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1 **Flow of Yield Stress and Carreau fluids through Rough-Walled Rock Fractures:**
2 **prediction and experiments**

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20

21 **Abstract**

22

23 Many natural phenomena in geophysics and hydrogeology involve the flow of non-
24 Newtonian fluids through natural rough-walled fractures. Therefore, there is considerable
25 interest in predicting the pressure drop generated by complex flow in these media under a
26 given set of boundary conditions. However, this task is markedly more challenging than the
27 Newtonian case given the coupling of geometrical and rheological parameters in the flow
28 law. The main contribution of this paper is to propose a simple method to predict the flow of
29 commonly used Carreau and yield stress fluids through fractures. To do so, an expression
30 relating the “*in-situ*” shear viscosity of the fluid to the bulk shear-viscosity parameters is
31 obtained. Then, this “*in-situ*” viscosity is entered in the macroscopic laws to predict the flow
32 rate-pressure gradient relations. Experiments with yield stress and Carreau fluids in two
33 replicas of natural fractures covering a wide range of injection flow rates are presented and
34 compared to the predictions of the proposed method. Our results show that the use of a
35 constant shift parameter to relate “*in-situ*” and bulk shear viscosity is no longer valid in the
36 presence of a yield stress or a plateau viscosity. Consequently, properly representing the
37 dependence of the shift parameter on the flow rate is crucial to obtain accurate predictions.
38 The proposed method predicts the pressure drop in a rough-walled fracture at a given
39 injection flow rate by only using the shear rheology of the fluid, the hydraulic aperture of the
40 fracture and the inertial coefficients as inputs.

41

42 1. Introduction

43

44 The flow of complex fluids through rough-walled rock fractures is involved in many
45 economically important industrial applications, such as soil remediation, hydrogeology or
46 Enhanced Oil Recovery (EOR) [*Radilla et al.*, 2013; *Tosco et al.*, 2013; *Coussot*, 2014].
47 Numerous complex fluids are shear-thinning, showing a decrease in shear viscosity as the
48 applied shear rate is increased. Shear-thinning fluids are extensively used in petroleum
49 engineering and soil remediation to improve the microscopic sweep of the reservoir through
50 stabilization of the injection front [*Lake*, 1989; *Silva et al.*, 2012; *Wever et al.*, 2011]. For
51 instance, shear-thinning drilling fluids containing the biopolymer xanthan [*Zhong et al.*,
52 2008; *Truex et al.*, 2015] and other polymers such as polyacrylamide [*Ball and Pitts*, 1984],
53 carboxymethylcellulose [*Zhang et al.*, 2016] and guar gum [*Hernández-Espriú et al.*, 2013]
54 are widely used in EOR.

55

56 In some cases, fluids with shear-rate dependent viscosity also present a yield stress, i.e. a
57 threshold value in terms of shear stress below which they do not flow. Many complex fluids
58 used in industrial applications exhibit yield stress behaviour, e.g. polymer solutions, waxy
59 crude oils, volcanic lavas, emulsions, colloid suspensions, foams, etc. [*Coussot*, 2005;
60 *Dimitriou and McKinley*, 2015; *Roustaei et al.*, 2016; *Talon et al.*, 2014; *Lavrov*, 2013;
61 *Coussot*, 2014]. Common examples of yield stress shear-thinning fluids are the slurries or
62 cement grouts injected to reinforce soils, the heavy oils or the drilling fluids injected into
63 rocks for the reinforcement of wells [*Lavrov*, 2013; *Coussot*, 2014]. Indeed, drilling fluids are
64 often designed so as to have a yield stress in order to prevent cutting from settling when
65 circulation stops [*Lavrov*, 2013]. Also, a number of fracturing fluids used in hydraulic

66 fracturing exhibit a yield stress designed to enhance proppant transport [*Talon et al.*, 2014;
67 Roustaei et al., 2016] and present shear-thinning behaviour [*Lavrov*, 2015; *Perkowska et al.*,
68 2016].

69

70 For these reasons, the flow of shear-thinning fluids in porous media, and in particular that of
71 yield stress fluids, has become a field of great research interest [*Chevalier et al.*, 2013;
72 *Chevalier et al.*, 2014; *Coussot*, 2014; *Talon et al.*, 2014; *Rodríguez de Castro et al.*, 2016].
73 However, although recent advances have been made [*Chevalier et al.*, 2013; *Chevalier et al.*,
74 2014], obtaining a macroscopic law to predict pressure drop as a function of flow rate has
75 proved to be a stumbling-block. Also, despite its broad interest, a serious lack of
76 experimental works involving the flow of yield stress fluids was reported by *Lavrov* [2013]
77 and *Coussot* [2014].

78

79 Inspired by the growing scope of industrial applications in which shear-thinning and yield
80 stress fluids are injected through rough-walled fractures, the objective of this work is to
81 present a simple method to predict the pressure losses generated during single-phase flow.
82 The accuracy of the resulting predictions is then evaluated through comparison with
83 experimental data. To do so, a series of flow experiments with concentrated aqueous
84 solutions of xanthan biopolymer presenting a yield stress were carried out by measuring the
85 pressure drop as a function of the injection flow rate during the flow through two replicas of
86 rough-walled natural fractures (granite and Vosges sandstone). Furthermore, previously
87 presented experimental data involving the flow of shear-thinning with no yield stress are also
88 compared with the predictions obtained with the proposed method.

89

90 The single-phase flow of incompressible Newtonian fluids through porous media is governed
91 by Darcy's law [*Darcy*, 1856]. In the case of one-directional steady flow through a horizontal
92 porous media, this law is written as:

93

$$\nabla P = \frac{\mu Q}{KA} = \frac{\mu}{K} u \quad (1)$$

94 $\nabla P = \frac{\Delta P}{L}$ being the pressure gradient, ΔP the absolute value of the pressure drop over a
95 distance L , Q the volumetric flow rate, A the cross-sectional area, $u = Q/A$ the average
96 velocity, μ the viscosity of the injected fluid, and K the intrinsic permeability. This model is
97 restricted to creeping flow in which inertial forces are negligible compared to viscous forces
98 [*Schneebeli*, 1955; *Hubbert*, 1956; *Scheidegger*, 1960; *Chauveteau and Thirriot*, 1967].
99 Nonlinearity of fluid flow stems from inertial pressure losses generated by the repeated
100 accelerations and decelerations due to rapid changes in flow velocity and direction along the
101 flow path. Both theoretical and empirical models taking into account the extra pressure losses
102 due to inertial effects were presented in the literature [*Miskimins et al.*, 2005]. The results of
103 these studies confirm the existence of a strong inertial regime and a weak inertial regime. The
104 nonlinear behaviors associated to those regimes can be described respectively by a quadratic
105 and a cubic function of the average velocity. Forchheimer's empirical law [*Forchheimer*,
106 1901] is commonly used to model the strong inertial regime through addition of a quadratic
107 flow rate term to Darcy's law:

108

109

$$\nabla P = \frac{\mu}{K}u + \beta\rho u^2 \quad (2)$$

110 where ρ is the fluid density and β is the inertial coefficient. Forchheimer's law has been
 111 experimentally validated [*Dullien and Azzam, 1973; Geertsma, 1974; MacDonald et al.,*
 112 *1979; Rasoloarijaona and Auriault, 1994; Javadi et al., 2014; Rodríguez de Castro and*
 113 *Radilla, 2016a*] and has found some theoretical justifications [*Cvetkovic, 1986; Giorgi, 1997;*
 114 *Chen et al., 2001*]. In the case of the weak inertial regime, which occurs at moderate values
 115 of the Reynolds number, deviations from the linear relationship between flow rate and
 116 pressure loss were shown to follow a cubic function of the mean velocity in the porous media
 117 [*Mei and Auriault, 1991; Firdaouss et al., 1997; Fourar et al., 2004; Rocha and Cruz, 2010*].

118

$$\nabla P = \frac{\mu}{K}u + \frac{d\rho^2}{\mu}u^3 \quad (3)$$

119

120 where d is a dimensionless inertial coefficient. Reynolds number can be specifically defined
 121 for weak inertia cubic law as [*Radilla et al., 2013; Rodríguez de Castro and Radilla, 2016a*].

122

$$Re_c = \sqrt{Kd} \frac{\rho Q}{\mu A} \quad (4)$$

123 Cubic law was obtained from numerical simulations in a 2D periodic porous medium
 124 [*Barrère et al., 1990; Fidarous and Guermond, 1995; Amaral Souto and Moyne, 1997*] and
 125 also by using the homogenization technique for isotropic homogeneous porous media [*Mei*

126 *and Auriault, 1991; Wodie and Levy, 1991*]. This law was shown to be in agreement with
127 experimental data [*Firdaous et al., 1997; Rodríguez de Castro and Radilla, 2016a*].

128

129 Using the asymptotic expansions method in a thin cylindrical channel with oscillating walls
130 and averaging over the channel diameter, *Buès et al. [2004]* and *Panfilov and Fouar [2006]*
131 presented a macroscopic flow equation which proved to be in good agreement with numerical
132 simulations in rectangular and cylindrical fractures at high flow rates. This flow equation was
133 expressed in the form of a full cubic law:

134

$$\nabla P = \frac{\mu}{K}u + \beta\rho u^2 + \frac{d\rho^2}{\mu}u^3 \quad (5)$$

135 where β and d are the inertial coefficients which may be positive or negative, depending on
136 the channel geometry. β and d were shown to be independent of the shear rheology of the
137 injected fluid in previous numerical [*Firdaouss et al., 1997; Yadzchi and Luding, 2012; Tosco*
138 *et al., 2013*] and experimental works [*Rodríguez de Castro and Radilla, 2016a; Rodríguez de*
139 *Castro and Radilla, 2016b*]. In this full cubic law, the quadratic term describes the pure
140 inertia effect caused by an irreversible loss of kinetic energy due to flow acceleration and the
141 cubic term corresponds to a cross viscous–inertia effect caused by the streamline deformation
142 due to inertia forces. This macroscopic flow equation is valid not only in the Darcian flow
143 regime but also, to some limited extent, for the non-Darcian flow regimes. β and d can be
144 obtained either through fitting to experimental data [*Dukhan et al., 2014; Rodríguez de*
145 *Castro and Radilla, 2016a, 2016b*] or through theoretical predictions obtained from porosity,
146 permeability and roughness of the porous medium [*Cornell and Katz, 1953; Geertsma, 1974;*
147 *Neasham, 1977; Noman and Archer, 1987; López, 2004, Agnaou et al., 2016*].

148

149 Analogously to the case of cubic law, Reynolds number can be defined for full cubic law as
150 [*Radilla et al.*, 2013; *Rodríguez de Castro and Radilla*, 2016a]:

151

$$\text{Re}_{fc} = \frac{K\beta\rho u}{\mu} \quad (6)$$

152

153 Previous experimental works demonstrated that Darcy's law fails to predict pressure drops in
154 fractures when inertial effects are relevant [*Zimmerman et al.*, 2004; *Radilla et al.*, 2013;
155 *Javadi et al.*, 2014; *Rodríguez de Castro and Radilla*, 2016a, 2016b]. *Zimmerman et al.*
156 [2004] presented experimental data on non-creeping flow through a rock fracture, showing
157 good agreement with Forchheimer's model. The same authors also proved, via numerical
158 solution of the Navier-Stokes equations, the existence of the weak inertia regime for
159 moderate values of Reynolds numbers. *Radilla et al.* [2013] modelled single-phase flow
160 experiments by means of the full cubic law and presented an elegant method to compare
161 fractures in terms of hydraulic behaviour versus flow regime using the intrinsic
162 hydrodynamic parameters. Besides, a geometrical model for non-linear fluid flow through
163 rough fractures was proposed and evaluated through numerical simulations by *Javadi et al.*
164 [2010]. More recently, *Roustaei et al.* [2016] numerically investigated the 2D-flow of a yield
165 stress fluid along an uneven fracture, showing that important Darcy-type flow law lead to
166 important errors in the case of short fractures due to self-selection of the flowing region and
167 the existence of fouling layers of unyielded fluid.

