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Theoretical approach to the masses of the elementary fermions

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Abstract

We made the hypothesis that, if spacetime is composed of small hypercubes of one Planck length edge, it exists elementary wavefunctions which are equal to $\sqrt{2}\exp(ix_j)$ if it corresponds to a space dimension or equal to $\sqrt{2}\exp(it)$ if it corresponds to a time dimension. The masses of fermions belonging to the first family of fermions are equal to integer powers of 2 (in eV/c^2) [1]. We make the hypothesis that the fermions of the 2nd and 3rd families are excited states of the fermions of the 1st family. Indeed, the fermions of the 2nd and 3rd families have masses equal to $2^n.(p^2)/2$ where n is an integer [1] calculated for the first family of fermions and p is another integer. p is an integer which corresponds to the excited states of the elementary wavefunctions (the energy of the excited elementary wave functions are equal to $p^2/2$; using normalized units).

Keywords:

1. Introduction

I make the assumption that our threedimensional universe is embedded in a four dimensional euclidean space. Time is a function of the fourth dimension of this space [2–4]. If we apply this hypothesis to particle physics, we may say that elementary particles are fourdimensional, threedimensional and twodimensional.

The coordinates (x, y, z, t) are not orthonormal.Indeed, time t evolves as $log(r)$ where r is the comoving distance in cosmology [2].

To obtain the first family of fermions from the Standard Model (i.e. quark up, electron, electron neutrino) one may say that [5]:

- the electron is fourdimensional (t, x, y, z)
- the quark is three dimensional (t, x, y).
- finally, the electronic neutrino is twodimensional (t, x) and x, y and z are equivalent.

To obtain the remaining fermions (elementary particles), one has to modify the quantum number p (similar to the quantum number of a particle in a box). Thus, the remaining fermions of the Standard Model may be seen as excited states of the first fermion family.

Straightforwardly, we make the following hypotheses:

- Spacetime has an underlying hypersquare array of edge length $\hbar$
- Elementary wave functions (in (x, y, z, t) space)are eigenfunctions of a particle in a square potential (reduced parameters) $\sqrt{2}exp(-ix)$ for space $\sqrt{2}exp(-it)$ for time
- The eigenvalues of the elementary wave functions are equal to $p^2/2$ (with p an integer number)

2. Masses of the electron, muon and tau

For electrons, we have

$$i\gamma^\mu \partial_\mu \psi = m\psi \quad (2)$$

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$$\begin{pmatrix} \gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_0 \end{pmatrix} \begin{pmatrix} \partial_1\Psi \\ \partial_2\Psi \\ \partial_3\Psi \\ \partial_1\Psi \\ \partial_2\Psi \\ \partial_3\Psi \\ \partial_1\Psi \\ \partial_2\Psi \\ \partial_3\Psi \\ \partial_0\Psi \end{pmatrix} = m. \begin{pmatrix} \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \end{pmatrix} \tag{1}$$

particle	Theoretical mass (eV/c^2)	Experimental mass (eV/c^2) [6]
electron	$2^{19} = 0.524MeV/c^2$	$0.511MeV/c^2$
muon	$2^{19}.20^2/2 = 104.8MeV/c^2$	$105.6MeV/c^2$
tau	$2^{19}.82^2/2 = 1.76GeV/c^2$	$1.78GeV/c^2$

Table 1: Theoretical and experimental masses of the electron, muon and tau

The Dirac matrices are representative of infinitesimal rotations within the wavefunction of a given elementary particle.

Using combinatorial analyzis we obtain equation (1) (using the fact that electrons are $4d$ [5] and that all space dimensions are equivalent).

There are 3 possibilities of arranging $\gamma_1, \gamma_2, \gamma_3$ (the Dirac matrices) over x, y and z (all space dimensions are equivalent) and one possibility to arrange σ_0 (temporal Pauli matrix: half of γ_0; because time does not go backward).

