Possibility theory and PROMETHEE II for decision aid in engineering design process

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Abstract:
In many design problems, the designer must select the design solution from among several alternative design solutions. In situations where relevant information to evaluate the design solutions are not available, the evaluation is imprecise and uncertain. Therefore, in order to make the right decision, it is necessary to take into account imprecision and uncertainty in the decision process. In this context, this article presents a multi-criteria decision support approach for the complete ranking of alternative design solutions. The proposed approach allows to take into account the imprecision and uncertainty related to the evaluation of the decision criteria. It is based on the possibility theory and PROMETHEE II ranking method. It is validated using an application dealing with the design of a technical bid solution (a couple of technical system and delivery process) during a bidding process.

Keywords: Multi-Criteria Decision Making (MCDM), Possibilistic dominance degree, PROMETHEE II, Uncertainty and imprecision, Technical bid elaboration.

1. INTRODUCTION

In many engineering design problems, several design solutions are relevant to the requirements Renzi et al. (2017). In such a situation, one of the most critical phase in the design process is to select the most interesting design solution from a panel of several potential solutions El Amine et al. (2016). The selection, most of the time, involves several conflicting criteria such as: cost, duration and technical performances. Moreover, the criteria may have different importance to the decision maker. All of these aspects of the design problems must be taken into account in the decision process Renzi et al. (2017).

In addition, in many situations, relevant information to evaluate the design solutions are not available. Due to this lack of information, the values of the decision criteria which characterize the design solutions are imprecise and uncertain (epistemic uncertainty and epistemic imprecision) Chapman et al. (2000). These imprecision and uncertainty are more important in the early phase of the design process where the design solutions are only partially designed El Amine et al. (2016). Therefore, in order to make the right decision when selecting the most interesting design solution, the uncertainty and imprecision related to the values of the decision criteria must be taken into account in the decision process Durbach and Stewart (2012). In this article, we consider the problem of the selection of a design solution in engineering design process where the evaluation of the design solutions is imprecise and uncertain.

In the scientific literature, many multi-criteria decision support approach which take into account uncertainty and imprecision, have been proposed (see the review in Durbach and Stewart (2012)). In many of these research work, in order to take uncertainty and imprecision into account in the decision process, a standard multi-criteria decision analysis method (e.g. PROMETHEE, AHP, ELECTRE, etc.) is associated with an uncertainty theory (probability theory, fuzzy set theory, possibility theory, etc.) Durbach and Stewart (2012). As a multi-criteria decision analysis approach, PROMETHEE II method is well-known to be effective and easy to implement Behzadian et al. (2010). Moreover, possibility theory is relevant to simultaneously deal with uncertainty and imprecision Dubois and Prade (1983). Therefore, in this article, we propose an approach based on possibility theory and PROMETHEE II method for the complete ranking of alternative design solutions in engineering design process.

In our previous work Sylla et al. (2017b), four possibilistic mono-criterion dominance relations have been proposed in order to compare two solutions with respect to a single decision criterion, and, to compute the possibilistic dominance relation between them. In the proposed approach, these four possibilistic dominance relations are used to compute a possibilistic dominance degree of a solution S_i over another one S_j. Then, based on this, PROMETHEE II ranking method is adapted in order : (i) to compare the potential solutions following all the decision criteria, and (ii) to compute a complete ranking of all the solu-

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tions while taking into account the degree of certainty or possibility of dominance.

The remainder of the article is structured as follows. The section 2, 3 and 4 present adequate background about PROMETHEE II method, the modeling of the imprecise and uncertain values of the decision criteria, and the four possibilistic mono-criterion dominance relations, respectively. In section 5, the proposed approach is detailed. In section 6, an illustrative application, dealing with the design of a technical bid solution (a couple of technical system and delivery process), is presented and discussed in order to show the validity and effectiveness of the proposed approach. Finally, section 7 presents conclusion and future research.

2. PROMETHEE II RANKING METHOD

The PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) methods belong to the family of outranking methods Behzadian et al. (2010). The first versions PROMETHEE I (for partial ranking) and PROMETHEE II (for complete ranking) have been developed by Brans et al. (1986). Since, several versions of PROMETHEE methods have been proposed, especially : PROMETHEE III (for ranking based on interval) and PROMETHEE IV (for ranking when the set of solutions is continuous) Behzadian et al. (2010). They are well-known to be simple and easy to implement compared to other MCDM methods Behzadian et al. (2010). In this article, PROMETHEE II ranking method is used in order to perform a complete ranking of design solutions in engineering design process. As the other outranking methods, PROMETHEE II is based on the pairwise comparison of potential solutions with respect to each decision criterion. Its main steps are presented in the following.