168

169 Several authors showed that the intrinsic permeability K and the cross-sectional area A used
170 in Eqs (1) to (5) can be written as functions of the hydraulic aperture of the fracture h
171 [*Witherspoon et al.*, 1980; *Brown*, 1987; *Zimmerman and Yeo*, 2000; *Brush and Thomson*,
172 2003]:

173

$$K = \frac{h^2}{12} \quad (7)$$

174

$$A = hw \quad (8)$$

175 The aperture distribution of rough-walled rock fractures always presents a strong
176 heterogeneity, due to the wide range of aperture sizes and the significant number of contact
177 points [*Witherspoon et al.*, 1980; *Xiong et al.*, 2011; *Javidi et al.*, 2014; *Wang et al.*, 2016].

178 The hydraulic behaviour through a fracture is known to be heavily dependent upon the
179 apertures distribution [*Isakov et al.*, 2001; *Javidi et al.*, 2014; *Wang et al.*, 2016]. This is
180 explained by the tendency of the fluid to flow through the paths with the largest apertures.
181 Moreover, within a given path, the hydraulic behaviour of fracture would be controlled by the
182 small apertures and constrictions [*Tsang and Tsang*, 1987; *Neuzil and Tracy*, 1981].

183

184 Several attempts have been made to obtain a macroscopic law linking the injection flow rate
185 to the resulting pressure drop during the flow of yield stress fluids in porous media [*Pascal*,
186 1983; *Al-Fariss and Pinder*, 1987; *Chase and Dachavijit*, 2005; *Coussot*, 2014]. A major
187 drawback of most available expressions is the existence of experimentally adjustable

188 parameters with no clear physical meaning as inputs, which impedes direct computational
189 predictions. As an alternative, some pore-network approaches have also been proposed [*Chen*
190 *et al.*, 2005; *Sochi and Blunt*, 2008]. The main advantage of pore-network models is that they
191 provide a reasonably realistic description of the reality in which the number of flow paths
192 through the porous media increases with the applied pressure gradient. Nonetheless, these
193 effects need experimental validation. Also, pore-network methods do not lead to analytical
194 ∇P - u expressions, which is particularly aggravating in applications involving a wide range of
195 injection flow rates. An important difference between Newtonian and complex fluids is the
196 coupling of geometrical and rheological parameters in the flow law [*Roustaei et al.*, 2016].
197 Indeed, a non-toxic method of porosimetry has been proposed, which is based on the
198 injection of yield stress fluids through porous media and takes advantage of the mentioned
199 coupling [*Ambari et al.*, 1990; *Malvault*, 2013; *Oukhlef et al.* 2014; *Rodríguez de Castro*,
200 2014; *Rodríguez de Castro et al.*, 2014; *Rodríguez de Castro et al.*, 2016a].

201

202 The literature survey conducted by *Lavrov* [2013] revealed the severe lack of research on
203 fracture flow of non-Newtonian fluids, especially regarding yield-stress fluids. The earlier
204 works of Di Federico [*Di Federico*, 1997; *Di Federico*, 1998; *Di Federico*, 2001] mainly
205 focused on defining and estimating the equivalent aperture for flow of a non-Newtonian fluid
206 in a variable aperture fracture, without experimental validation. Also, *Silliman* [1989]
207 provided different aperture estimates for variable aperture fractures. Only a few experimental
208 works exist for the flow of yield stress fluids in porous media [*Al-Fariss and Pinder*, 1987;
209 *Chase and Dachavijit*, 2005; *Chevalier et al.*, 2013; *Chevalier et al.*, 2014; *Rodríguez de*
210 *Castro*, 2016a], and the ranges of variation of u are usually narrow. These experimental
211 works show that the relationship between ∇P and u is of the same form as the constitutive
212 equation of the fluid, i.e. $\nabla P = \nabla P_0 + Cu^n$ with ∇P_0 being the critical pressure gradient below

213 which no flow occurs, n being the flow index of the fluid and C being a parameter that
214 depends on the porous medium and the boundary conditions. *Chevalier et al.* [2014] used an
215 NMR imaging technique to show that the velocity density distribution of a yield stress fluid
216 flowing through a packed bed was similar to that of a Newtonian fluid due to the minor role
217 played by the constitutive equation of the fluid in rapidly varying pore geometry. On the basis
218 of these results, the latter authors propose explicit (but complex) expressions with physical
219 meaning to calculate ∇P_0 and C . Concerning multiphase flow, *Boronin et al.*, [2015]
220 developed a model for the displacement of yield-stress fluids in a vertical Hele-Shaw cell and
221 used it to investigate the joint effect of viscous fingering, yielding and gravitational slumping,
222 showing that unyielded fluid zones develop as a result of viscous fingering generated when a
223 yield stress fluid is displaced by a low-viscosity Newtonian one.

224

225 Recently, *Rodríguez de Castro and Radilla* [2016a] conducted non-Darcian flow experiments
226 of shear-thinning fluids without yield stress in rough-walled fractures, showing that the
227 inertial pressure losses do not depend on fluid's rheology. These authors proposed a method
228 to predict the pressure losses generated during non-Darcian shear-thinning flow from the
229 values of K , γ and β obtained during creeping Newtonian flow and the shear-viscosity
230 parameters of the fluid. Their predictions showed good agreement with experimental data.
231 However, a major drawback was the use of an experimentally obtained shift parameter to
232 relate the apparent viscosity of the fluid in the porous medium to its bulk viscosity. Indeed,
233 the calculation of the mentioned shift parameter involved carrying out preliminary Darcian
234 shear-thinning flow experiments, so predicting its value is of considerable interest. Another
235 interesting prospect consisted in extending this prediction method to the case of shear-
236 thinning fluids with yield stress. The same authors also proposed a simple method to predict
237 non-Darcian flow of Carreau fluids through packed beads [*Rodríguez de Castro and Radilla*,

238 2016b]. However, the flow of yield stress fluids was not tackled in these works and no
239 estimate of the shift parameter relating the apparent viscosity of the fluid in the porous
240 medium to its bulk viscosity was provided either.

241

242 Many applications require the flow rate in a fracture to be predicted from the applied pressure
243 gradient and known fracture size and fluid rheology. In particular, understanding the flow of
244 drilling fluids with yield stress through a rough-walled fracture is of vital importance in order
245 to design the additives used to stop the fluid loss when a fracture is hit during drilling
246 [Lavrov, 2013]. In this work, a simple approach is proposed to extend Darcy's and full cubic
247 laws to the case of yield stress and Carreau fluids. In order to achieve this goal, flow
248 experiments with concentrated aqueous polymer solutions have been conducted using
249 replicas of natural fractures. Particular attention will be paid to investigating how yield stress
250 affects the relationships between flow rate and pressure losses in rough-walled rock fractures.

251

252 **2. Predicting the flow of yield stress fluids and Carreau fluids in porous media**

253

254 The shear-thinning behaviour of semi-dilute polymer solutions widely used in EOR and soil
255 remediation is commonly represented by the empirical Carreau model [Carreau, 1972] based
256 on molecular network theory [Sorbie, 1989; López *et al.*, 2003; Rodríguez de Castro *et al.*,
257 2016b]. The Carreau equation is often presented as $\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = [1 + (\lambda\dot{\gamma})^2]^{\frac{n-1}{2}}$, where μ is the
258 viscosity at a given shear rate $\dot{\gamma}$, μ_0 and μ_∞ are the zero shear rate and infinite shear rate
259 viscosities, respectively, n is the power-law index, and λ is the time constant. n is inferior to
260 unity for shear-thinning fluids. The values of μ_0 , μ_∞ , n and λ are determined by the polymer

261 concentration under given pressure and temperature conditions. In the region far from the low
 262 shear viscosity plateau, i.e. when $\dot{\gamma} \gg \frac{1}{\lambda}$, Carreau's law leads to the following expression
 263 [*Rodríguez de Castro and Radilla, 2016a*]:

$$\mu \approx \mu_{\infty} + (\mu_0 - \mu_{\infty})\lambda^{n-1}\dot{\gamma}^{n-1} = \mu_{\infty} + c\dot{\gamma}^{n-1} \quad (9)$$

264 with $c = (\mu_0 - \mu_{\infty})\lambda^{n-1}$. Given that all the shear rates involved in the flow experiments with
 265 Carreau fluids analysed in this work are sufficiently high, only the high shear rates version of
 266 Carreau's equation (Eq. 9) will be considered subsequently.

267 Some concentrated polymer solutions present a yield stress, as shown in previous works
 268 [*Song et al., 2006; Carnali, 1991; Withcomb and Macosko, 1978; Economides and Nolte,*
 269 *2000; Khodja, 2008; Benmouffok-Benbelkacem et al., 2010*]. The steady-state shear flow of
 270 concentrated polymer solutions has been proved to be well described by the Herschel–
 271 Bulkley law [*Herschel and Bulkley, 1926*]. This empirical law can be written as follows:

272

$$\begin{cases} \tau = \tau_0 + a\dot{\gamma}^n & \text{for } \tau > \tau_0 \\ \dot{\gamma} = 0 & \text{for } \tau \leq \tau_0 \end{cases} \quad (10)$$

273 where τ_0 is the yield stress, a is the consistency and n is the flow index of the fluid. In the
 274 case of shear-thinning yield stress fluids, n is inferior to unity. The three parameters are
 275 generally obtained by fitting the data obtained by measuring the shear rate $\dot{\gamma}$ as a function of
 276 the applied shear stress τ using a rheometer.

277

278 A practical approach to study the flow of complex fluids with shear-rate-dependent viscosity
 279 through a porous medium consists in defining an equivalent viscosity μ_{eq} as being the

280 quantity that must replace the viscosity in Darcy's law to result in the same pressure drop
 281 actually measured [Tosco *et al.*, 2013]. In the case of a rectangular fracture, μ_{eq} is expressed
 282 as:

$$\mu_{eq} = K \frac{\nabla P}{u} = \frac{h^2}{12} \frac{\nabla P}{u} \quad (11)$$

284 It should be noted that both inertial and viscous effects are encompassed in μ_{eq} . In order to
 285 analyse the viscous effects separately, the “*in situ*” shear viscosity μ_{pm} in the porous medium
 286 must be calculated. To do such calculation from the constitutive equation of the fluid, an
 287 apparent shear rate in the porous medium has to be determined first. The apparent shear rate
 288 $\dot{\gamma}_{pm}$ of shear-thinning fluids flowing through a porous medium can be defined by dividing the
 289 mean velocity u by a characteristic microscopic length of the porous media [Chauveteau,
 290 1982; Sorbie *et al.*, 1989; Perrin *et al.*, 2006; Tosco *et al.*, 2013; Rodríguez de Castro *et al.*,
 291 2016b]. This microscopic length is usually taken as $\sqrt{K\varepsilon}$ with ε being the porosity of the
 292 porous medium. From the definition of cross-sectional area (Eq. 7), it is expected that
 293 porosity is close to unity in the particular case of fractures. Therefore, $\dot{\gamma}_{pm}$ can be defined as:

$$\dot{\gamma}_{pm} = \alpha \frac{u}{\sqrt{K}} = \alpha \frac{2\sqrt{3}u}{h} \quad (12)$$

295
 296 where α is an empirical shift factor known to be a function of both the bulk rheology of the
 297 fluid and the porous media [Chauveteau, 1982; Sorbie *et al.*, 1989; López *et al.*, 2003; López,
 298 2004; Comba *et al.*, 2011]. Previous research showed that $\dot{\gamma}_{pm}$ corresponds to the wall shear

299 rate in the average pore throat diameter [*Chauveteau and Zaitoun, 1981; Chauveteau, 1982;*
300 *Sheng, 2011*].

