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Firms’ Labor Market Power and Aggregate Instability∗

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Abstract: This work proposes to study the emergence of aggregate instability, in the form of macroeconomic fluctuations due to the volatility of agents’ expectations, caused by imperfect competition on the labor market. We consider that firms have some monopsony power which is introduced by a) considering that firms face a finite elasticity of labor supply, b) there is a finite number of firms operating under Cournot competition on the labor market. We show that given a free-entry and zero profit conditions, we obtain local indeterminacy when the elasticity of the sectoral labor supply is sufficiently low and factors are substitutable enough. We illustrate numerically our results with some empirical estimates for OECD countries and we conclude that expectation-driven fluctuations is obtained for plausible values.

Keywords: Monopsony power, local indeterminacy, expectation-driven business cycles.

Journal of Economic Literature Classification Numbers: C62, E32, J42.

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1 Introduction

Although the economic effect of monopsony and oligopsony power in the labor market are well known and studied for some particular labor market, as heavily documented in Manning [19], the macroeconomic consequences of firms’ market power in the labor market through monopsony are less investigated. One potential reason is that the view of a unique firm on the labor market at the aggregate level seems un-plausible. Yet, several arguments tend justify the case for monopsony power on the labor market outside the one firm-one market case. From a theoretical point of view, a group of firms may not face an infinitely-elastic labor supply as in the standard view of labor market but an upward-sloping supply.\footnote{Note that monopsony power also arises in presence of frictions on the labor markets. This is another way to consider upward-sloping labor supply.} Also, a labor market may be characterized by a finite number of firms which implies a form of concentration on the demand side. Recent empirical contributions of Azar et al. [1], [2] and also Webber [21] supports these features and points out that monopsony power and its macroeconomic effects are more significant than we previously thought.

The present paper proposes to account for firms’ market power in the labor market in an otherwise standard general equilibrium model and shed light on a new and less expected effect: monopsony power may cause aggregate instability and be a source of expectation-driven fluctuations. A similar mechanism has been found by Dos Santos Ferreira and Lloyd-Braga [9], [10] and Jaimovich [15] where imperfect competition in the product markets and an endogenous markup on price leads to an economy with local indeterminacy and sunspot equilibria.\footnote{The significance of firms’ market power in the product markets at the aggregate level also received recent supports by empirical studies (see for instance De Loecker and Eeckhout [8]).} Different from these contributions that assume a perfectly competitive labor market, we exploit the symmetry between the markup on price and the markdown on wage\footnote{Both markup and markdown have a similar effect on the real wage paid to the workers which is lower than the marginal product of labor and is decreasing with firms’ market power.} to exhibit this original effect of monopsony power.

To introduce firms’ market power, we consider that labor supplied by the households is an imperfect substitute across different labor markets and in which firms are large enough, but low with respect to the economy. As a result, we capture two sources of monopsony power: on the one hand, firms face an upward-sloping labor supply at the individual level, on the other hand, there is some concentration of firms on the labor markets. We assume that firms operate under Cournot competition on labor markets and that the number of firms is
endogenous by introducing a symmetric free-entry condition. Accordingly, firms’ market power, measured by the size of the markup on wage, depends on the degree of substitutability of labor supply across markets and the level of economic activity. In particular, firms have a significantly large market power when the substitutability is low and the markup on wage is decreasing with the level of activity as suggested by empirical evidence (see Jaimovich [15] and Jaimovich and Floetotto [16] for evidence of countercyclical markup, and Hirsch et al. [12] for empirical evidence of a countercyclical monopsony power). The rest of the model is standard with respect to the literature: we consider infinitely-lived households with separable preferences supplies an infinitely-elastic amount of total labor and choose an optimal plan over time for capital and consumption. Firms have access to a constant return to scale general production allowing for capital and labor substitution and choose their production plan under monopolistic competition on the product markets.

We show that given a minimal degree of monopolistic competition and a high enough substitutability of labor and capital, local indeterminacy arises if the elasticity of substitution of labor across markets is low enough, i.e. when monopsony power is large. Through a numerical illustration, we conclude that this outcome is plausible for a large set of OECD countries. The reason for the emergence of expectation-driven fluctuations in this economy follows that if there is an optimistic expectation about the future, say households anticipate an increase of the wage rate in the future, there is a boom in investment leading to new profit opportunities, and entry of new firms in the market. Since the markdown is countercyclical, the creation of new firms decreases marginal cost due to a more intense competition which shifts production up and makes the initial expectation self-fulfilling.

The rest of the paper is organized as follows. Section 2 presents the model and define the equilibrium solution of agents’ behaviour. In Section 3, we focus on the benchmark case where only oligopsony power is considered and study the local dynamics of this benchmark economy. Section 3 contains our main results, provides a numerical illustration to discuss the plausibility of our conclusions and gives some economic intuition. In Section 5, we draw some concluding comments and all the proofs are given in the Appendix.