Given: a finite set S = {S_1, S_2, ..., S_m} (m $\geq$ 2) of m possible solutions; a finite set C = {C_1, C_2, ..., C_n} (n $\geq$ 2) of n decision criteria. Each decision criterion C_k is characterized with a weight w_k which corresponds to its relative importance with regard to the other decision criteria. Each solution S_i is evaluated following every decision criterion. The evaluation of a solution S_i following a decision criterion C_k is noted S_i^k.

Step 1: For each pair of potential solutions (S_i,S_j), the preference degree of the solution S_i over the solution S_j with respect to each decision criterion C_k, noted $P_k(S_i,S_j)$, is computed. A preference degree $P_k(S_i,S_j)$ is a score between [0,1] which expresses how a solution S_i is preferred to another one S_j with respect to the criterion C_k. Its computation is based on the evaluation S_i^k and S_j^k of the solutions S_i and S_j, and, it is done using a preference function $F_k(S_i,S_j)$.

$$P_k(S_i, S_j) = F_k(S_i^k, S_j^k) \tag{1}$$

Step 2: For each pair of potential solutions (S_i,S_j), the overall preference degree of the solution S_i over the solution S_j with respect to all the decision criterion, noted $\pi(S_i, S_j)$, is computed. $\pi(S_i, S_j)$ is calculated as the weighted sum of the preference degree $P_k(S_j,S_i)$. The weight of each decision criterion, noted w_k, is supposed known. It represents the relative importance of the decision criterion C_k with regard to the other criteria.

$$\pi(S_i, S_j) = \sum_{k=1}^{n} (P_k(S_i, S_j) * w_k) \tag{2}$$

Step 3: Based on the overall dominance degree, the positive, negative and net outranking flows are computed for each potential solution S_i. The positive outranking flow of S_i, noted $\phi^+(S_i)$, indicates how a solution S_i is preferred over all the other solutions whereas the negative outranking flow of S_i, noted $\phi^-(S_i)$, indicates how the other solutions are preferred over the solution S_i. The net outranking flow of S_i, noted $\phi(S_i)$, takes into account both the positive and negative outranking flows. Is is calculated as the difference between the positive outranking flow and the negative flow. The net outranking flow is used to compute a complete ranking of the potential solutions. The solutions are ranked from the best to the worst. The more the net outranking flow of a solution S_i is higher, the more it is preferred to the other solutions.

$$\phi^+(S_i) = \frac{1}{m-1} \sum_{S_j \in S / S_i \succ S_j} (\pi(S_i, S_j)) \tag{3}$$

$$\phi^-(S_i) = \frac{1}{m-1} \sum_{S_j \in S / S_j \succ S_i} (\pi(S_j, S_i)) \tag{4}$$

$$\phi(S_i) = \phi^+(S_i) - \phi^-(S_i) \tag{5}$$

In this article, the values of the decision criteria are imprecise and uncertain. They are modeled as possibility distributions (see section 3). Therefore, in section 5, the four possibilistic mono-criterion dominance relations, proposed in our previous work Sylla et al. (2017b) and recalled in section 4, are used to compare two potential solutions S_i and S_j with respect to a single decision criterion and to compute the possibilistic dominance degree of the solution S_i over the solution S_j. As the preference degree in standard PROMETHEE II method, the possibilistic dominance degree expresses how the solution S_i is preferred to the solution S_j. Thus, the approach proposed in this article is based on possibilistic dominance degree instead of preference degree in a standard PROMETHEE II approach. In the next section 3, the way the values of the decision criteria are modeled is presented.

3. MODELING OF THE EVALUATION OF THE DECISION CRITERIA AS POSSIBILITY DISTRIBUTIONS

In scientific work, many framework have been used to model the imprecise and uncertain values of the decision criteria Durbach and Stewart (2012). In this article, possibility theory is used to model the evaluation of a solution S_i with regard to a criterion C_k as a possibility distribution $\mu_{S_i^k}$. The way the values are modeled is shown in Fig. 1 and described in the following. The possible values of the decision criterion C_k are represented as interval of values. The interval [a,d] is the evaluation domain of the decision criterion. The interval [b,c] is the most possible

values. Their possibility is equal to 1. The possibility to have a value out of this interval is equal to e. The value of e is computed using the equation 6, where $\rho_{[b,c]}$ is the confidence in (or certitude of) the interval [b,c]. This confidence may be computed using a well-defined method as proposed in Sylla et al. (2017a) or may be provided by an expert with sufficient experience about the design solutions.