301

302 The usual approach to determine the value α consists in overlaying the porous medium μ_{eq}
303 vs. $\dot{\gamma}_{app}$ with the bulk μ_{eq} vs. $\dot{\gamma}$ curves as closely as possible and noting the scale change in
304 shear rate required to obtain the best fit. This criterion to select α was proposed by *Sorbie et*
305 *al.* [1989] as a pragmatic alternative to the original one previously proposed by *Chauveteau*
306 [1982], and was subsequently used by other authors [*González et al., 2005; Amundarain et*
307 *al., 2009*]. It should be noted that a good overlay between both curves is only possible in the
308 low flow rates region where no significant inertial effects occur, assuming no wall slip [*Tosco*
309 *et al., 2013; Rodríguez de Castro and Radilla, 2016a*]. Keeping in mind the objective to
310 propose a prediction method, expressions for the calculation of α must be provided so as to
311 avoid the need to perform α -determination experiments.

312

313 In the case of Carreau fluids flowing at moderate and high shear rates μ_{pm} can be obtained
314 from Eqs. (9) and (12):

315

$$\mu_{pm,Carreau} = \mu_{\infty} + c \left(\alpha \frac{2\sqrt{3}u}{h} \right)^{n-1} \quad (13)$$

316 Analogously, in the case of Herschel-Bulkley fluids, μ_{pm} can be obtained from Eqs. (10) and
317 (12):

318

$$\mu_{pm,ysf} = \frac{\tau_0 h}{\alpha 2\sqrt{3}u} + a \left(\alpha \frac{2\sqrt{3}u}{h} \right)^{n-1} \quad (14)$$

319

320 Although Eqs. (12-14), which are based on the bundle-of-capillaries model, were originally
 321 proposed for the flow of non-Newtonian fluids through packed beads, the apparent viscosity
 322 was found to correlate reasonably well in porous media with complex pre geometry and
 323 topology [*Sorbie et al.*, 1989].

324 Let us focus now on the determination of the wall shear rate in rectangular channels. For the
 325 steady 2D-flow of an incompressible fluid through a rectangular channel, the wall shear stress
 326 τ_w is related to the pressure gradient ∇P as follows [*Pipe et al.*, 2008]:

327

$$\tau_w = \frac{wh}{2(w+h)} \nabla P \quad (15)$$

328

329 For the calculation of ∇P in Eq. (15), the fractures will be modelled as being rectangular
 330 channels of width w and depth h . As explained above, $\dot{\gamma}_{pm}$ corresponds to the wall shear rate
 331 in the average pore throat diameter [*Chauveteau and Zaitoun*, 1981; *Chauveteau*, 1982;
 332 *Sheng*, 2011]. In the case of a rough-walled fracture, the average pore throat diameter can be
 333 assimilated to the hydraulic aperture. Therefore, $\dot{\gamma}_{pm}$ can be interpreted as the wall shear rate
 334 in a section of aperture h . The wall shear stress in a section of aperture h can be calculated
 335 from Eq. (15), by using Eqs. (1) and (13) for the case of Carreau fluids:

336

337

$$\tau_w = \frac{6w}{(w+h)} \frac{u}{h} [\mu_\infty + c\dot{\gamma}_{pm}^{n-1}] = \frac{6w}{(w+h)} \frac{u}{h} \left[\mu_\infty + 2^{n-1} 3^{\frac{n-1}{2}} c \left(\alpha \frac{u}{h} \right)^{n-1} \right] \quad (16)$$

338

339 And using Eqs. (1) and (14) for the case of Herschel-Bulkley fluids, Eq. (15) can be written

340 as:

$$\tau_w = \frac{6w}{(w+h)} \frac{u}{h} \left(\frac{\tau_0}{\dot{\gamma}_{pm}} + a\dot{\gamma}_{pm}^{n-1} \right) = \frac{\sqrt{3}w}{(w+h)\alpha} \left[\tau_0 + 2^n 3^{\frac{n}{2}} a \left(\alpha \frac{u}{h} \right)^n \right] \quad (17)$$

341

342 For a constant viscosity fluid, the wall shear rate is given by $\dot{\gamma}_{w,Newtonian} = \frac{6u}{h}$. However, for

343 incompressible flows of liquids with a shear-rate-dependent viscosity, the calculation of $\dot{\gamma}_w$ is

344 more complex given that the velocity profile is no longer parabolic [*Pipe et al.*, 2008]. An

345 apparent shear rate $\dot{\gamma}_{app}$ can thus be defined as:

346

$$\dot{\gamma}_{app} = \frac{6u}{h} \quad (18)$$

347

348 The true wall shear rate can be found using the Weissenberg–Rabinowitsch–Mooney

349 equation [*Macosko*, 1994; *Pipe et al.*, 2008]:

350

$$\dot{\gamma}_w = \frac{\dot{\gamma}_{app}}{3} \left[2 + \frac{d(\ln \dot{\gamma}_{app})}{d(\ln \tau_w)} \right] \quad (19)$$

351

352 Therefore, the next equation can be obtained from Eqs. (16), (18) and (19) for a Carreau
353 fluid:

$$\dot{\gamma}_{w,Carreau} = \frac{2u}{h} \left(2 + \frac{2\sqrt{3}\alpha h^n \mu_\infty u + 2^n 3^{n/2} ah(\alpha u)^n}{2\sqrt{3}\alpha h^n \mu_\infty u + 2^n 3^{n/2} ahn(\alpha u)^n} \right) \quad (20)$$

354

355 Analogously, the next equation can be obtained from Eqs. (17), (18) and (19) for a Herschel-
356 Bulkley fluid:

$$\dot{\gamma}_{w,ysf} = \frac{2u}{h} \left(2 + \frac{a + 2^{-n} 3^{-n/2} \tau_0 \left(\alpha \frac{u}{h} \right)^{-n}}{an} \right) \quad (21)$$

357

358 For a Carreau fluid, $\dot{\gamma}_{pm} = \dot{\gamma}_{w,Carreau}$, so Eqs. (12) and (20) lead to the following expression:

359

$$\alpha = \frac{6\sqrt{3}\alpha h^n \mu_\infty u + 2^n 3^{\frac{n}{2}} ah(1 + 2n)(\alpha u)^n}{6\alpha h^n \mu_\infty u + 2^n 3^{\frac{n+1}{2}} ahn(\alpha u)^n} \quad (22)$$

360

361 In the case of a Herschel-Bulkley fluid ($\dot{\gamma}_{pm} = \dot{\gamma}_{w,ysf}$), Eqs. 12 and 21 lead to:

362

$$\alpha = \frac{1}{\sqrt{3}} \left(2 + \frac{a + 2^{-n} 3^{-n/2} \tau_0 \left(\alpha \frac{u}{h} \right)^{-n}}{an} \right) \quad (23)$$

363

364 From Eqs. (22) and (23), it can be deduced that α is not a constant parameter in the case of
 365 Carreau fluids and yield stress fluids, but depends on u . For the simpler case of a power-law
 366 fluid ($\tau_0=0$), Eq. (23) leads to:

367

$$\alpha = \frac{1}{\sqrt{3}} \left(2 + \frac{1}{n} \right) \quad (25)$$

368 which becomes $\alpha = \sqrt{3}$ for a Newtonian fluid. Therefore, α is a constant parameter only if
 369 $\tau_0 = 0$ and $\mu_\infty = 0$.

370

371 Given that α depends on u for both Carreau and yield stress fluids, Eqs. (22) and (23) are
 372 only relevant in the cases $\mu_\infty = 0$ and $\tau_0 = 0$, respectively. For $\mu_\infty \neq 0$ and $\tau_0 \neq 0$, Eq. (19)
 373 becomes:

374

$$\dot{\gamma}_w = \frac{\dot{\gamma}_{app}}{3} \left[2 + \frac{\frac{d(\ln \dot{\gamma}_{app})}{du} du}{\frac{\partial(\ln \tau_w)}{\partial u} du + \frac{\partial(\ln \tau_w)}{\partial \alpha} d\alpha} \right] = \frac{\dot{\gamma}_{app}}{3} \left[2 + \frac{\frac{d(\ln \dot{\gamma}_{app})}{du}}{\frac{\partial(\ln \tau_w)}{\partial u} + \frac{\partial(\ln \tau_w)}{\partial \alpha} \frac{d\alpha}{du}} \right] \quad (26)$$

375 where α is a function of u .

376

377 Therefore, for a Carreau fluid, Eq. (20) becomes:

$$\dot{\gamma}_{w,Carreau} = \frac{2u}{h} \left[2 + \frac{\alpha \left(2\sqrt{3}\mu_{\infty}u\alpha + 2^n 3^{\frac{n}{2}} ah \left(\frac{u\alpha}{h} \right)^n \right)}{2\sqrt{3}\mu_{\infty}\alpha^2 + 2^n 3^{\frac{n}{2}} ahn\alpha \left(\frac{u\alpha}{h} \right)^n + 2^{n+1} 3^{\frac{n}{2}} ah(n-1)u \left(\frac{u\alpha}{h} \right)^n \frac{\partial\alpha}{\partial u}} \right] \quad (27)$$

378

379 Also, for a Herschel-Bulkley fluid, Eq. (21) becomes:

380

$$\dot{\gamma}_{w,ysf} = \frac{2u}{h} \left[2 + \frac{\alpha \left(\tau_0 + 2^n 3^{\frac{n}{2}} a \left(\frac{u\alpha}{h} \right)^n \right)}{2^n 3^{\frac{n}{2}} a n \alpha \left(\frac{u\alpha}{h} \right)^n - 2u \left(\tau_0 - 2^n 3^{\frac{n}{2}} a (n-1) \left(\frac{u\alpha}{h} \right)^n \right) \frac{\partial\alpha}{\partial u}} \right] \quad (28)$$

381

382 Consequently, the following differential equation has to be solved in order to determine α as

383 a function of u for a Carreau fluid:

384

$$\alpha = \frac{1}{\sqrt{3}} \left[2 + \frac{\alpha \left(2\sqrt{3}\mu_{\infty}u\alpha + 2^n 3^{\frac{n}{2}} ah \left(\frac{u\alpha}{h} \right)^n \right)}{2\sqrt{3}\mu_{\infty}\alpha^2 + 2^n 3^{\frac{n}{2}} ahn\alpha \left(\frac{u\alpha}{h} \right)^n + 2^{n+1} 3^{\frac{n}{2}} ah(n-1)u \left(\frac{u\alpha}{h} \right)^n \frac{\partial\alpha}{\partial u}} \right] \quad (29)$$

385

386 And for a Herschel-Bulkley fluid:

387

$$\alpha = \frac{1}{\sqrt{3}} \left[2 + \frac{\alpha \left(\tau_0 + 2^n 3^{\frac{n}{2}} a \left(\frac{u\alpha}{h} \right)^n \right)}{2^n 3^{\frac{n}{2}} a n \alpha \left(\frac{u\alpha}{h} \right)^n - 2u \left(\tau_0 - 2^n 3^{\frac{n}{2}} a (n-1) \left(\frac{u\alpha}{h} \right)^n \right) \frac{\partial\alpha}{\partial u}} \right] \quad (30)$$

388

389 Eqs. (29) and (30) can be numerically solved within a given range of u to obtain the relation

390 between α and u . Then, the obtained relation can be used in Eq. (13) for a Carreau fluid and

391 in Eq. (14) for a Herschel-Bulkley fluid to obtain $\mu_{pm,Carreau}$ and $\mu_{pm,ysf}$, respectively. Once

