Reachability and Coverage Planning for Connected Agents*

Extended Abstract

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Unmanned autonomous vehicle assisted information gathering missions have quickly picked up interest. Indeed, the advances on drones are making this type of missions possible. Thus, we study multi-agent path planning problems, namely reachability and coverage, for such missions with a connectivity constraint. This version of the multi-agent path planning asks to generate a plan, a sequence of steps, for a group of agents that are to stay connected during the missions while satisfying the specified goal.

In this paper, we study the complexity of the coverage and reachability problems for a cooperation of agents with a connectivity constraint which restrain their movement. We identify a class of topological graphs which allows one to reduce the complexity of the decision problems from PSPACE-complete to LOGSPACE. We show, on the other hand, that the bounded versions of the previous problems are NP-complete.

Additional Key Words and Phrases: Multi-Agent Path Planning; Complexity Theory; Connectivity; Coverage; Reachability

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Introduction. Search and rescue missions are difficult to undertake and achieve. In fact, beyond the time constraint of such tasks in order to retrieve the endangered individuals, the place to search can be large as well as hazardous. Thus, unmanned autonomous vehicles (UAV) have caught interest for their ability to be used in unsafe conditions, without (or less) allocation of experts. Furthermore, multiple UAVs can be used to cover a large portion of the place quicker than the experts could in most cases. This approach is used for decision-making and, thus, the UAVs are connected with a base at which an expert can analyze the reported data.

The goal of this work is to study, complexity wise, the problems associated with the usage of connected UAVs. The settings and type of missions we consider are closely linked to the requirements described by the use of UAVs for search and rescue missions. Hence, we examine the complexity of generating reachability and coverage plans of multiple connected agents on topological graphs.

Topological graphs are composed of two types of edges, namely movement and communication edges. The movement edges describe the allowed movements between two nodes. A communication edge enables two agents to communicate from their respective nodes. A plan is a sequence of configurations of agents on the topological graph. For a plan to be valid, it needs to keep the agents connected with the base, which is a specific node of the graph. An agent can be directly

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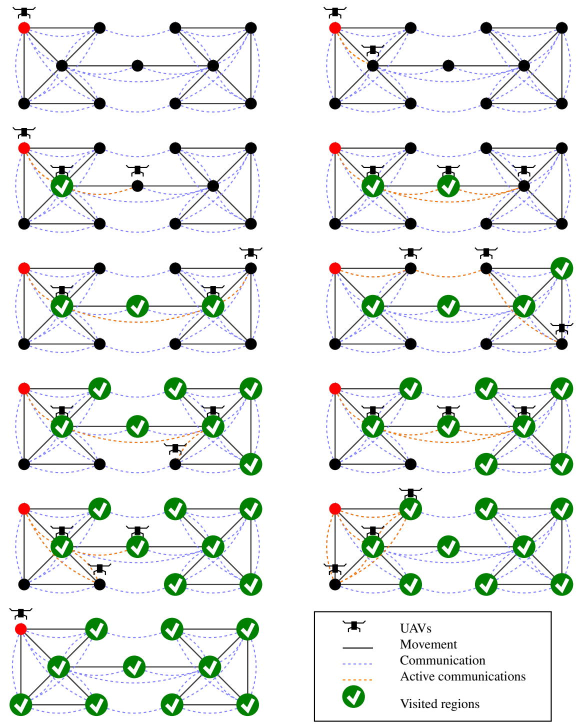

Fig. 1. Example of a mission execution.

connected to the base with a communication edge or it can be connected via other agents who act as relays. Thanks to such relays, agents can cooperatively reach locations behind obstacles which are not directly connected to the base.

Two major types of objectives arise from search and rescue missions. Either an agent is required to reach a specific location in order to sense it, or the agents need to spread over the place to examine several locations. UAVs can, for instance, be used for locating victims of an avalanche by quickly covering as much ground as possible. We explore, in this paper, the complexity of the so-called *Reachability* problem and the *Coverage* problems along with their bounded versions, *bReachability* and *bCoverage*.

In Figure 1, we depict a coverage plan execution. The red node denotes the base of the topological graph from which the agents start and need to stay connected with.

The multi-agent path planning (MAPP) with connectivity constraint was introduced in [3], in which that the existence of a bounded plan for the reachability of a configuration is shown NP-hard. Recently, Tateo et al. showed that the existence of a connected plan for the reachability is PSPACE-complete in [7]. We extend both of the latter works by considering the connected coverage planning and subclasses of topological graphs on which we can obtain more efficient algorithms for the previously stated problems.