$$\mathrm{e} = 1 - \rho_{[b,c]} \tag{6}$$

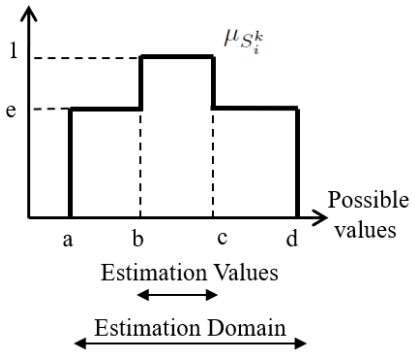

Fig. 1. Modeling of the values of a decision criterion

4. POSSIBILISTIC MONO-CRITERION DOMINANCE RELATIONS

In many real world decision making problems, the values of the decision criteria are imprecise and uncertain. Therefore, the comparison of the potential solutions leads to the comparison of imprecise and uncertain numbers Durbach and Stewart (2012). In Dubois and Prade (1983), four indexes have been proposed for the comparison of two imprecise and uncertain numbers modeled as possibility distributions. They are effectively able to indicate the possibility and necessity of a possibility distribution $\mu_{S_i^k}$ to be greater or smaller than another one $\mu_{S_j^k}$ Dubois and Prade (1983). They are presented in the following. For the seek of simplicity and clarity, the computation method of these four indexes are not detailed here in this article. For more details, please consult Dubois and Prade (1983).

Given two possibility distributions $\mu_{S_i^k}$ and $\mu_{S_j^k}$:

- Possibility Of Dominance (POD) : provides the possibility of the possibility distribution $\mu_{S_i^k}$ to be smaller than (or equal to) the possibility distribution $\mu_{S_j^k}$.
- Possibility of Strict Dominance (PSD) : provides the possibility of the possibility distribution $\mu_{S_i^k}$ to be smaller than the possibility distribution $\mu_{S_j^k}$.
- Necessity Of Dominance (NOD) : provides the necessity of the possibility distribution $\mu_{S_i^k}$ to be smaller than (or equal to) the possibility distribution $\mu_{S_j^k}$.
- Necessity of Strict Dominance (NSD) : provides the necessity of the possibility distribution $\mu_{S_i^k}$ to be smaller than the possibility distribution $\mu_{S_j^k}$.

Based on these four indexes (POD, PSD, NOD and NSD), in our previous work Sylla et al. (2017b), four possibilistic mono-criterion dominance relations have been proposed in order to compare two solutions S_i and S_j with respect to a single decision criterion, and to compute the possibilistic dominance relations of S_i over S_j. They are presented in the following.

Given: two potential solutions S_i and S_j; their evaluation on a criteria C_k, noted $\mu_{S_i^k}$ and $\mu_{S_j^k}$, respectively; and their dominance indexes represented in the two vectors $I_{S_i \succ S_j} = [POD\, PSD\, NOD\, NSD]$ for the solution S_i and $I_{S_j \succ S_i} = [POD\, PSD\, NOD\, NSD]$ for the solution S_j. The notation $S_j \succ S_i$ means that the solution S_i dominates (or is preferred to) the solution S_j.

Certain Dominance (CD): In this situation, the two possibility distributions $\mu_{S_i^k}$ and $\mu_{S_j^k}$ are completely disjoints. One possibility distribution $\mu_{S_i^k}$ is certainly smaller than the other one $\mu_{S_j^k}$. As a consequence, the solution S_i dominates the other one S_j with certainty. The value of the dominance index NSD of the vector $I_{S_i \succ S_j}$ is equal to 1 as shown in the equation 7.

$$I^k_{S_i \succ S_j}(4) = 1 \tag{7}$$

Strong Possibility of Dominance (SPD): In this situation, the two possibility distributions $\mu_{S_i^k}$ and $\mu_{S_j^k}$ overlap. However, the four dominance indexes (POD, PSD, NOD and NSD) indicate that one possibility distribution $\mu_{S_i^k}$ is generally smaller than the other one $\mu_{S_j^k}$. Therefore, it is not certain that the solution S_i dominates the solution S_j, however it is strongly possible. This relation is formalized in the equation 8.

$$[I^k_{S_i \succ S_j}(4) < 1] \wedge [\forall\, t \in \{1, ..., 4\}; D^k_{S_i \succ S_j}(t) > D^k_{S_j \succ S_i}(t)] \tag{8}$$