392 $\mu_{pm,Carreau}$ and $\mu_{pm,ysf}$ have been determined, they can be entered in Eq. (1), Eq. (2), Eq. (3)
 393 and Eq. (5), leading to the extension of Darcy's law (Eq. 31), Forchheimer's law (Eq. 32) and
 394 full cubic law (Eq. 33) to Carreau and yield stress fluids:

395

$$\nabla P = \frac{\mu_{pm}}{K} u = \frac{12\mu_{pm}}{h^2} u \quad (31)$$

396

$$\nabla P = \frac{\mu_{pm}}{K} u + \beta \rho u^2 = \frac{12\mu_{pm}}{h^2} u + \beta \rho u^2 \quad (32)$$

397

$$\nabla P = \frac{\mu_{pm}}{K} u + \beta \rho u^2 + \frac{\gamma \rho^2}{\mu_{pm}} u^3 = \frac{12\mu_{pm}}{h^2} u + \beta \rho u^2 + \frac{\gamma \rho^2}{\mu_{pm}} u^3 \quad (33)$$

398

399 It is remarked that $\frac{12\mu_{pm}}{h^2} u = \frac{2\sqrt{3}\tau_0}{h} \frac{1}{\alpha} + \frac{2^{n+1}3^{\frac{n+1}{2}}a}{h^{n+1}} \alpha^{n-1} u^n = C_1 \frac{1}{\alpha} + C_2 \alpha^{n-1} u^n$ in the case of
 400 Herschel-Bulkley fluids, with $C_1 = \frac{2\sqrt{3}\tau_0}{h}$ and $C_2 = \frac{2^{n+1}3^{\frac{n+1}{2}}a}{h^{n+1}}$. Therefore, Eqs. (31) to (33)
 401 present a limiting pressure gradient [Roustaei *et al.*, 2016] of value $C_1 \frac{1}{\alpha}$. In the preceding
 402 expressions, μ_{pm} corresponds to $\mu_{pm,Carreau}$ or $\mu_{pm,ysf}$ depending on the type of fluid being
 403 considered.

404

405

406

407 It should be noted that elongational flows during the injection of solutions of polymers
408 presenting a certain degree of flexibility through porous media are known to induce extra
409 pressure losses with respect to pure shear flow [*Rodríguez et al.*, 1993; *Müller and Sáez*,
410 1999; *Nguyen and Kausch*, 1999; *Seright et al.*, 2011; *Amundarain et al.*, 2009). These extra
411 pressure losses were attributed to the formation of transient entanglements of polymer
412 molecules due to the action of the extensional component of the flow. In the present
413 approach, we first hypothesize that the differences between the total pressure drops measured
414 during the flow of the investigated complex fluids through rough-walled rock fractures and
415 the viscous pressure drop as predicted from the shear viscosity of the fluid can be explained
416 in terms of inertial effects generated in the porous medium flow. This hypothesis is then
417 validated through analysis of the experimental results.

418

419 **3. Materials and Methods**

420

421 In this section, we present the experimental procedure and the materials used to carry out the
422 flow experiments with a yield stress fluid specifically performed in the framework of the
423 present study. However, the proposed method to predict ∇P as a function of u in rough-
424 walled fractures is also compared with previously presented experimental data [*Rodríguez de*
425 *Castro and Radilla*, 2016a] in order to assess its efficiency in the case of Carreau fluids.

426

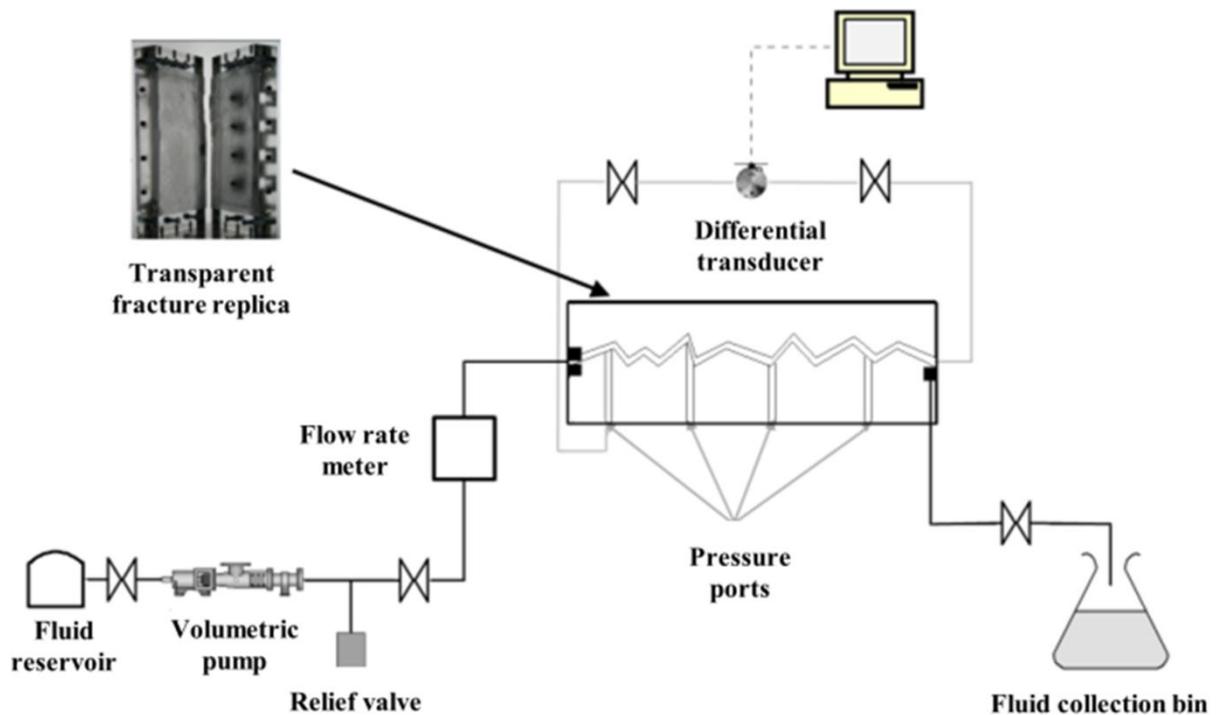
427 **3.1. Experimental setup and procedure**

428

429 A series of experiments was conducted injecting a concentrated aqueous polymer solution
430 through two transparent epoxy resin replicas of natural rough-walled rock fractures. The
431 original fractures used in this work are a Vosges sandstone sample with dimensions 26 cm
432 long and $w = 14.8$ cm wide, and a granite sample with dimensions 33 cm long and $w = 15.5$
433 cm wide. Details of the fabrication process of these fracture replicas can be found elsewhere
434 [Isakov *et al.*, 2001; Nowamooz *et al.*, 2013]. The aperture maps of both fractures obtained by
435 Nowamooz *et al.* [2013] have been included as supporting information of the present article
436 (Figures S₁ and S₂), showing the high spatial variability. The latter authors analysed in detail
437 the aperture variability and distribution of the fractures using an image processing procedure
438 based on the attenuation law of Beer-Lamber. They showed that the smallest apertures are
439 located at the centre and the largest apertures are located near the inlet and the outlet of the
440 fractures. Moreover, the apertures of the Vosges sandstone fracture are more variable at
441 lower half than at the upper half, while the spatial variability appears to be relatively high
442 across the entire granite fracture area. This results in a more heterogeneous aperture map for
443 the granite fracture. Moreover, Nowamooz *et al.* [2013] showed that the spatial variability of
444 the fracture aperture field, especially the constricted areas at the centre of the fractures,
445 resulted in the creation preferential paths for the flow of the fluid. These effects are expected
446 to be more important in the case of shear-thinning fluids and yield stress fluids as the pressure
447 loss sensitivity to aperture is higher (shear viscosity depends on the local aperture) [Roustaei
448 *et al.*, 2016].

449
450 Two different configurations were used depending on the involved flow rates. For the lowest
451 flow rates, ranging from 0.06 L/h to 6 L/h, the injection circuit was open and the fluid was
452 injected through the fractures at the selected flow rate using a dual piston pump (Prep Digital
453 HPCL pump, A.I.T., France). For the highest flow rates, ranging from 9 L/h to 250 L/h, the

454 circuit was closed. In this case, the fluid was injected from a tank situated upstream of the
 455 fracture using a volumetric pump (EcoMoineau M Series, PCM, France), and its flow rate
 456 was measured with a positive displacement flow meter (Model LSM45, Oval, Japan). The
 457 injected fluid was continuously recirculated to the upstream tank after passing through the
 458 fracture. A differential pressure sensor (DP15 Variable Reluctance Pressure Sensor,
 459 Validyne, USA) was used to measure the pressure drop over a distance of $L = 20.5$ cm in the
 460 case of the Vosges sandstone fracture and $L = 27$ cm in the case of the granite fracture. A
 461 sketch of the experimental setup is shown in Figure 1. The range of the piston pump was
 462 from 6×10^{-3} to 6 L/h with an accuracy of $\pm 2\%$ while the volumetric pump was able to
 463 provide flow rates ranging from 0 to 300 L/h. The range of the flow meter installed at the
 464 outlet of the volumetric pump was from 7 to 500 L/h with an accuracy of $\pm 1\%$ and the range
 465 of the pressure sensor was adjusted by installing different membranes from 0-1400 Pa to 0-
 466 56000 Pa with an accuracy of $\pm 0.3\%$ of the full scale
 467



468

469 **Figure 1.** Sketch of the experimental setup used in the present experiments.

470

471 The procedure followed in our experiments was similar to the one followed by *Rodríguez de*
472 *Castro and Radilla* [2016a], but the covered range of injection flow rates was considerably
473 wider. In this procedure, the fractures were saturated with CO₂ (more water-miscible gas than
474 air) prior to saturation with polymer solution in order to avoid air trapping during the
475 experiments. Once saturated with polymer solution, a set of forty-five different flow rates
476 ranging from 0.06 to 250 L/h were imposed for the flow through the fracture and the
477 corresponding pressure drops were measured. It can be observed that the range of u used in
478 this work is significantly wider than those used in some preceding works (Sabiri and Comiti,
479 1994), which permits a better assessment of the proposed prediction methods (over ~ 3.6
480 orders of magnitude). Each step was repeated four times and the uncertainty related to the
481 repeatability of the pressure drop and the accuracy of the involved instruments was calculated
482 as $\pm 2\sigma$, with σ being an estimate of the relative standard deviation of the measurements (95%
483 confidence interval). The room temperature during the experiments was $20^{\circ}\text{C} \pm 1$.

484

485 **3.2. Fluid Properties**

486

487 Filtered water and a xanthan gum aqueous solution with polymer concentration $C_p = 7000$
488 ppm were used as injected fluid in the present experiments. Xanthan gum is an important
489 industrial biopolymer commonly obtained through fermentation of *Xanthomonas campestris*
490 bacteria [*Garcia-Ochoa et al.*, 2000; *Palarinaj and Javarman*, 2011; *Wadhai and Dixit*,
491 2011]. This biopolymer is widely used as viscosity-enhancing additive in the food and
492 cosmetics industries, as zerovalent iron for groundwater remediation and as part of the
493 formulation of drilling muds in EOR [*Garcia-Ochoa et al.*, 2000; *Amundarain et al.*, 2009;

494 *Palarinaj and Javarman, 2011; Wadhai and Dixit, 2011; Xin et al., 2015*]. In solution state,
495 an isolated xanthan macromolecule is more or less rigid and is of typically 1 μm of contour
496 length [*Mongruel and Cloitre, 2003*] and a transverse size of approximately 2 nm. Song
497 [2007] presented additional information about the chemical composition, structure and other
498 physico-chemical properties of this biopolymer. Xanthan gum solutions are one of the main
499 examples of inelastic, shear-thinning fluids in contrast to linear flexible polymers as
500 polyacrylamide [*Jones and Walters 1989; Sorbie 1991a*] which are highly viscoelastic. Due to
501 the stiffness of its molecule, xanthan semidilute aqueous solutions develop a high viscosity
502 level and a very pronounced shear-thinning behavior. Therefore, xanthan gum solutions have
503 been reported to present an apparent yield stress [*Song et al., 2006; Carnali, 1991; Withcomb*
504 *and Macosko, 1978; Khodja, 2008; Benmouffok-Benbelkacem et al., 2010*] even if strictly
505 speaking, they should be referred to as pseudo-yield stress fluids. The Herschel–Bulkley
506 model [*Herschel and Bulkley, 1926*] has been proved to describe the steady-state shear flow
507 of concentrated xanthan gum solutions [*Song et al., 2006; Rodríguez de Castro et al., 2014,*
508 2016a].