2 The Model

This section describes the details of an infinite horizon economy characterized by oligopsonistic labor markets and presents the subsequent equilibrium conditions.
2.1 Households’ behavior

We consider an economy populated by a large number of identical infinitely-lived agents. We assume without loss of generality that the total population is constant and normalized to one. At each point in time, an agent supplies an infinitely-elastic amount of labor \( l(t) \in [0, \bar{l}] \), with \( \bar{l} > 1 \) his time endowment and chooses the allocation of hours worked in labor market \( i \in I, l(i, t) \). She derives her intertemporal utility from consumption \( c(t) \) and disutility from labor \( l(t) \) according to the instantaneous utility function \( U(c_t, l_t) \) such that:

\[
U(c_t, l_t) = \ln(c(t)) - Bl(t)
\]

Where \( B \) is a scaling constant. Households face the following budget constraint:

\[
\dot{k}(t) + c(t) + \delta k(t) = \frac{w(t)}{p(t)}l(t) + \frac{r(t)}{p(t)}k(t)
\]

with \( k(t) \) the capital stock, with initial condition \( k(0) = k_0 \), \( w(t) \) the wage rate, \( r(t) \) the rental rate of capital and \( \delta > 0 \) the depreciation rate of the capital stock, \( p(t) \) the price index. Denote the discount rate by \( \rho > 0 \) and \( \lambda \) the shadow price of capital, the current-value Hamiltonian writes:

\[
H = \ln(c(t)) - Bl(t) + \lambda(t)\left[ \frac{w(t)}{p(t)}l(t) + \frac{r(t)}{p(t)}k(t) - \delta k(t) - c(t) \right]
\]

The first-order conditions are:

\[
c(t)^{-1} = \lambda(t)
\]

\[
B = \lambda(t)\frac{w(t)}{p(t)}
\]

\[
\dot{\lambda}(t) = (\rho + \delta) - \frac{r(t)}{p(t)}
\]

\[
\dot{k}(t) = \frac{w(t)}{p(t)}l(t) + \frac{r(t)}{p(t)}k(t) - \delta k(t) - c(t)
\]

Any solution needs also to satisfy the transversality condition:

\[
\lim_{t \to +\infty} e^{-\rho t} \lambda(t)k(t) = 0
\]

We assume that households’ labor supply is imperfectly substitutable among many labor markets indexed by \( i \). In particular, the households’ total labor supply takes the form:
where $\gamma \in (0, +\infty)$ is the elasticity of substitution of hours worked across markets $i$. Note that we omit the time variable for all the variable depending on $i$ since they are going to be characterized by static solutions.\(^4\) This specification is considered by Berger, Herkenhoff and Mongey [6], Horvath [13] and Huang and Meng [14] while Cardi and Restout [7] and Katayama and Kim [18] follows an identical formulation but restricts to a two sector economy. When $\gamma$ tends to infinity, the utility cost of modifying hours worked in labor markets $i$ toward another is low and workers favors move their whole time endowment to the labor market with the highest wage. Hence, firms adjust their labor demand to attract workers which leads to wage equalization across all labors markets. In such case, as will be shown below, firms’ labor market power is minimal and close to perfect competition. In constrast, when $\gamma$ is finite and low, a minimal disutility of labor is obtained when workers share an equal working time between the different markets disregarding wage differences: this configuration implies a large labor market power for the firms.

Households’ labor income satisfies $w(t)l(t) = \int_0^1 w(i)l(i)di$. Given the latter and equation (9), the optimal allocation of sectoral labor supply is:

$$\frac{l(i)}{l(t)} = \left( \frac{w(i)}{w(t)} \right)^{\gamma}$$

with $w(t)$, the wage index:

$$w(t) = \left( \int_0^1 w(i)^{1+\gamma}di \right)^{\frac{1}{1+\gamma}}$$

A possible interpretation of equation (10) can be done as a labor supply in market $i$ relative to total labor supply. When $\gamma$ tends to 0, then the households inelastically supply a share of their total labor supply to market $i$, irrespective the difference of the wage of this market with the wage index. If $\gamma$ tends toward infinity, labor supply is infinitely elastic between sectors and freely moves between them leading to wage equalization as labor is perfectly substitutable across markets. Hence, $\gamma$ can be interpreted as a wage elasticity of labor.

\(^4\)The assumption of a continuum of labor market is a mere simplification. It is easily shown that our results do not depend on a discrete number of labor markets. As a result, under the present formulation, labor markets can be seen as local geographical markets but could be also seen as sectoral labor markets if a discrete formulation is introduced.
2.2 The production structure

2.2.1 Firms’ decisions

There is a final good $y(t)$ produced using $n$ intermediate goods $y_j(t)$ according to a CES technology:

$$y(t) = \left( \sum_{j=1}^{n} y_j(t)^{\frac{\phi - 1}{\phi}} \right)^{\frac{\phi}{\phi - 1}}$$  \hspace{1cm} (12)

where $\phi > 0$ is the elasticity of substitution between the intermediate goods. Denote the price of the intermediate good by $p_j(t)$ and $p = \left( \sum_{j=1}^{n} p_j(t)^{1-\phi} \right)^{1/(1-\phi)}$, the demand of the intermediate good is given by:

$$\frac{y_j(t)}{y(t)} = \left( \frac{p_j(t)}{p(t)} \right)^{-\phi}$$  \hspace{1cm} (13)

In the industry $j$, there is a unique firm $j$ which is small with respect to the whole economy and produce the intermediate good $y_j(t)$. Each firm $j \in J$ has access to a constant return to scale technology $y_j(t) = Af(k_j(t), l_j(t)) = Al_j(t)f(a_j(t))$ with $f(a_j(t))$ increasing and concave, $f(0) = 0$ and $f(+\infty) = +\infty$ and where $a_j(t) = \frac{k_j(t)l_j(t)}{l_j(t)}$, the intensive stock of capital used by firm $j$. We impose the following assumption about the production function:

**Assumption 1.** The function $f(a_j(t))$ is twice differentiable, increasing and concave and satisfies $f'(0) = +\infty$, $f'(+\infty) = 0$

We define $s(a_j) = \frac{a_jf'(a_j)}{f(a_j)}$, the elasticity of the production with respect to the intensive stock of capital and $\sigma(a_j) = \frac{-(1-s(a_j))f'(a_j)}{a_jf''(a_j)}$, the capital-labor elasticity of substitution. The firm also faces a fixed cost $F$. Furthermore, since there are $n(i, t)$ firms within each labor market $i$ and a unique firm on the $n(t)$ intermediate good markets, we assume Cournot competition over labor but monopolistic competition on the product markets. It follows that firm $j$ takes into account the demand for good $j$ and the labor supply in labor market $i$ as respectively given by equations (10) and (13). Hence, the profit maximization program is given by:
\[
\pi_j = \max_{y_j, l_j, k_j} \quad p(y_j(t))y_j(t) - w(l(i))l_j(t) - r(t)k_j(t)
\]
\[
\text{s.t.}\quad y_j(t) = A l_j(t)f(a_j(t)) - F
\]
\[
p(y_j(t)) = p(t) \left( \frac{y_j(t)}{y_j(t)} \right)^{\frac{\gamma}{\gamma-1}}
\]
\[
w(l(i)) = w(t) \left( \frac{l(i)}{l(t)} \right)^{\frac{1}{\gamma}}
\]
\[
l(i) = \sum_{j=1}^{n} l_j(t)
\]

The first-order conditions are:
\[
\frac{r(t)}{p_j(t)} = (1 - \varepsilon_p)Af'(a(t))
\]
\[
\frac{w(i)}{p_j(t)} = \frac{(1-\varepsilon_p)}{(1+\varepsilon_w)} A[f(a(t)) - a(t)f'(a(t))]
\]
\[
y_j(t) = A l_j(t)f(a_j(t)) - F
\]

with \(\varepsilon_p = \frac{1}{\phi}\) and \(\varepsilon_w = \frac{\partial w}{\partial l} = \frac{1}{\gamma \sum_j l_j}\) which are respectively the markup rule on price and the markdown on the wage rate. It follows that the markup decreases when the substitutability of the intermediate goods while and the markdown decreases when the wage elasticity of labor supply in market \(i\) increases.

### 2.3 Symmetric Equilibrium

We consider a symmetric equilibrium \(n(t) = n(i, t)\) with the following aggregate variables \(k(t) = n(t)k_j(t), l(t) = l(t) = n(t)l_j(t), y(t) = n(t)y_j(t), p(t) = p_j(t)\) and \(w(t) = w(t)\). It follows that the markdown on the wage is given by \(\varepsilon_w \equiv \varepsilon_w(n(t)) = \frac{1}{n(t)}\). Note that the latter expression implies that the markdown evolves in an opposite direction with respect to the number of firms: a higher number of firms implies \(ceteris paribus\) a tougher competition and reduces the markdown on wage which increases, in return the wage rate.

Given a symmetric equilibrium, we introduce a free-entry restriction which leads to the usual zero profit condition:
\[
n(t)l(t) \left[ f(a(t)) - \frac{w(t)}{p(t)} - \frac{r(t)}{p(t)} a(t) \right] = F
\]

Using the latter condition, the definition of the production technology, \(y(t) = Al(t)f(a(t)) - F\), and the first-order conditions (15), we find\(^5\)

\(^5\)From now on, it is assumed that \(n(t)\) evolves continuously.
\[ A(t)f(c(t)) = \frac{[1+\varepsilon_w(n(t))]n(t)F}{[1-s][1-\varepsilon_p]r_w(n(t))\varepsilon_k(1+\varepsilon_w(n(t))]} \]
\[ y(t) = \frac{(1-\varepsilon_p)[1+\varepsilon_w(n(t))]n(t)F}{[1-s][1-\varepsilon_p]r_w(n(t))\varepsilon_k(1+\varepsilon_w(n(t))]} \] (17)

From these expressions and the first-order conditions, the interest rate and the wage rate write:
\[ \frac{r(t)}{p(t)} = \frac{s(a(t))[1+\varepsilon_w(n(t))]y(t)}{[1(t)+s(a(t))\varepsilon_w(n(t))]k(t)} \]
\[ \frac{w(t)}{p(t)} = \frac{|1-s(a(t))|y(t)}{[1-s(a(t))\varepsilon_w(n(t))]k(t)} \] (18)

Where the aggregate price index \( p(t) \) is normalized to unity from now on. In the next section, we define the intertemporal equilibrium of this economy and derive steady state conclusions.