Weak Possibility of Dominance (WPD): In this situation, the two possibility distributions $\mu_{S_i^k}$ and $\mu_{S_j^k}$ overlap and all the four indexes are not consistent for the ranking of the two possibility distributions. However, most of them indicate that one possibility distribution $\mu_{S_i^k}$ is generally smaller than the other one $\mu_{S_i^k}$. As a consequence, it is not certain that the solution S_i dominates the solution S_j, however it is weakly possible. This relation is formalized in the equations 9, 10, 11 and 12.

$$\begin{aligned}[\exists\, t \in \{1, ..., 4\} : I^k_{S_i \succ S_j}(t) \leq I^k_{S_j \succ S_i}(t)] \\ \wedge [\forall\, l \neq t : I^k_{S_i \succ S_j}(l) > I^k_{S_j \succ S_i}(l)]\end{aligned} \tag{9}$$

$$\begin{aligned}[\forall\, t \in \{1, 4\} : I^k_{S_i \succ S_j}(t) = I^k_{S_j \succ S_i}(t)] \\ \wedge [\forall\, l \neq t : I^k_{S_i \succ S_j}(l) > I^k_{S_j \succ S_i}(l)]\end{aligned} \tag{10}$$

$$\begin{aligned}[\forall\, t \in \{1, 4\} : I^k_{S_i \succ S_j}(t) > DI^k_{S_j \succ S_i}(t)] \\ \wedge [\forall\, l \neq t : I^k_{S_i \succ S_j}(l) = I^k_{S_j \succ S_i}(l)]\end{aligned} \tag{11}$$

$$[\exists\, t \in \{1, ..., 4\} : I^k_{S_i \succ S_j}(t) > I^k_{S_j \succ S_i}(t)] \\ \wedge [\forall\, l \neq t : I^k_{S_i \succ S_j}(l) = I^k_{S_j \succ S_i}(l)] \tag{12}$$

INDifference (IND): In this situation, the two possibility distributions $\mu_{S_i^k}$ and $\mu_{S_j^k}$ strongly overlap and all the four indexes are not consistent for the ranking of the two possibility distributions. In addition, in the contrary to the previous situations, none of the two possibility distributions have more dominance indexes which indicate that it is generally smaller than the other one. Therefore, the two solutions S_i and S_j are indifferent. This relation is formalized in the equations 13, 14 and 15.

$$[\forall\, t \in \{1, ..., 4\} : I^k_{S_i \succ S_j}(t) = I^k_{S_j \succ S_i}(t)] \tag{13}$$

$$[\forall\, t \in \{1, 4\} : I^k_{S_i \succ S_j}(t) = I^k_{S_j \succ S_i}(t)] \\ \wedge\, [I^k_{S_i \succ S_j}(2) > I^k_{S_j \succ S_i}(2)] \\ \wedge [I^k_{S_i \succ S_j}(3) < I^k_{S_j \succ S_i}(3)] \tag{14}$$

$$[\forall\, t \in \{1, 4\} : I^k_{S_i \succ S_j}(t) = I^k_{S_j \succ S_i}(t)] \\ \wedge\, [I^k_{S_i \succ S_j}(2) < I^k_{S_j \succ S_i}(2)] \\ \wedge [I^k_{S_i \succ S_j}(3) > I^k_{S_j \succ S_i}(3)] \tag{15}$$

In the following section, these four mono-criterion dominance relations are used to compare two solutions S_i and S_j with respect to a single decision criterion C_k, and to compute the dominance degree of S_i over S_j.

5. PROMETHEE II RANKING METHOD BASED ON POSSIBILISTIC DOMINANCE DEGREE

The proposed approach follows the three main steps of PROMETHEE II ranking method presented in section 2. They are described in the following.

5.1 Computation of the possibilistic dominance degrees

The potential solutions are compared with respect to a single decision criterion. For each decision criterion C_k and for each pair of potential solutions (S_i,S_j), the possibilistic dominance degree of the solution S_i over the solution S_j, noted $\delta(S_i, S_j)$, is computed based on the equations presented in section 4. The computed dominance degree $\delta(S_i, S_j)$ takes into account the possibility and necessity of dominance of the solution S_i over the solution S_j, but also, the possibility and necessity of dominance of the solution S_j over the solution S_i. Therefore, in this article, we consider that the possibilistic dominance degree (*CD*, *SPD* and *WPD*) are unidirectional. Meaning that once the dominance of a solution S_i over the solution S_j is equal to *CD*, *SPD* or *WPD*, the dominance degree of S_j over S_i is not relevant. It is noted *NR*. The function used to compute the possibilistic dominance degrees is presented in Algorithm 1.