509

510 Sixty litres of polymer solution were prepared by dissolving xanthan gum in filtered water
511 containing 400 ppm of NaN_3 as a bactericide. The xanthan gum powders were progressively
512 dissolved in water while gently mixing with a custom-made overhead device. Once prepared,
513 the polymer solution was characterized by means of a stress controlled rheometer (ARG2, TA
514 Instruments) equipped with cone-plate geometry at a constant temperature of $19^\circ\text{C} \pm 1$,
515 following a procedure previously presented in the literature [*Rodríguez de Castro et al., 2014,*
516 2016a, 2016b]. The obtained rheograms are provided as supporting information (Figure S3).
517 Eq. (10) was used to fit the rheograms following the procedure presented by *Rodríguez de*
518 *Castro et al. [2014]* and obtaining $\tau_0 = 7.4 \text{ Pa}$, $a = 0.37 \text{ Pa s}^n$ and $n = 0.52$. A viscosity of

519 0.001 Pa s was measured for the solvent (water) and the densities ρ of both the water and the
520 xanthan gum solution were taken as 1000 kg/m^3 .

521

522 Moreover, a set of effluent fluid samples were collected at the outlet of the fractures after
523 injection at the highest flow rate. The effluent rheograms were determined and compared to
524 that of the inflowing fluid in order to assess polymer degradation and retention of the
525 polymer on the fracture walls. No significant difference was observed between the
526 rheograms, so polymer degradation and significant polymer retention were proved to be
527 negligible. Moreover, no air macro bubbles were observed in the injected fluid. Also, the
528 rheograms of a degassed fluid sample and an undegassed fluid sample were measured and
529 compared in order to evaluate the influence of residual air micro bubbles, showing no
530 significant difference.

531

532 The Carreau fluids used in the non-Darcian shear-thinning flow experiments in rough-walled
533 fractures performed by [Rodríguez de Castro and Radilla, 2016a] were three xanthan gum
534 aqueous solutions with polymer concentrations of 200 ppm, 500 ppm and 700 ppm,
535 respectively. The corresponding rheological parameters used in Eq. (9) for these fluids were
536 [$c = 4.8 \times 10^{-3} \text{ Pa s}^n$, $\mu_\infty = 1.1 \times 10^{-3} \text{ Pa s}$, $n = 6.6 \times 10^{-1}$] for $C_p = 200 \text{ ppm}$, [$c = 2.4 \times 10^{-3} \text{ Pa}$
537 s^n , $\mu_\infty = 1.1 \times 10^{-3} \text{ Pa s}$, $n = 5.8 \times 10^{-1}$] for $C_p = 500 \text{ ppm}$ and [$c = 4.2 \times 10^{-3} \text{ Pa s}^n$, $\mu_\infty = 1.1 \times$
538 10^{-3} Pa s , $n = 5.2 \times 10^{-1}$] for $C_p = 700 \text{ ppm}$.

539

540 **4. Results**

541

542 The flow experiments were conducted for both fluids (water and yield stress fluid) and were
543 repeated four times. For each fluid, a total of a hundred and eighty (four repetitions for each
544 of the forty-five flow rates) were completed. The hundred and eighty measures for a given
545 fluid-fracture pair were considered to be an experimental set.

546

547 **4.1. Non-Darcian flow of a Newtonian fluid: obtaining K , γ and β from experiments**

548

549 The experimental sets of ∇P as a function of u for water injection ($C_p = 0$ ppm) through both
550 fractures are included as supporting information (Figure S4). Higher pressure losses were
551 obtained for the less permeable fracture (Vosges sandstone), as expected, and non-linear
552 relations between u and ∇P were observed in both cases stemming from inertial effects at high
553 flow rates. It is known that directly fitting Eq. (5) to the whole set of data results in
554 overestimation of permeability [*Du Plessis and Masliyah, 1988; Dukhan et al., 2014*].
555 Indeed, by fitting the whole set of data to the polynomial law, a part of the pressure drop
556 would be attributed to inertial effects even at the lowest flow rates, which is not realistic.
557 Consequently, the viscous pressure loss would be underestimated leading to permeability
558 overestimation. To avoid this issue, the procedure proposed by *Rodríguez de Castro and*
559 *Radilla [2016a]* was followed to determine h and K in the present experiments. This
560 procedure is divided into two-steps:

561 1) In this step, the hydraulic apertures h_j obtained by only using the first j experimental data
562 (starting with the lowest flow rates) are calculated by minimizing the sum $\sum_{i=1}^j \left(\nabla P_i -$

563 $\left(\frac{12Q_i\mu}{h_j^3w}\right)^2$ for $j = 1\dots N$, with N being the number of experimental data and μ being the
 564 measured dynamic viscosity of water at the room temperature (0.001 Pa s).

565

566 2) Then, the quality of the N fits obtained by using the N values of h_j calculated in the

567 preceding step is evaluated by using the merit function $F(j) = \frac{\sum_{i=1}^j \left| \frac{\nabla P_i - \frac{12Q_i\mu}{h_j^3w}}{\nabla P_i} \right|}{j}$ for $j = 1\dots N$.

568 After that, the value of j minimizing $F(j)$ was determined. The corresponding h_j value was
 569 selected as the hydraulic aperture of the fracture from which K was calculated using Eq. (7).

570

571 The obtained values for the granite fracture were $K = 6.1 \times 10^{-8} \text{ m}^2$ ($\pm 2\%$) and $h = 8.5 \times 10^{-4}$
 572 m ($\pm 2\%$), while for the Vosges sandstone fracture the computed values were $K = 2.1 \times 10^{-8}$
 573 m^2 ($\pm 1\%$) and $h = 5.0 \times 10^{-4} \text{ m}$ ($\pm 1\%$). Once permeability was determined, the $(Q_i, \nabla P_i,)$ data
 574 were fitted to a full cubic law (Eq. 5) through a standard least squares method using the value
 575 of K calculated in the previous step and obtaining the values of d and β . The computed values
 576 were $d = 2.5 \times 10^{-5}$ ($\pm 5\%$) and $\beta = 0 \text{ m}^{-1}$ for the granite fracture, and $d = 2.2 \times 10^{-5}$ ($\pm 2\%$) and
 577 $\beta = 1.5(\pm 2\%) \text{ m}^{-1}$ for the granite fracture. Percentages represent $\pm 2\sigma$, with σ being an
 578 estimate of the relative standard deviation of the measurements (95% confidence interval).

579

580 **4.2. Equivalent and shear viscosity relations**

581

582 Eq. (30) was numerically solved within the involved range of u for both fractures using an
 583 implicit Runge-Kutta method. From (23), it can be deduced that α becomes the constant
 584 value $\frac{1}{\sqrt{3}}\left(2 + \frac{1}{n}\right)$ for Herschel-Bulkley shear-thinning fluids ($0 < n < 1$) flowing at very high

585 values of u , i.e. when $u \gg \frac{\tau_0 h^n}{2^n 3^{n/2} \alpha^n}$. Given that the shift parameter is known to be greater than
586 unity [Chauveteau, 1982; Sorbie *et al.*, 1989; López, 2003; Comba *et al.*, 2011], the
587 preceding condition will be respected if $u \gg u^* = \frac{\tau_0 h^n}{2^n 3^{n/2}}$. Consequently, the boundary
588 condition $\alpha (u = 10^5 u^*) = \frac{1}{\sqrt{3}} \left(2 + \frac{1}{n}\right)$ was used to numerically solve Eq. (30). The resulting α
589 versus u functions are presented in Figure 2(b) and 1(d).

590

591 Analogously, Eq. (29) was numerically solved within the range of u used by Rodríguez de
592 Castro and Radilla [2016a] for both fractures. From (22), it can be deduced that α becomes
593 the constant value $\sqrt{3}$ for Carreau shear-thinning fluids ($0 < n < 1$) flowing at very high

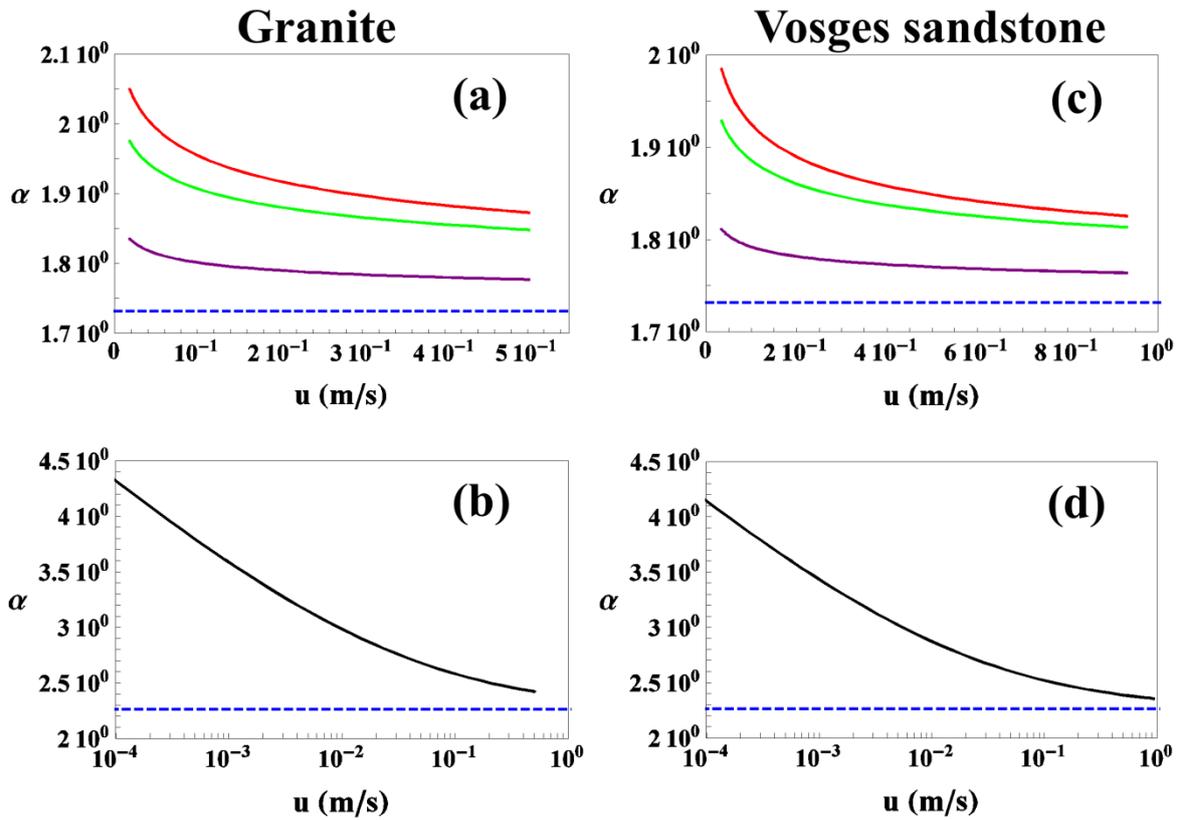
594 values of u , i.e. when $u \gg \left[\frac{2^n \alpha^{n-1} h^{1-n}}{6\mu_\infty} \text{Max} \left(3^{\frac{n-1}{2}}, 3^{\frac{n+1}{2}} c \right) \right]^{\frac{1}{1-n}}$. Since the shift parameter is
595 known to be greater than unity, the preceding condition will be respected if $u \gg$

596 $u^* = \left[\frac{2^n h^{1-n}}{6\mu_\infty} \text{Max} \left(3^{\frac{n-1}{2}}, 3^{\frac{n+1}{2}} c \right) \right]^{\frac{1}{1-n}}$. Therefore, the boundary condition $\alpha (u = 10^5 u^*) = \sqrt{3}$

597 was used to numerically solve Eq. (29). The resulting α versus u functions are presented in
598 Figure 2(a) and 1(c).