The large matrix M (see equation (1)) containing all combinations has a dimension $9X4 + 2 = 38$. We see that, with the coordinate vectors $\sqrt{2}exp(-it)$ and $\sqrt{2}exp(-ix)$ (eigenfunctions of a particle in a square potential), we have to multiply the modified Dirac equation by the Jacobian corresponding to these new coordinates. This Jacobian is equal to $\sqrt{2}^{38}$ where 38 is the dimension of the large matrix [1]. We multiply the mass of the first particle of this family by the eigenvalues of the eigenfunctions (of the particle).

We decompose the eigenvalues into prime numbers. The number of eigenvalues for the ground state (electron) is 38 (the dimension of the large matrix M). For the other particles, we take into account the spinor $(1, 0)^T$ corresponding to the σ_0 Pauli matrix. So except for the electron, there are 37 eigenvalues for each particle.

- The mass of the electron is equal to $\sqrt{2}^{38}$ $=2^{19}eV/c^2=2^{19}.(\frac{1}{2}^2)^{19}.(\frac{2^2}{2})^{19}$ = $0.524MeV/c^2 \approx$ *0.511* MeV/c^2
- The mass of the muon is equal to $2^{19}.20^2/2$=$2^{19}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{5^2}{2}.(\frac{1}{2}^2)^{16}.(\frac{2^2}{2})^{16}$ = $104.8MeV/c^2 \approx$ *105.6* MeV/c^2
- The mass of the tau is equal to $2^{19}.82^2/2$=$2^{19}.\frac{41^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.(\frac{1}{2}^2)^{17}.(\frac{2^2}{2})^{17}$ = $1.76GeV/c^2 \approx$ *1.78* GeV/c^2

The values in italic are the experimental masses [6].

We see that for the tau particle, one of the eigenvalues ($\frac{41^2}{2}$) is much larger than the others. This may explain the short lifetime of this particle.

The masses (theoretical and experimental) of the electron, muon and tau are summarized in Table 1.

3. Masses of the quarks

For quarks, we have

$$i\gamma^\mu \partial_\mu \psi = m\psi \tag{4}$$

The Dirac matrices are representative of infinitesimal rotations within the wavefunction of a given elementary particle.

Using combinatorial analyzis we obtain equation (3) (using the fact that quarks are $3d$ [5] and that all space dimensions are equivalent).

There are 3 possibilities of arranging $\gamma_1, \gamma_2, \gamma_3$ (the Dirac matrices) over x, y and z (all space dimensions are equivalent). There is one possibility to arrange σ_0

$$\begin{pmatrix} \gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_0 \end{pmatrix} \begin{pmatrix} \partial_1\Psi \\ \partial_2\Psi \\ \partial_3\Psi \\ \partial_0\Psi \\ \partial_1\Psi \\ \partial_2\Psi \\ \partial_3\Psi \\ \partial_0\Psi \\ \partial_1\Psi \\ \partial_2\Psi \\ \partial_3\Psi \\ \partial_0\Psi \end{pmatrix} = m \begin{pmatrix} \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \end{pmatrix} \quad (3)$$

quark	Theoretical mass (eV/c^2)	Experimental mass (eV/c^2) [6]
up	$2^{21} = 2.09Mev/c^2$	$2.2MeV/c^2$
down	$2^{21}.2^2/2 = 4.19MeV/c^2$	$4.7MeV/c^2$
strange	$2^{21}.9^2/2 = 84.9MeV/c^2$	$96MeV/c^2$
charm	$2^{21}.36^2/2 = 1.35GeV/c^2$	$1.27GeV/c^2$
bottom	$2^{21}.63^2/2 = 4.16GeV/c^2$	$4.18GeV/c^2$
top	$2^{21}.405^2/2 = 171.9GeV/c^2$	$173GeV/c^2$

Table 2: Theoretical and experimental masses of the quarks family.

(temporal Pauli matrix; half of γ_0, because time does not go backward) for each combination of spatial Dirac matrices (x, y; x, z and y, z). We have to take into account that the quarks are 3 dimensional. So, the matrix M containing all combinations has a dimension equal to $9X4 + 3X2 = 42$.

We see that, with the coordinate vectors $\sqrt{2}exp(-it)$ and $\sqrt{2}exp(-ix)$ (eigenfunctions of the underlying hypersquare array), we have to multiply the modified Dirac equation by the Jacobian corresponding to these new coordinates. This Jacobian is equal to $\sqrt{2}^{42}$ where 42 is the dimension of the matrix [1].