### 2.4 Intertemporal equilibrium and steady state analysis

Recall that \( a(t) = \frac{k(t)}{l(t)} \). We solve the system (17) to get \( n(k(t), y(t)) \) and \( l(k(t), y(t)) \). It follows that \( \varepsilon_w(n(t)) = \varepsilon_w(k(t), y(t)) \) and \( a(t) = a(k(t), y(t)) = \frac{k(t)}{l(k(t), y(t))} \), from which we derive \( s(a(t)) = s(k(t), y(t)), w(t) = w(k(t), y(t)), \)
\( r(t) = r(k(t), y(t)) \) and \( \sigma(a(t)) = \sigma(k(t), y(t)) \).6

We combine the first-order conditions (4)-(5) and obtain the equilibrium value of consumption:
\[ Bc(k(t), y(t)) = \frac{(1-s(k(t), y(t)))y(t)}{[1+s(k(t), y(t))]\varepsilon_w(k(t), y(t))]l(k(t), y(t))} \] (19)

We differentiate with respect to time the first-order condition (4) and (19) to get:
\[ \frac{\dot{\lambda}(t)}{\lambda(t)} = \varepsilon_{cy} \frac{\dot{y}(t)}{y(t)} + \varepsilon_{ck} \frac{\dot{k}(t)}{k(t)} \] (20)

with \( \varepsilon_{cy} = \frac{\partial c(k(t), y(t))}{\partial y(t)} \frac{y(t)}{k(t)} \) \( \frac{\partial c(k(t), y(t))}{\partial k(t)} \) and \( \varepsilon_{ck} = \frac{\partial c(k(t), y(t))}{\partial k(t)} \frac{k(t)}{c(k(t), y(t))} \). We substitute this expression in equation (6) to write the Euler equation:
\[ \frac{\dot{y}(t)}{y(t)} = \frac{\left(s(k(t), y(t))(1+\varepsilon_w(k(t), y(t)))y(t)\right)}{k(t)(1+s(k(t), y(t)))\varepsilon_w(k(t), y(t))} - \left(\rho + \delta \right) - \varepsilon_{ck} \frac{\dot{k}(t)}{k(t)} \] (21)

From the households’ budget constraint, we obtain the aggregat resource constraint:
\[ \dot{k}(t) = y(t) - \delta k(t) - c(k(t), y(t)) \] (22)

6In Appendix (5.3), we obtain the elasticities of \( l(t) \) and \( n(t) \) with respect to \( k(t) \) and \( y(t) \), \( \varepsilon_{l,k}, \varepsilon_{l,y}, \varepsilon_{n,k} \) and \( \varepsilon_{n,y} \).
An intertemporal equilibrium is a path \{k(t), y(t)\}_{t \geq 0} with k(0) = k_0 > 0, satisfying equations (21)-(22) and the transversality condition (8).

A steady state is a solution \((k^*, a^*, c^*, y^*, n^*)\), satisfying:
\[
c^* = y^* - \delta k^*
\]
\[
s(a^*)(1+\varepsilon_w(n^*))y^* = (\rho + \delta)
\]
\[
A \frac{k^*}{n^*} f(a^*) = \frac{[1+\varepsilon_w(n^*)][1+\varepsilon_w(n^*)]n^*F}{[1-(s(a^*))s_w(n^*)]+s_p(1+\varepsilon_w(n^*))}
\]
\[
y^* = \frac{[1-(s(a^*))s_w(n^*)]n^*F}{[1-(s(a^*))s_w(n^*)]+s_p(1+\varepsilon_w(n^*))}
\]
\[
B c^* = \frac{(1-s(a^*)n^*)a^*y^*}{[1+s(a^*)s_w(n^*)]n^*F}
\]

with \(a^* = k^*/l^*\), \(s(a^*) = \frac{a^* f'(a^*)}{f(a^*)}\) and \(\varepsilon_w(n^*) = \frac{1}{n^*}\). Solving the previous system gives:
\[
c^* = \frac{[\rho+\delta(1-s)+\rho(s(a^*))s_w(n^*)][1-\varepsilon_w(n^*)]}{[1-(s(a^*))s_w(n^*)]+s_p(1+\varepsilon_w(n^*))}
\]
\[
k^* = \frac{s(a^*)(1+\varepsilon_w(n^*))[1-\varepsilon_w(n^*)]}{[1-(s(a^*))s_w(n^*)]+s_p(1+\varepsilon_w(n^*))}
\]
\[
A f'(a^*) = \frac{(\rho+\delta)}{[1-\varepsilon_p]}
\]
\[
y^* = \frac{(1-\varepsilon_p)(1+s(a^*)s_w(n^*)]+s_p(1+\varepsilon_w(n^*))}{[1-(s(a^*))s_w(n^*)]+s_p(1+\varepsilon_w(n^*))}
\]
\[
B f'(a^*) = \frac{(\rho+\delta)^2 a^*(1-s(a^*))}{s(a^*)(1-\varepsilon_p)}
\]

We can now derive a first result about the steady state:

**Proposition 1.** Under Assumption 1, a steady state \((k^*, a^*, c^*, y^*, n^*)\) exists and is unique.

*Proof:* see Appendix 5.1

Exploiting the uniqueness of the steady state, we also find:

**Proposition 2.** Define the steady state share of labor income as \(\frac{w^* y^*}{y^*} = \frac{(1-s(a^*))}{[1+s(a^*)s_w(n^*)]n^*F}\). Then, an increase in monopsony power (due to a decrease in the elasticity of substitution of labor \(\gamma\) or a decrease in \(n^*\)) decreases the equilibrium number of firms and the steady state share of labor income.