Preliminaries. As mentioned earlier, we study reachability and coverage problems on subclasses of topological graphs. A *directed* topological graph is a tuple $G = \langle V, \rightarrow, \cdots\rangle$, V is a set of nodes, $\rightarrow \subseteq V \times V$ is a set of directed movement

<table>
<tr><th></th><th>Reach</th><th>Cover</th><th>bReach</th><th>bCover</th></tr>
<tr><td>Directed</td><td>PSPACE-c</td><td>PSPACE-c</td><td rowspan="3">NP-c
[Hollinger]</td><td rowspan="5">NP-c</td></tr>
<tr><td>nc</td><td rowspan="2">PSPACE-c
[Tateo et al.]</td><td>PSPACE-c</td></tr>
<tr><td>Undirected</td><td>?</td></tr>
<tr><td>sm</td><td rowspan="2">in LOGSPACE</td><td rowspan="2">in LOGSPACE</td><td>NP-c</td></tr>
<tr><td>cc</td><td>in LOGSPACE</td></tr>
</table>

Fig. 2. Complexity results.

edges and $\cdots \subseteq V \times V$ is a set of undirected communication edges. The subclasses of topological graphs are named and stated as follows:

- A *neighbor-communicable* topological graph (nc) is a topological graph in which $v \rightarrow v'$ implies $v \cdots v'$.
- A *sight-moveable* topological graph (sm) is a topological graph with $\rightarrow$ undirected and reflexive and whenever $v \cdots v'$ there exists a sequence $\rho = \langle \rho_1, \ldots, \rho_n \rangle$ of nodes such that $v = \rho_1$, $v' = \rho_n$, $v \cdots \rho_i$ and $\rho_i \rightarrow \rho_{i+1}$ for all $1 \leq i < n$.
- A *complete-communication* topological graph (cc) is a topological graph in which $\rightarrow$ is undirected and reflexive, and $\cdots = V \times V$.

Intuitively, a neighbor-communicable topological graph makes sure that if an agent can move to another node in one step then both nodes are communicating. In sight-moveable ones, the communication is restricted to line-of-sight communication and the agents cannot communicate through obstacles. A complete-communication topological graph can model a situation where a hovering agent connects all agents to the base in the whole area.

On these classes of topological graphs, we study the reachability of a particular configuration of agents in the graph, denoted *Reachability*. We also study the problem of coverage of the graph, we consider that a plan is covering if and only if every nodes of the graph are visited during the plan by at least one agent, denoted *Coverage*. Furthermore, we study the bounded versions of the previous problems, respectively denoted *bReachability* and *bCoverage*, where a bound on the size of a plan is part of the input, and is encoded in unary. These are the decisions problems associated to the optimization problems for computing minimal plans.

The results stated later consider that the agents are anonymous and, thus, two configurations of agents on the graph are *equivalent* if one is a reordering of the other. Furthermore, we do not consider head-on- and meet- collisions. Hence, two agents can be located at the same node or they can travel on the same edge in opposite direction during a step of the plan.

Known results. The following theorem is extracted from the original work of Hollinger and Singh in which the bounded connected reachability problem is studied and proved to be NP-hard on undirected topological graph. Thus, we can state this lower bound on directed topological graphs.

THEOREM 1 ([3]). *$bReachability_{dir}$ is NP-hard.*

Tateo et al. shown that the existence of a plan for the reachability is PSPACE-complete on directed graphs and discuss that this result extends to directed topological graphs with a base.

THEOREM 2 ([7]). *$Reachability_{dir}$ is PSPACE-complete.*

Contributions. We extend previously known results and show that the coverage problems are PSPACE-complete on directed and nc-graphs, and remarkably show that the complexity is reduced to LOGSPACE for sm- and cm-graphs. Bounded reachability and coverage problems do however remain NP-complete for sm-graphs. All our results are summarized in Fig. 2.

To obtain the LOGSPACE algorithm, we use the result of Reingold who showed that the undirected s-t connectivity problem is in LOGSPACE [6]. Since LOGSPACE $\subseteq$ NC, it admits polylogarithmic time algorithm on a parallel machine with a polynomial number of processors [2]. Although the bounded problems on sight-moveable topological graphs are intractable, it might be possible to use, for instance, SAT solvers, or obtain polynomial-time approximation algorithms. We lack the complexity result for undirected topological graphs, which is shown by a question mark in the table.

Related work. Coverage planning has been applied to other areas such as lawn mowing and floor cleaning [1]. This problem was addressed with analytic techniques [8, 10]. We advocate for the use of formal methods for provable guarantees on the behaviors; see *e.g.* [4, 9]. The coverage problem presented is closely related to the traveling salesman problem (TSP) or more precisely, its generalization, multiple TSP. An overview of TSP and its extensions can be found in [5]. However, to the best of our knowledge, a connected version of mTSP or VRP has not been studied.

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