We multiply the mass of the first particle of the quarks family by the eigenvalues of the eigenfunctions (of the particle). We decompose the eigenvalues into prime numbers. The number of eigenvalues for the ground state (quark up) is 42 (the dimension of the large matrix M, see equation (3)). For the other quarks, we take into account the spinor $(1, 0)^T$ corresponding to the 3 σ_0 Pauli matrices. So except for the quark up, there are 39 eigenvalues for each quark.

- The quark up has a mass equal to $\sqrt{2}^{42} = 2^{21} eV/c^2 = 2^{21}.(\frac{1}{2}^2)^{21}.(\frac{2^2}{2})^{21} = 2.09MeV/c^2 \approx$ *2.2 MeV/c^2*
- The quark down has a mass equal to $2^{21}.\frac{2^2}{2} = 2^{21}.\frac{2^2}{2}.(\frac{1}{2}^2)^{19}.(\frac{2^2}{2})^{19} = 4.19MeV/c^2 \approx$ *$4.7MeV/c^2$*
- The quark strange has a mass equal to $2^{21}.\frac{9^2}{2} = 2^{21}.\frac{3^2}{2}.\frac{3^2}{2}.\frac{2^2}{2}.(\frac{1}{2}^2)^{18}.(\frac{2^2}{2})^{18} = 84.9MeV/c^2 \approx$ *96 MeV/c^2*
- The quark charm has a mass equal to$2^{21}.\frac{36^2}{2} = 2^{21}.\frac{3^2}{2}.\frac{3^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}(\frac{1}{2}^2)^{16}.(\frac{2^2}{2})^{16} = 1.35GeV/c^2 \approx$ *$1.27GeV/c^2$*
- The quark bottom has a mass equal to $2^{21}.\frac{63^2}{2} = 2^{21}.\frac{3^2}{2}.\frac{3^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{7^2}{2}.(\frac{1}{2}^2)^{17}.(\frac{2^2}{2})^{17} = 4.16GeV/c^2 \approx$ *$4.18GeV/c^2$*
- The quark top has a mass equal to $2^{21}.\frac{405^2}{2} = 2^{21}.\frac{3^2}{2}.\frac{3^2}{2}.\frac{3^2}{2}.\frac{3^2}{2}.\frac{5^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.\frac{2^2}{2}.(\frac{1}{2}^2)^{15}.(\frac{2^2}{2})^{15} = 171.9GeV/c^2 \approx$ *$173GeV/c^2$*

The values in italic are the experimental masses [6].

The theoretical and experimental masses of the quarks family are summarized in table 2.

4. Masses of the neutrinos

Up to now, there is no theoretical propagation equation for the neutrinos.

particle	Theoretical mass	Experimental mass: upper limit [6]
electron neutrino	$2eV/c^2$	$2.5eV/c^2$
muon neutrino	$412^2 = 169.7keV/c^2$	$170keV/c^2$
tau neutrino	$3937^2 = 15.4MeV/c^2$	$18MeV/c^2$

Table 3: Theoretical masses of the neutrinos and upper limits of experimental masses

If we use the eigenvalues of the elementary wave functions like for quarks and electrons, muons and taus, we may write:

- The mass of the electron neutrino is equal to $2eV/c^2$
- The mass of the muon neutrino is equal to $2.\frac{412^2}{2} = 412^2 eV/c^2 = 169keV/c^2$
- The mass of the tau neutrino is equal to $2.\frac{3937^2}{2} = 3937^2 eV/c^2 = 15.4MeV/c^2$

Hence, we found theoretical values of the masses of the neutrinos which are in good agreement with the experimental masses.

5. Conclusion

We found the theoretical values of masses for all the elementary fermions (1st, 2nd and 3rd families of fermions). These theoretical masses are in good agreement with the experimental masses (the differences between theoretical and experimental masses are less than 10% except for the quarks down and strange). There is a possibility to analyze the symmetries of these particles and compare them to the symmetries of the Standard Model.

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