599

600 It can be noted that the relation between α and u strongly depends on polymer concentration
601 as shown in Figure 2. Indeed, the dependence of α on u is weaker for the low polymer
602 concentration as expected from their less pronounced shear-thinning behaviour. It is also
603 remarked that this dependence of α on u is less significant as u increases and α approaches
604 the limit value $\lim_{u \rightarrow \infty} \alpha(u)$. This implies that assuming a constant value of α should lead to
605 acceptable levels of accuracy in the prediction of the ∇P - u relations within the high- u region.



607

608 **Figure 2.** $\alpha(u)$ functions as numerically obtained from Eqs. (29) and (30). (a,c) correspond to
 609 the Carreau fluids used by *Rodríguez de Castro and Radilla* [2016a]. (b,d) correspond to the
 610 7000 ppm solution used in the present experiments. Solid lines represent the computed $\alpha(u)$
 611 functions and dashed lines represent $\lim_{u \rightarrow \infty} \alpha(u)$. Purple lines correspond to the 200 ppm
 612 Carreau fluid, green lines to the 500 ppm Carreau fluid, red lines to the 700 ppm Carreau
 613 fluid and black lines to the 7000 ppm yield stress fluid.

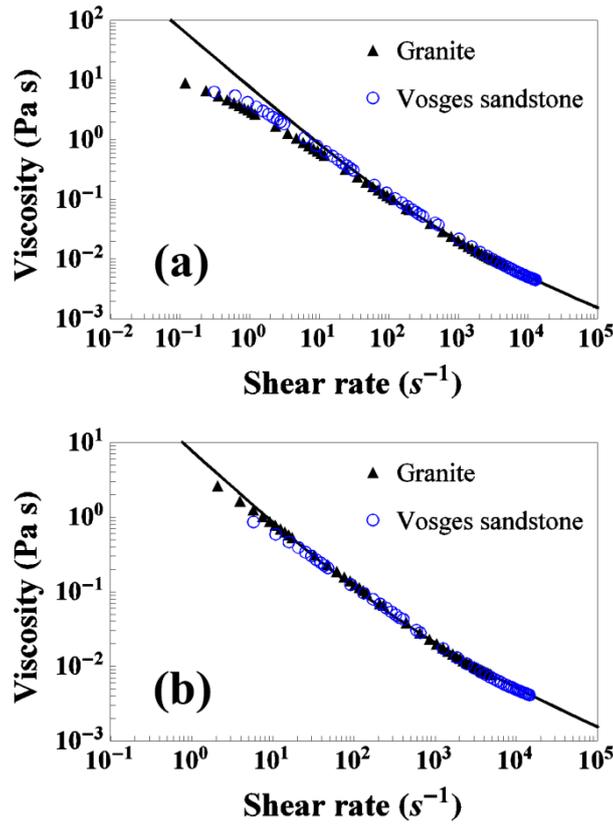
614

615 $\mu_{pm,ysf}$ was computed for the flow of the 7000 ppm solution through each fracture using Eq.
 616 (13). Two different approaches were followed: 1) the constant value $\alpha = \frac{1}{\sqrt{3}} \left(2 + \frac{1}{n} \right)$ was used
 617 in Eq. (13) and 2) the $\alpha(u)$ function obtained as explained above was used in Eq. (13). The
 618 results of both approaches are presented in Figure 3, together with μ_{eq} as obtained with Eq.

619 (11) from the ∇P - u measurements. In this figure, it can be observed that μ_{eq} is close to
620 $\mu_{pm,ysf}$ at high values u for both the constant α and the variable- α methods. However, this is
621 not the case at low and moderate values of u for which μ_{eq} approaches clearly better $\mu_{pm,ysf}$
622 with the variable- α method. It should be highlighted that xanthan gum may induce a depleted
623 layer close to pore walls with a lesser concentration in that region. This produces an apparent
624 wall slip which leads to a reduced average viscosity in the pores, mainly at low values of u
625 [*Chauveteau*, 1982; *Sorbie*, 1991b]. However, in the case of the present fractures, the
626 dimensions of the macromolecules is negligible with respect to the fracture apertures so this
627 effect is not observed and μ_{eq} is very close to $\mu_{pm,ysf}$ even at low values of u . This shows that
628 that the effect of fluid-solid interactions (e.g. polymer mechanical degradation and apparent
629 wall slip) on the relationship between viscosity and shear rate is negligible [*González et al.*,
630 2005; *Amundarain et al.*, 2009; *Rodríguez de Castro et al.*, 2016b]. Also, it is expected that
631 $\mu_{eq} > \mu_{pm,ysf}$ at high values of u in the presence of important inertial effects [*Tosco et al.*,
632 2013; *Rodríguez de Castro and Radilla*, 2016a]. The fact that no important deviation of μ_{eq}
633 with respect to $\mu_{pm,ysf}$ is observed in the present experiments reflects that inertial effects are
634 not significant. Moreover, Figure 3 shows that the shear rates involved in the flow through
635 the Vosges sandstone fracture are higher than those involved in the flow through the granite
636 fracture. This is coherent with the highest values of u and the lowest permeability of the
637 Vosges sandstone fracture.

638

639



640

641 **Figure 3.** μ_{eq} and $\mu_{pm,ysf}$ for the yield stress fluids used in the present experiments. Symbols
 642 represent μ_{eq} and solid lines represent μ_{pm} . (a) Corresponds to $\alpha = \frac{1}{\sqrt{3}} \left(2 + \frac{1}{n} \right)$. (b)
 643 Correspond to the $\alpha(u)$ functions presented in Figure 2.

644

645 It should be noted that the two-parameter power law model used in most of the preceding
 646 works dealing with shear-thinning fluids [Chhabra and Srinivas, 1991; Rao and Chhabra,
 647 1993; Sabiri and Comiti, 1994; Smit and du Plessis, 1997; Tiu et al. 1997; Machac et al.,
 648 1998; Chhabra et al., 2001; Broniarz-Press et al., 2007] is not appropriate to study non-
 649 Darcian flow as the involved shear rates are high and close to the upper Newtonian plateau of
 650 viscosity [Woudberg et al., 2006; Fayed et al., 2016], which is not taken into account by this
 651 model. In contrast, the empirical Carreau model [Carreau, 1972] can accurately predict the
 652 variation in the viscosity at all shear rates and is known to successfully represent the shear-

653 thinning behaviour of xanthan gum semi-dilute solutions [Sorbie et al., 1989; López et al.,
654 2003; Rodríguez de Castro et al., 2016b; *Rodríguez de Castro and Radilla*, 2016a]. Although
655 Herschel-Bulkley model does not include an upper Newtonian plateau viscosity, there is less
656 concern in the case of this type of fluids. Indeed, as can be observed in figures 3 and S₃, the
657 high levels of viscosity presented by the concentrated solutions injected in the present
658 experiments are far from the upper plateau in all cases. This is in contrast with the results of
659 *Rodríguez de Castro and Radilla* [2016a] for less concentrated xanthan gum solutions.

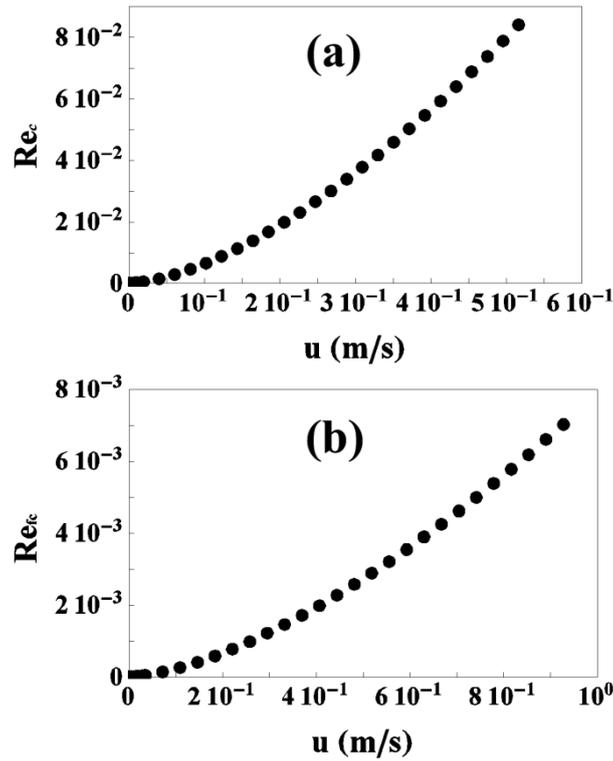
660

661 **4.3. Effects of yield stress on Reynolds number**

662

663 In previous works, it was shown that Reynolds number is not directly proportional to u for
664 shear-thinning fluids, in contrast to the Newtonian case [*Rodríguez de Castro and Radilla*,
665 2016a, 2016b]. Indeed, according to Eqs. (13) and (14), an increase in u implies a decrease in
666 viscosity which implies in turn an extra increase in Reynolds number. In this work, the effect
667 of yield stress on the Re - u relationship was also analysed. To do so, the Reynolds numbers
668 obtained for the imposed values of u were calculated through Eq. (4) in the case of the granite
669 fracture and Eq. (6) in the case of the Vosges sandstone fracture. μ_{pm} was used for the
670 calculation of Reynolds number. It is highlighted that μ_{pm} accounts only for viscous effects
671 and is consistent with the definition of Reynolds number as the ratio of inertial to viscous
672 forces, in contrast to μ_{eq} that accounts also for inertial effects. The results are presented in
673 Figure 4. In this figure, it can be observed that Reynolds number is close to zero at low values
674 of u for the flow of the yield stress fluid in both fractures.

675



676

677 **Figure 4.** (a) Re_c vs. u for the granite fracture (b) Re_{fc} vs. u for the Vosges sandstone fracture.

678

679 From Figure 4, one can also deduce that the non-linear dependence of Re on u previously
 680 reported for Carreau fluids is also observed for shear-thinning yield stress fluids.