*Proof:* see Appendix 5.2

We use the scaling parameters \(A, B > 0\) to ensure the existence of a normalized steady state (NSS) with \(a^* = 1\) and \(n^* = \bar{n}\), an arbitrary number of firms, which remains invariant with respect to preferences and technological parameters. Hence, we derive:
Proposition 3. Under Assumption 1, the solution \((k^*, 1, c^*, y^*, \bar{n})\) of the system (24) is a NSS if and only if \(A = A^*\) and \(B = B^*\) with

\[
A^* = \frac{(\rho + \delta) \sigma^2(1 - \epsilon_p)(1 - \epsilon_p)}{\sigma(1 - \epsilon_p)(1 - \epsilon_p)(1 - \epsilon_p)}
\]

\[
B^* = \frac{(\rho + \delta)^2 (1 - s(1)) [(1 - s(1))(1 - s(1)) + \epsilon_p (1 + \epsilon_p)(\bar{n})]]}{\sigma(1 - \epsilon_p)(1 - \epsilon_p)(1 - \epsilon_p)(1 - \epsilon_p)}
\]

This normalization procedure allows to use parameters of interest \(\varepsilon_w \equiv \varepsilon_w(\bar{n})\), \(\sigma \equiv \sigma(1)\) and \(s \equiv s(1)\) that are independent of the steady state.

We devote the next section to study the stability of properties of the model. In particular, we analytically derive the main results of this paper, that an economy concerned by a strong monopsony power on the labor market may experience expectation-driven fluctuations through local indeterminacy, then we discuss the likelihood of our results through a quantitative exercise.

3 Expectation-driven Fluctuations and Aggregate Instability

To study the local dynamics, we linearize the system (21)-(22) around the NSS. The characteristic polynomial of the Jacobian matrix is given by \(P(\lambda) = \lambda^2 - \text{Tr} \lambda + \text{Det}\) with:

\[
\text{Det} = \frac{(\rho + \delta) V(\varepsilon_w)}{P(\varepsilon_w)}
\]

\[
\text{Tr} = \rho - \frac{(\rho + \delta) \epsilon_p \varepsilon_w (1 - s)}{P(\varepsilon_w)}
\]

where:

\[
V(\varepsilon_w) = a_v \varepsilon_w^2 + b_v \varepsilon_w + c_v,
\]

\[
P(\varepsilon_w) = a_p \varepsilon_w^2 + b_p \varepsilon_w + c_p
\]

\[
a_v = \left[1 + \frac{s p}{(\rho + \delta)(1 - s)} + \frac{\epsilon_p}{(1 - s)(1 - \epsilon_p)}\right], \quad a_p = \left[1 - \frac{2s(1 - \epsilon_p)}{\sigma} - \frac{s \epsilon_p}{\sigma(1 - s)}\right]
\]

\[
b_v = \left[2 + \frac{\epsilon_p}{(1 - s)(1 - \epsilon_p)}\right], \quad b_p = \left[\frac{2s(1 - \epsilon_p)}{\sigma^2} + \frac{2s \epsilon_p}{\sigma(1 - s)} - \varepsilon_p\right]
\]

\[
c_v = \frac{2s p}{(1 - s)(1 - \epsilon_p)}, \quad c_p = -\frac{s \epsilon_p}{\sigma^2(1 - s)}
\]

Given that we have one predetermined variable, the necessary and sufficient conditions to obtain a sink, and therefore local indeterminacy, are \(D > 0\) and \(\text{Tr} < 0\) while the NSS displays saddle-path stability if \(D < 0\) and is a source if and only if \(D > 0\) and \(\text{Tr} > 0\). Furthermore, a Hopf bifurcation occurs when
$D > 0 \text{ and } Tr = 0$. In that case, the economy displays persistent fluctuations through the emergence of period cycles.

Note that $\varepsilon_w = \frac{1}{\bar{n}}$ so that, for any given $\bar{n}$, the parameter $\varepsilon_w \in (0, +\infty)$ is varying continuously through the parameter $\gamma$ which measures the substitutability of labor supply across industries and reflects the degree of firms’ monopsony power.

We derive first a proposition that ensures saddle-path stability of the NSS:

**Proposition 4.** Under Assumption 1, there exists $\sigma > 0$ such that the NSS is always locally determinate if $\sigma < \sigma^*$.

*Proof:* see Appendix 5.5

After deriving when local indeterminacy is ruled out, the following Proposition establishes our main results:

**Proposition 5.** Under Assumption 1, there is a critical value $\sigma > 0$, such that for any given value $\sigma > \sigma^*$ and $\varepsilon_p > 0$, there exists $\varepsilon_w$ and $\varepsilon_w^* \in (0, \varepsilon_w)$ such the NSS is a saddle-path if and only if $\varepsilon_w \in (0, \varepsilon_w^*)$ is a sink if and only if $\varepsilon_w \in (\varepsilon_w^*, \varepsilon_w)$, undergoes a Hopf bifurcation if $\varepsilon_w = \varepsilon_w^*$ and is a source if and only $\varepsilon_w > \varepsilon_w^*$.

