centrum voor Wiskunde en Informatica (1999)
8. Battigalli, P.: Rationalizability in infinite dynamic games with incomplete information. Res. Econ. **57**(1), 1–38 (2003)
9. Battigalli, P., Siniscalchi, M.: Rationalization and incomplete information. B.E. J. Theoret. Econ. **3**, 1–46 (2003)
10. Bernheim, B.D.: Rationalizable strategic behaviour. Econometrica **52**(4), 1007–1028 (1984)
11. Crawford, V., Sobel, J.: Strategic information transmission. Econometrica **50**(6), 1431–1451 (1982)
12. Dekel, E., Siniscalchi, M.: Epistemic game theory (chapter 12). In: Aumann, R.J., Hart, S. (eds.) Handbook of Game Theory with Economic Applications, vol. 4, pp. 619–702. Elsevier Publications, Cambridge (2015)
13. Franke, M.: Semantic meaning and pragmatic inference in non-cooperative conversation. In: Icard, T., Muskens, R. (eds.) ESSLLI 2008-2009. LNCS (LNAI), vol. 6211, pp. 13–24. Springer, Heidelberg (2010). doi:10.1007/978-3-642-14729-6_2
14. Gaiffman, H.: On local and non-local properties. In: Proceedings of Herbrand Symposium, Logic Colloquium 1981. North Holland (1982)
15. Glazer, J., Rubinstein, A.: On optimal rules of persuasion. Econometrica **72**(6), 119–123 (2004)
16. Grädel, E., Thomas, W., Wilke, T. (eds.): Automata, Logics, and Infinite Games: A Guide to Current Research [Outcome of a Dagstuhl Seminar, February 2001]. LNCS, vol. 2500. Springer, Heidelberg (2002)
17. Harsanyi, J.C.: Games with incomplete information played by Bayesian players, parts i-iii. Manag. Sci. **14**, 159–182 (1967)
18. Hellman, Z., Levy, Y.: Bayesian games with a continuum of states. Technical report, Bar Ilan University (2013)
19. Kechris, A.: Classical Descriptive Set Theory. Springer, New York (1995)
20. Libkin, L.: Elements of Finite Model Theory. Springer, New York (2004)
21. McNaughton, R., Papert, S.: Counter-free automata. In: Research Monograph, vol. 65. MIT Press, Cambridge (1971)
22. Nash, J.: Non-cooperative games. Ann. Math. **54**(2), 286–295 (1951)
23. Perrin, D., Pin, J.E.: Infinite Words - Automata, Semigroups, Logic and Games. Elsevier, Cambridge (1995)
24. Spence, A.M.: Job market signaling. J. Econ. **87**(3), 355–374 (1973)
25. Venant, A., Asher, N.: Dynamics of public commitments in dialogue. In: Proceedings of 11th International Conference on Computational Semantics, pp. 272–282. Association for Computational Linguistics, London, April 2015
26. Asher, N., Venant, A.: Ok or not ok? In: Semantics and Linguistic Theory 25. Cornell University Press (2015)