681 Furthermore, as reflected in the same figure, there is a threshold value in terms of u below
 682 which Re is very close to zero for the injection of yield stress fluids. This threshold value

683 arises from the yield stress of the fluid. In fact, for a yield stress fluid, viscosity approaches
 684 infinity at low shear rates leading to very low values of Re . Also, the critical value of Re for

685 the transition to non-Darcian regime was reported to be close to $Re_c = 0.3$ for the granite
 686 fracture and $Re_{fc} = 0.05$ for the Vosges sandstone fracture [Rodríguez de Castro and Radilla,

687 2016a]. As can be seen in Figure 4, the Re obtained for the present experiments are lower
 688 than these critical values in both fractures, so no important inertial effects are expected. The

689 ratio between inertial and viscous pressure losses was calculated from Eq. (33) as $\frac{\Delta P_{inertial}}{\Delta P_{viscous}} =$

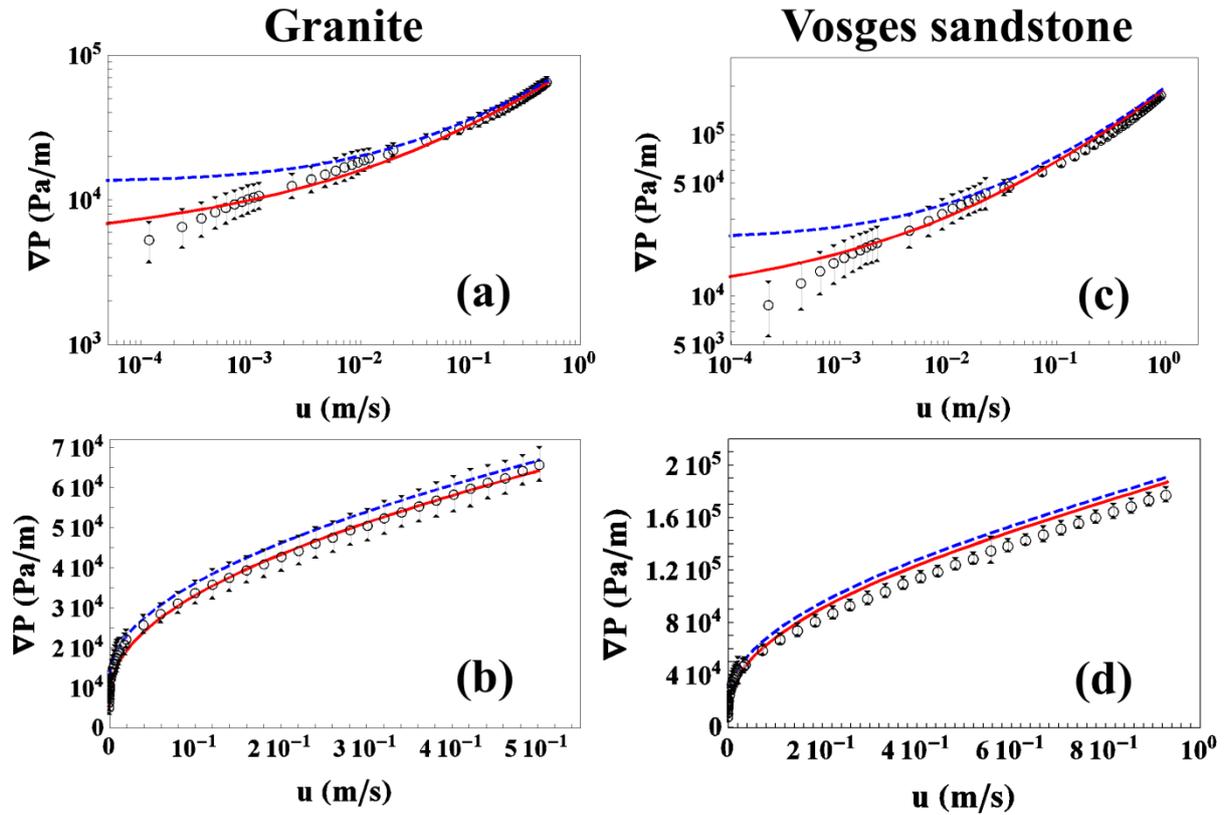
690 $\frac{\beta \rho u + \frac{\gamma \rho^2}{\mu_{pm}} u^2}{\frac{12 \mu_{pm}}{h^2}}$ leading to values of $\frac{\Delta P_{inertial}}{\Delta P_{viscous}} < 7.1 \times 10^{-3}$ for the granite fracture and $\frac{\Delta P_{inertial}}{\Delta P_{viscous}} < 3.1$
691 $\times 10^{-2}$ for the Vosges sandstone fracture. This confirms that inertial pressure losses are not
692 relatively important, in contrast to the experiments with Carreau fluids performed by
693 [Rodríguez de Castro and Radilla, 2016a].

694

695 **4.4. Experimental validation of the proposed prediction methods**

696

697 Eq. (33) was used to predict the relation between ∇P and u for the injection of the 7000 ppm
698 solution through the fractures. The β and d values in Eqs. (2-5) do not depend on polymer
699 concentration as shown by Rodríguez de Castro and Radilla [2016a, 2016b], so the values
700 obtained from water injection (subsection 4.1) were used. The obtained predictions are
701 presented in Figure 5 together with the experimental results of measurements performed in
702 the present work. In this figure, the errors bars correspond to a 95% confidence interval as
703 explained in subsection 3.1. The results are presented in a log-log scale in order to allow
704 visibility of all the range of measurements and in a linear scale so as to show that the form of
705 the curves is the same as that of the rheogram of a yield stress fluid (Figure S3). From these
706 results, the accuracy of the proposed methods for the prediction of ∇P as a function of u
707 during the flow of yield stress fluids through rough-walled fractures can be assessed. Figure 5
708 shows that the variable- α approach provides more accurate predictions within the low and
709 moderate u regions, which is in agreement with the arguments presented above. However, a
710 less important difference is obtained between both methods for the highest values of u . It is
711 observed that the variable- α method successfully predicts the ∇P - u relationship for the flow
712 of the yield stress fluid through both fractures, even though the obtained predictions are
713 slightly less accurate in the case of the Vosges sandstone.



715

716 **Figure 5.** ∇P as a function of u corresponding to (a,b) Granite and (c,d) Vosges sandstone
 717 fractures. Symbols represent experimental data, red solid lines represent predictions using Eq.
 718 (33) with the $\alpha(u)$ functions presented in Figure 2 and blue dashed lines represent predictions
 719 using Eq. (33) with $\alpha = \frac{1}{\sqrt{3}} \left(2 + \frac{1}{n} \right)$.

720

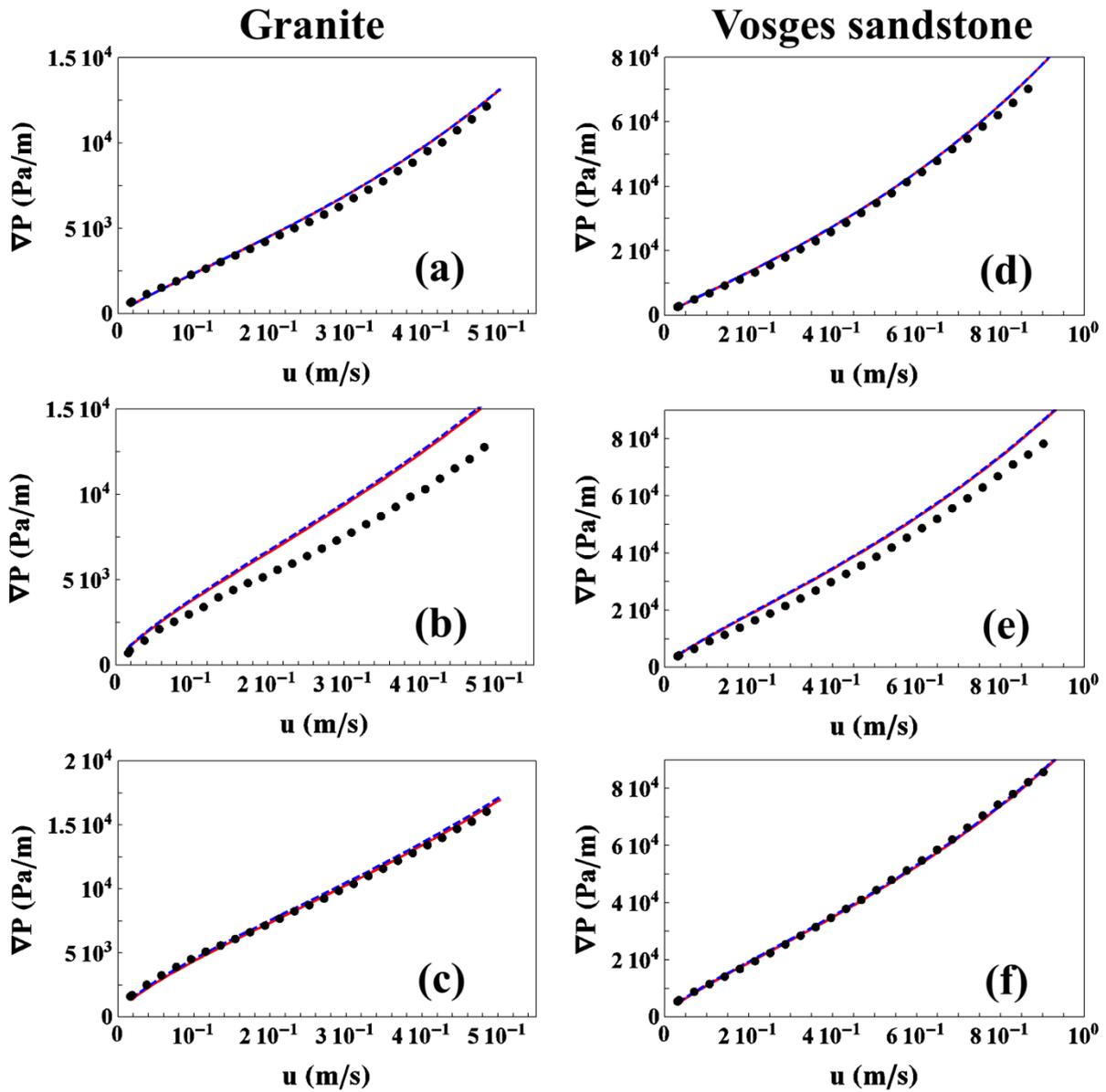
721 With the objective of assessing the accuracy of the proposed predictions in the case of
 722 Carreau fluids, Eq. (33) was also used to predict the u - ∇P relations for the injection of the
 723 three Carreau fluids used by *Rodríguez de Castro and Radilla* [2016a], and the results were
 724 compared to their experimental data in Figure 6. The average deviations between predictions
 725 and experiments corresponding to the whole range of explored u for all tested fluids and
 726 fractures are included as supporting information of this article (Table S1). As can be seen in

727 Figure 6, the predictions coming from both methods (constant- α and variable- α) methods are
728 almost identical in all cases. This is explained by the proximity of all the α values to the limit
729 value $\sqrt{3}$ (Figure 2) within the range of imposed u . Furthermore, the low and moderate u
730 regions were not explored by [Rodríguez de Castro and Radilla, 2016a], while it is precisely
731 in these regions where more important differences are expected between both approaches.
732 However, the predictions obtained for the covered range of u is in very good agreement with
733 the experimental data, apart from the 500 ppm – granite pair which will need further study.
734 Moreover, it is also remarked that Eq. (33) successfully takes into account the inertial effects,
735 which are important for the flow of the injected Carreau fluids at the involved values of u .

736

737

738



739

740 **Figure 6.** ∇P as a function of u corresponding the injection of the three Carreau fluids used
 741 by *Rodríguez de Castro and Radilla* [2016a] through (a,b,c) Granite and (d,e,f) Vosges
 742 sandstone fractures. Symbols represent experimental data, red solid lines represent
 743 predictions using Eq. (33) with the $\alpha(u)$ functions presented in Figure 2 and blue dashed lines
 744 represent predictions using Eq. (33) with $\alpha = \sqrt{3}$.