*Proof:* see Appendix 5.6

From this Proposition, we derive that local indeterminacy and expectation-driven fluctuations require a low enough $\gamma$ (i.e. a high monopsonic power from the firms) given that the capital and labor are substitute enough. It is worth noticing that the condition $\varepsilon_w > \varepsilon_w^*$ and $\sigma > \sigma^*$ implies an upward-sloping labor demand as in Benhabib and Farmer [4]. Note that this condition is also equivalent to a positive consumption-output equilibrium relationship as it implies $\varepsilon_{cy} > 0$. The economic mechanism for expectation-driven fluctuations goes as follows. Departing from the steady state, Assume that households have an expectation of a future wage increase and anticipate therefore the labor supply to increase. This makes future interest rate more attractive which leads to a soar in investment. Since the capital stock in the present is predetermined, households must increase their labor supply to provide the necessary additional output. This leads to the creation of new firms and reduces the markdown which results in an increase in the current wage. The initial expectation is

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7 see Beaudry, Galizia and Portier [5] for a recent investigation about the empirical relevance of cyclical fluctuations.
therefore self-fulfilling.

To discuss the plausibility of our results, we consider a crude but indicative calibration of the deep parameters our model. We consider the following:

<table>
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<th>Parameters</th>
<th>$s$</th>
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<th>$\delta$</th>
<th>$\varepsilon_p$</th>
<th>$\sigma$</th>
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<td>0.001</td>
<td>0.02</td>
<td>0.05</td>
<td>(0.6,2.5)</td>
</tr>
</tbody>
</table>

The first three parameters have been chosen according on a quarterly basis and so that to match an annual return of capital of 8 per cent. We also consider a small value of $\varepsilon_p$ as a benchmark case and to avoid strong deviations from perfect competition on the product markets. Considering a particular value for the capital-labor elasticity of substitution is more problematic: while most recent contributions conclude that capital and labor are complement ($\sigma < 1$), the highest estimates found in the literature focus on a value of $\sigma$ between 1.25 and 3.25 (see Duffy and Papageorgiou [11] and Karagiannis et al. [17]). We first compute the value of $\sigma = 0.58$ implying that for most of the empirical value of $\sigma$, the relevant condition is $\sigma > \sigma_0$.

Given this condition, we find that local indeterminacy occurs for any value of the markdown $\varepsilon_w \in (1, 42, 2.27)$ if $\sigma = 0.8$ and $\varepsilon_w \in (0.6, 1.16)$ if $\sigma = 1.5$.

Assuming $\bar{n} = 2$, we plot the critical values of $\gamma$ leading to local indeterminacy for the relevant range of $\sigma$ in Figure 3. The lower line is the values of the Hopf bifurcation while the upper line represents the necessary condition on $\gamma$.

As shown by figure 3, local indeterminacy occurs for any $\gamma \in (0.22, 0.35)$ if $\sigma = 0.8$ and $\gamma \in (0.72, 1.70)$ if $\sigma = 2.5$. Furthermore, the interval between the lower and upper bound gets wider as $\sigma$ increases. Furthermore, an increase in $\varepsilon_p$ shifts both curve downward. Such values have to be compared with known estimates of $\gamma$ which highly depends on how this parameters is interpreted. If the interpretation of the individual wage elasticity of labor supply is assumed, the literature review by Reichling and Whalen [20] concludes of a range $\gamma \in (0.27, 0.53)$ with a central tendency around 0.4. The investigations made by Berger, Hergenkoff and Mongey [6] and Katayama and Kim [18] find a value of $\gamma$ around unity for the recent years.

A detailed country estimation is provided by Cardi and Restout [7] in their two sector framework. Using panel techniques, they find that the average value of $\gamma$ for main OECD countries is around 0.62, with a minimum of 0.216 and a
maximum of 1.82.\footnote{The following countries are considered: Belgium, Denmark, Finland, Great Britain, Germany, Ireland, Italy, Japan, Korea, Netherland, Spain, Sweden, US over the period 1971-2007.} We conclude therefore that monopsony power on the labor market is likely to be a source of destabilization, even with a minimal degree of imperfect competition on the product market.

\section*{4 Concluding Comments}

This paper considers firms' monopsony power on the labor market as a potential source of expectation-driven fluctuations in a dynamic general equilibrium model. While the effect of monopsony power on wage and participation are well document, its consequence on dynamic properties, and in particular for the emergence of local indeterminacy, is less, if not, studied. To investigate such properties, we considered a simple model where households’ labor supply is imperfectly substitutable across labor markets implying that firms in a labor market face an upward-sloping labor supply. As a result, firms exert a market power by paying a wage that is lower than the marginal product of labor. Given minimal assumptions, we show that local indeterminacy arises as soon as this
market power is large enough, capital and labor are substitutes and we find that our results are plausible according a numerical illustration.

These conclusions stands within a wide range of literature. First, it completes and converges to the conclusion of Dos Santos Ferreira and Lloyd-Braga [9], [10] and Jaimovich [15] who study the emergence of local indeterminacy and expectation-driven fluctuations under oligopolistic competition and free-entry. It also contributes to the general equilibrium modelling of firms’ labor market power in imperfectly competitive economy as recently renewed by Azar and Vives [3] or Berger, Herkenhoff and Mongey [6]. While the consideration of imperfect substitutability of labor supply has attracted more and more contributions, in particular in the DSGE literature (e.g. Cardi and Restout [7], Horvath [13], Huang and Meng [14], Katayama and Kim [18]), most of them still consider a perfectly competitive labor market ruling out firms’ market power on this market and leaving aside potentially significant effects at the macroeconomic level.

5 Appendix

5.1 Proof of Proposition 1

From the third equation of the system (24) and under Assumption 1, we notice that there is a unique \( a^* \). Hence, all the terms in the last equation varies with \( n^* \) only and we can show that the left-hand side tends to zero when \( n^* = 0 \) while it tends to \(+\infty\) as \( n \) becomes arbitrary large. Furthermore, the left-hand side is strictly increasing with respect to \( n^* \) implying that \( n^* \) is also unique. Since \( k^*, c^* \) and \( y^* \) depends only on \( a^* \) and \( n^* \), the rest of the system admits a unique solution. \(\square\)

5.2 Proof of Proposition 2

Consider the expression for the share of labor income, \( \alpha = \frac{(1-s(a^*))}{(1+s(a^*)\varepsilon_w(n^*))} \). An increase in the market power means that the markdown on wage \( \varepsilon_w \) increases. We easily obtain that the derivative of the latter expression is given by:

\[
\frac{da}{d\varepsilon_w} = \frac{-(1-s(a^*)s(a^*)}{[(1+s(a^*)\varepsilon_w(n^*))^2} \tag{26}
\]

Although it is trivial to show that \( \varepsilon_w \) is decreasing with respect to \( n^* \) or \( \gamma \), there is also a steady state relationship between the \( n^* \) and \( \gamma \). From the last
equation of the system (24), we can derive implicitly that \( \frac{dn^*}{d\gamma} > 0 \). We conclude therefore:

\[
\frac{d\alpha}{d\gamma} = \frac{(1 - s(a^*)s(a^*)) \left(1 + \frac{dn^*}{d\gamma}\right)}{[(1 + s(a^*)\varepsilon_w(n^*))^2 \gamma n^*)^2]
\]  

(27)

5.3 Derivation of the main elasticities

Using equations in (17) and the definition of \( s = \frac{af'(a)}{f(a)} \) with \( a = \frac{k}{\sigma} \), we get:

\[
\varepsilon_{ly} = \frac{[1 + \varepsilon_w] \Delta + (1 - s)(1 - \varepsilon_p)\varepsilon_w}{(1 - s)E(1 + \varepsilon_w)\Delta}
\]

\[
\varepsilon_{lk} = -s\left[E(1 + \varepsilon_w) + (1 - \frac{1}{2})\varepsilon_w\right] \\
\varepsilon_{ny} = \frac{\Delta + (1 - \frac{1}{2})s\varepsilon_w(1 - \varepsilon_p)}{E\Delta}
\]

\[
\varepsilon_{nk} = -s\left(1 - \frac{1}{2}\right)\varepsilon_w(1 + \varepsilon_w)
\]

\[
E = \left[1 - (1 - \frac{1}{\sigma})\frac{s\varepsilon_w}{1 + s\varepsilon_w} + \frac{(1-s)\varepsilon_w}{(1+s\varepsilon_w)\Delta}\right]
\]

\[
\Delta = \left[(1-s)(1-\varepsilon_p)\varepsilon_w + \varepsilon_p(1 + \varepsilon_w)\right]
\]

We also derive the elasticities for \( w, r \) and \( s = \frac{af'(a)}{f(a)} \) with \( a = \frac{k}{\sigma} \), as a function of the above elasticities. It follows:

\[
\varepsilon_{wy} = 1 - \varepsilon_{ly} + \frac{s\varepsilon_w}{1 + s\varepsilon_w}\varepsilon_{ny} + \frac{s(1 + \varepsilon_w)}{1 + s\varepsilon_w}(1 - \frac{1}{\sigma})\varepsilon_{ly}
\]

\[
\varepsilon_{wk} = -\varepsilon_{lk} + \frac{s\varepsilon_w}{1 + s\varepsilon_w}\varepsilon_{nk} - \frac{s(1 + \varepsilon_w)}{1 + s\varepsilon_w}(1 - \frac{1}{2})(1 - \varepsilon_{lk})
\]

\[
\varepsilon_{ry} = 1 - \frac{(1-s)\varepsilon_w}{(1+s\varepsilon_w)(1+\varepsilon_w)}\varepsilon_{ny} - \frac{(1-s)}{(1+s\varepsilon_w)}(1 + \varepsilon_w)(1 - \frac{1}{\sigma})\varepsilon_{ly}
\]

\[
\varepsilon_{rk} = -1 - \frac{(1-s)\varepsilon_w}{(1+s\varepsilon_w)(1+\varepsilon_w)}\varepsilon_{nk} + \frac{(1-s)}{(1+s\varepsilon_w)}(1 - \frac{1}{\sigma})(1 - \varepsilon_{lk})\varepsilon_{ny} = -(1-s)(1 - \frac{1}{\sigma})\varepsilon_{ly}
\]

\[
\varepsilon_{sk} = (1-s)(1 - \frac{1}{\sigma})(1 - \varepsilon_{lk})
\]