745

746

747 **5. Discussion**

748

749 As explained in subsection 3.2., the concentrated xanthan gum solutions used in our
750 experiments present an apparent yield stress, so they should be referred to as pseudo-yield
751 stress fluids. In this regard, *Lipscomb and Denn* [1984] showed that the classical lubrication
752 approximation, which essentially assumes that flow is locally fully-developed, can be applied
753 to this type of fluids, while it cannot be successfully applied to ideal yield stress fluids with a
754 real yield stress. These authors argued that rigid plug regions should not exist in complex
755 geometries according to classical lubrication. Indeed, this theory predicts that plug-velocity
756 changes slowly as aperture varies, so the plug region cannot be truly unyielded for continuity
757 reasons. The latter is known as lubrication paradox [*Lipscomb and Denn*, 1984; *Frigaard and*
758 *Ryan*, 2004; *Lavrov*, 2013]. However, as showed by *Lipscomb and Denn* [1984], both real
759 and pseudo-yield stress fluids may exhibit a near-plug-like region in a complex flow field.
760 For fully developed flows, the depth of the plug as a function of ∇P is given by [*Lipscomb*
761 *and Denn*, 1984]:

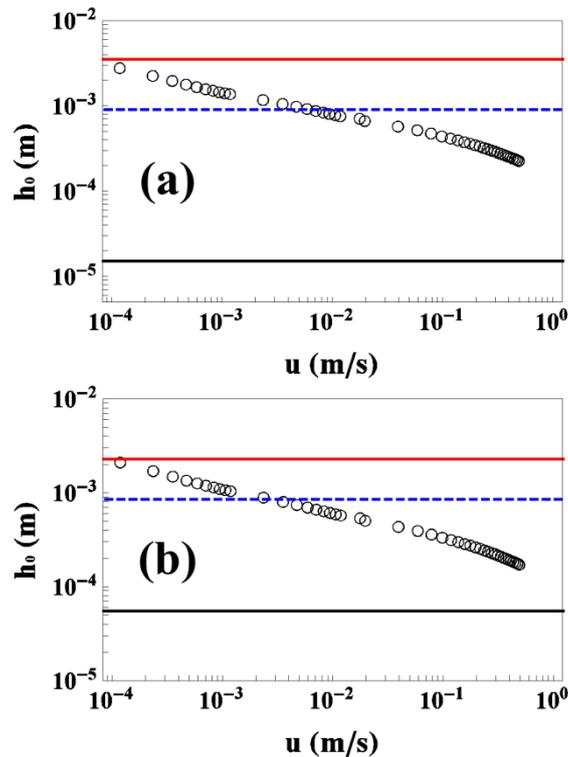
762

$$h_0 = \frac{2\tau_0}{\nabla P} \tag{34}$$

763 As a first simple approach similar to the one presented by *Auradou* [2008], we will assume
764 that the fracture space can be modelled as being a bundle of parallel rectangular canals of
765 length L , width w_i and aperture h_i , with $w_i \gg h_i$. h_i is expected to vary along the flow paths.
766 However, given the strong dependence of ∇P on the canal aperture in the case of shear-
767 thinning fluids, we will also assume that the pressure drop along a percolating path is located

768 exclusively in the section of smallest aperture h_0 . By doing so, it can be deduced that the
 769 fluid will flow through a percolating path only if the minimum local aperture is superior to h_0
 770 as given by Eq. (34). Therefore, the number of percolating pathways is expected to increase
 771 as ∇P and u increase. h_0 has been presented as a function of u for the present flow
 772 experiments with yield stress fluids in Figure 7. This figure shows that h_0 decreases with u ,
 773 as expected. Indeed, only the pathways with the highest apertures participate in the flow at
 774 the lowest values of u , while pathways with smaller values of h_0 are progressively
 775 incorporated as u increases. According to this simple approach, the flow pathways including
 776 the minimum apertures of the fracture would not participate in the flow, even at the highest u ,
 777 so unyielded fluid regions would exist (in agreement with *Frigaard and Ryan [2004]*).

778



779

780 **Figure 7.** h_0 as a function of u for the flow of the yield stress fluid through the (a) granite
 781 fracture and the (b) Vosges sandstone fracture. The experimental data are represented as void

782 symbols. Red solid lines represent the maximum fracture apertures, black solid lines
 783 represent the minimum fracture apertures and blue dashed lines represent the average fracture
 784 apertures as measured by *Nowamooz et al.* [2013] (see complementary figures).

785

786 If we focus on the Darcian flow of a yield stress fluid ($\mu_{pm} = \mu_{pm,ysf}$), Eq. (31) gives the
 787 pressure gradient through a rough-walled fracture of hydraulic aperture h as a function of the
 788 Herschel-Bulkley law parameters: τ_0 , a and n . This equation can be re-written as:

789

$$\nabla P = \frac{C_1}{\alpha} + C_2 \alpha^{n-1} u^n \quad (35)$$

790

791 with $C_1 = \frac{2\sqrt{3}\tau_0}{h}$ and $C_2 = \frac{2^{n+1}3^{\frac{n+1}{2}}a}{h^{n+1}}$.

792

793 In the high flow rates region, i.e. when $u \gg \frac{\tau_0 h^n}{2^n 3^{n/2}}$, α can be considered a constant value $\alpha =$

794 $\frac{1}{\sqrt{3}} \left(2 + \frac{1}{n} \right)$ and Eq. (31) leads to:

795

$$\nabla P = \nabla P_0 + C u^n \quad (36)$$

796

797 with $\nabla P_0 = \frac{6n\tau_0}{h(2n+1)}$ and $C = \frac{6an(\frac{2+4n}{hn})^n}{h+2hn}$. This is in agreement with the results of *Talon et al.*
798 [2014], who stated that u scales linearly as $(\nabla P - \nabla P_0)$ in the case of a Bingham fluid ($n = 1$)
799 flowing at high u through a one-dimensional channel. Also, *Nash and Rees* [2017] showed
800 that the manner in which flow begins once the threshold pressure gradient is exceeded
801 strongly depends on the channel size distribution of the porous media. The same authors
802 [*Talon et al.*, 2014; *Nash and Rees*, 2017] proved that ∇P_0 is higher than the actual threshold
803 pressure, which is consistent with our results given that α increases as u tends to zero (Figure
804 2). *Roustaei et al.* [2016] numerically showed that unyielded plug regions appear close to the
805 fracture wall and in the deeper layers (fouling layers) when injecting yield stress fluids in
806 short fractures, especially at low values of u . These researchers showed that Darcy-type flow
807 laws are limited to $H/L \ll 1$, H being a half of the difference between the maximum and the
808 minimum aperture of the fractures. In the case of the granite sandstone used in the present
809 work $H/L = 6.2 \times 10^{-3}$ while $H/L = 5.5 \times 10^{-3}$ for the granite sandstone as shown in supporting
810 figures, so a Darcy-type approach is expected to be valid.

811

812 Lavrov [2015] developed analytical solutions for the flow of truncated power law fluids
813 through smooth-walled fractures. Truncated power-law fluids, unlike Carreau fluids, enable a
814 closed-form solution for the flow between plane parallel walls while exhibiting more realistic
815 behaviour than simple power-law fluids for commonly used polymer solutions. However,
816 truncated power-law fails to model the real behaviour of these complex fluids at shear rates
817 lying within the transition region between the shear-thinning region and the upper Newtonian
818 plateau. Therefore, this model is not expected to provide accurate predictions in the wide
819 range of shear-rates explored in the present experiments.

820

821 One may wonder whether the proposed procedure is simpler than performing a numerical
822 solution to the actual flow equations, without invoking a bundle-of-capillaries approximation.
823 In this sense, it should be highlighted that performing a numerical solution to the actual flow
824 equations would imply using the size distribution of the flow paths as an input for the model.
825 This information on the size distribution of the flow paths is rarely available in real
826 applications, while the average aperture of the fracture can be more easily estimated or
827 measured from water flow experiments. It is reminded that the objective of this work is to
828 present a simple method to predict the pressure drop for the flow of shear-thinning fluids
829 through tough-walled rock fractures. Therefore, using hardly accessible inputs as needed to
830 perform a numerical solution to the actual flow equations is not a valid approach.

831

832 Also, it is noted that in our experiments with yield stress fluids, the total pressure drop
833 through the fractures was successfully predicted from the values of K , γ and β obtained from
834 water injection without any significant deviation. Therefore, elongational viscosity effects
835 have been shown to be negligible in the case of the present experiments with yield stress
836 fluids as they were with the Carreau fluids used by [Rodríguez de Castro and Radilla,
837 2016a].

838 6. Summary and conclusions

839

840 A simple method to extend Darcy's law, weak inertia cubic law and full cubic law to the flow
841 of yield stress fluids and Carreau fluids in rough-walled natural fractures has been presented
842 in the present work. In this method, the values of the shift parameter α between the μ_{pm}
843 measured in the rheometer and the μ_{eq} observed during the flow in the porous media is
844 predicted through identification of the apparent shear rate with the maximum wall shear rate
845 in a section with aperture h . The inputs of the method are only the shear rheology parameters
846 of the fluid, the hydraulic aperture of the fracture and the inertial coefficients γ and β . On the
847 basis of our results, an efficient protocol to predict ∇P as a function of u is proposed here:

848 1) Determine the shear-rheology parameters of the fluid: (τ_0, a, n) for Herschel-Bulkley
849 fluids or (μ_∞, c, n) for Carreau fluids.

850 2) Measure h , β and γ from Newtonian-flow experiments. Alternatively, h can be deduced
851 from the aperture distribution [Zimmerman *et al.*, 1991], which can be obtained
852 through image analysis [Nowamooz *et al.*, 2013].

853 3) Calculate the values of $\alpha(u)$

854 3.1) When low and moderate values of u are involved, solve the differential
855 equation (29) or (30) to obtain $\alpha(u)$.

856 3.2) When only high values of u are involved ($u \gg \frac{\tau_0 h^n}{2^n 3^{n/2}}$ for yield stress

857 fluids or $u \gg \left[\frac{2^n h^{1-n}}{6\mu_\infty} \text{Max} \left(3^{\frac{n-1}{2}}, 3^{\frac{n+1}{2}} c \right) \right]^{\frac{1}{1-n}}$ for Carreau fluids), use a constant

858 value $\alpha = \frac{1}{\sqrt{3}} \left(2 + \frac{1}{n} \right)$ for Hershel-Bulkley fluids or $\alpha = \sqrt{3}$ for Carreau fluids.

859 4) Use Eq. (13) or (14) to calculate $\mu_{pm,Carreau}$ Or $\mu_{pm,ysf}$

860 5) Use Eq. (33) to calculate ∇P as a function of u , with $\gamma = 0$ in the case of
861 Forchheimer's law (strong inertia regime), $\beta = 0$ in the case of a cubic law (weak
862 inertia regime), $\beta = 0$ and $\gamma = 0$ in the case of Darcy's law (creeping flow).

863

864 Flow experiments of yield stress fluids covering a wide range of u (~ 3.6 orders of
865 magnitude) have been performed and compared with the predictions of the proposed method,
866 showing good agreement. It has been observed that the existence of a yield stress reduces
867 significantly the value of Reynolds, so the inertial effects are negligible within the explored
868 range of u . Consequently, Darcy's law provide accurate u - ∇P predictions in contrast to the
869 case of less concentrated solutions with no yield stress [Rodríguez de Castro and Radilla,
870 2016a]. Also, the experimental results obtained in the non-Darcian shear-thinning flow
871 experiments through rough-walled fractures conducted by [Rodríguez de Castro and Radilla,
872 2016a] have been compared with the predictions of the proposed method, showing good
873 agreement also in the case of Carreau fluids. It should be noted that good predictions of the
874 pressure drop-flow rate relations are obtained by only using the global parameters h , β and γ
875 as inputs. Therefore, no significant effects of the aperture distributions of the fractures have
876 been observed.

877

878 The variable- α approach leads to a very good overlap between μ_{pm} and the μ_{eq} over the wide
879 range of u investigated in this work. Our results can be included in computational studies of
880 large-scale nonlinear flow in fractured rocks, as suggested in the works of *Javadi et al.*
881 [2014]. These conclusions must now be extended to other types of rough-walled rock
882 fractures.

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884

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887

888 Supporting data are included as four figures and a table in SI files; any additional data may be
889 obtained from the authors (antonio.rodriquezdecastro@ensam.eu).

890

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