(29)
5.4 Derivation of the determinant and the trace of the Jacobian matrix

Note that from equation (19), we find that \( \varepsilon_{cy} = \varepsilon_{wy} \) and \( \varepsilon_{ck} = \varepsilon_{wk} \). Using the expressions given in (29), we can now linearize the system (21)-(22) in the neighborhood of the steady state to obtain:

\[
\begin{pmatrix}
\frac{dy}{dt} \\
\frac{dk}{dt}
\end{pmatrix} = J \begin{pmatrix}
\frac{dy}{dt} \\
\frac{dk}{dt}
\end{pmatrix}
\] (30)

with \( J \) the Jacobian matrix given by:

\[
J = \begin{pmatrix}
\frac{(\rho + \delta)\varepsilon_{cy} - \frac{\varepsilon_{wk}(1-\varepsilon_{wy})}{\varepsilon_{wy}}}{\varepsilon_{wy}} & \frac{y}{\varepsilon_{wy}} \left[ \frac{(\rho + \delta)\varepsilon_{cy} + \frac{\varepsilon_{wk}(\delta + \varepsilon_{wy})}{\varepsilon_{wy}}}{\varepsilon_{wy}} \right] \\
(1 - \varepsilon_{wy}) & -(\delta + \varepsilon_{wk} \frac{\varepsilon_{wc}}{k})
\end{pmatrix}
\]

After some tedious computations, the trace and the determinant of the Jacobian matrix are obtained:

\[
V(\varepsilon_w) = a_v\varepsilon_w^2 + b_v\varepsilon_w + c_v, \quad P(\varepsilon_w) = a_p\varepsilon_w^2 + b_p\varepsilon_w + c_p
\]

\[
a_v = \left[ 1 + \frac{sp}{\rho + \delta (1-s)} \right], \quad a_p = \left[ 1 - \frac{2s(1-s)\varepsilon_p}{\sigma} - \frac{s\varepsilon_p}{\sigma(1-s)} \right]
\]

\[
b_v = \left[ 2 + \frac{\varepsilon_p}{(1-s)(1-\varepsilon_p)} \right], \quad b_p = \left[ \frac{2s(1-s)\varepsilon_p}{\sigma} + \frac{2s\varepsilon_p}{\sigma(1-s)} - \varepsilon_p \right]
\]

\[
c_v = \frac{2\varepsilon_p}{(1-s)(1-\varepsilon_p)}, \quad c_p = \frac{-s\varepsilon_p}{\sigma(1-s)}
\]

5.5 Proof of Proposition 4

Consider the Determinant. The numerator of the determinant is a polynomial of degree two, \( V(\varepsilon_w) \) with positive coefficient \( a_v, b_v, c_v > 0 \). The two roots have the same sign since their product share the same sign than \( a_v > 0 \) and are negative since the sign of sum of the two roots satisfies \( \frac{b_v}{a_v} < 0 \). Because \( V(0) > 0 \), we conclude that \( V(\varepsilon_w) \) is always positive for any \( \varepsilon_w > 0 \).

The denominator of the Determinant is also a polynomial of degree two. For any \( \sigma < \sigma = \frac{s}{(1-s)} [(1-s)(1-\varepsilon_p) + s\varepsilon_p] \), we have \( a_p, b_p, c_p < 0 \) and therefore, \( \frac{c_p}{a_p} > 0 \) and \( \frac{-b_p}{a_p} < 0 \), meaning that both roots of \( P(\varepsilon_w) \) are also negative. Because \( P(\varepsilon) < 0 \), we find \( P(\varepsilon_w) < 0 \) for any \( \varepsilon_w > 0 \) and the Determinant is therefore always negative.
5.6 Proof of Proposition 5

From the previous Proposition and the Trace, we derive that a necessary condition for local indeterminacy is $P(\varepsilon_w) > 0$ and it requires $\sigma > \overline{\sigma}$. In such case, we find that $a_p > 0$ while $b_p, c_p < 0$ and one of the roots is positive while the other is negative. Since $P(\varepsilon_w)$ is convex, there exists therefore a critical point $\varepsilon_w > 0$ such that we get $P(\varepsilon_w) < 0$ for any $\varepsilon_w \in (0, \varepsilon_w)$ and the Determinant is negative while $P(\varepsilon_w) > 0$ for any $\varepsilon_w \in (\varepsilon_w, +\infty)$ which leads to a positive Determinant.

Note that for any $\varepsilon_p = 0$, the Trace is positive and equal to $\rho$. We assume therefore $\varepsilon_p > 0$. The Trace writes:

$$Tr = \frac{H(\varepsilon_w)}{P(\varepsilon_w)}$$

with $H(\varepsilon_w) = a_h \varepsilon_w^2 + b_h \varepsilon_w + c_h$ and $a_h = \rho a_p > 0$, $b_h = \left[ \frac{2s\varepsilon_p}{\sigma(1-s)} + \frac{2s(1-\varepsilon_p)}{\sigma} + \rho + \delta(1-s) + s\rho \right] > 0$ and $c_h = \rho c_p < 0$. Similarly to $P(\varepsilon_w)$, the polynomial $H(\varepsilon_w)$ has one positive root, say $\bar{\varepsilon}_w$ and one negative and non relevant root, and is negative for any $\varepsilon_w \in (0, \varepsilon_w)$ and positive other.

Hence, without loss of generality, for any $\varepsilon_w \in (\varepsilon_w, \bar{\varepsilon}_w)$, the Trace is negative and the steady state is locally indeterminate (i.e. a sink) while the Trace is positive and the NSS is a source for any $\varepsilon_w > \bar{\varepsilon}_w$. Furthermore, the critical point $\varepsilon_w$ is a Hopf bifurcation and in its neighborhood, the NSS displays a limit cycle.

\[\square\]

References