5.2 Computation of the overall dominance degrees

For each pair of solutions (S_i,S_j), the overall dominance degree of the solution S_i over the solution S_j with respect to all the decision criteria is computed using the equation 16.

Algorithm 1 $\delta_k(S_i, S_j)$

```
if (equation 7 is TRUE) then
    return CD          # Sj dominates Si with certainty
else if (equation 8 is TRUE) then
    return SPD  # Sj dominates Si, not certainty, but
    with a strong possibility
else if ((equation 9 is TRUE) ∨ (equation 10 is TRUE)
∨ (equation 111 is TRUE) ∨ (equation 112 is TRUE))
then
    return WPD  # Sj dominates Si, not certainty, but
    with a weak possibility
else if ((equation 13 is TRUE) ∨ (equation 14 is TRUE)
∨ (equation 15 is TRUE)) then
    return IND          # Si and Sj are indifferent to
    each other
else
    return NR                  # Si is dominated by Sj
end if
```

$$\pi'(S_i, S_j) = \sum_{C_k \in C/S_i \succ S_j} (\delta'_k(S_i, S_j) * w_k) \tag{16}$$

$\delta'_k(S_i, S_j)$ is the numerical score corresponding to the possibilistic dominance degree of S_i over S_j computed in section 5.1. The numerical values of the possibilistic dominance degrees are computed using the Table 1. For example, if the dominance degree of a solution S_i over another one S_j is *SPD*, then the numerical value $\delta'_k(S_i, S_j)$ is equal to *0,66*.

Table 1. Numerical values of the possibilistic dominance degrees

$\delta_k(S_i, S_j)$	$\delta'_k(S_i, S_j)$
IND	0
WPD	0.33
SPD	0.66
CD	1

5.3 Computation of the outranking flows and the complete ranking of the potential solutions

In order to compute the net outranking flow, the positive and negative outranking flow are first calculated using the equation 17 and 18.

$$\phi^+(S_i) = \frac{1}{m-1} \sum_{S_j \in S/S_i \succ S_j} (\pi'(S_i, S_j)) \tag{17}$$

$$\phi^-(S_i) = \frac{1}{m-1} \sum_{S_j \in S/S_j \succ S_{ji}} (\pi'(S_j, S_i)) \tag{18}$$

Then, the net outranking flow is computed as the difference of the positive outranking flow and the negative outranking flow (see equation 5 in section 2). The potential design solutions are ranked based upon their net outranking flow. The most interesting solution is the one that has the highest net outranking flow.

6. ILLUSTRATIVE APPLICATION

This application concerns the design of a technical bid solution (a couple of technical system/delivery process) during a bidding process. In the context of a bidding process, in order to propose an offer to a customer, a bidder must design several technical bid solutions that satisfy the customer's requirements. Then, based on several criteria, they must select the most interesting one to propose to the customer. In general, the time allowed by the customers to the suppliers to transmit their offers is short Kromker (1998). Therefore, the bidders do not have sufficient time to perform a detailed design of their offers (couples systems/delivery process). The technical bid solutions are only partially designed. As a consequence, relevant knowledge to evaluate them are less available. Due to this lack of knowledge the values of the decision criteria which characterize the technical bid solutions are imprecise and uncertain. Therefore, in order to make the right decision when selecting the most interesting technical bid solution, the uncertainty and imprecision related to the values of the decision criteria must be taken into account in the decision process. In this design problem, nine technical bid solutions have been designed using a configuration software. They are evaluated on two decision criteria : (i) the cost of the technical bid solutions, and (ii) the duration of the delivery processes. Based on these criteria, the bidder must select one technical bid solution from this panel of nine potential solutions. The detail about the decision problem is presented in Table 2.

Table 2. The nine technical bid solutions (S)

S	Cost(K$) Weight = 0.75					Duration(Weeks) Weight = 0.25				
	a	b	c	d	ρ	a	b	c	d	ρ
S1	64	66	73	84	0,7	63	65	71	73	0,7
S2	63	67	74	83	0,5	52	53	56	60	0,6
S3	67	80	82	84	0,8	57	60	63	65	0,7
S4	72	76	79	85	0,9	55	57	58	59	0,9
S5	87	88	89	90	0,7	64	66	67	70	0,6
S6	67	68	75	81	0,8	53	54	57	58	0,9
S7	86	87	90	92	0,5	68	70	72	75	0,65
S8	68	73	76	85	0,8	49	55	58	65	0,8
S9	69	78	80	82	0,7	60	61	64	65	0,8

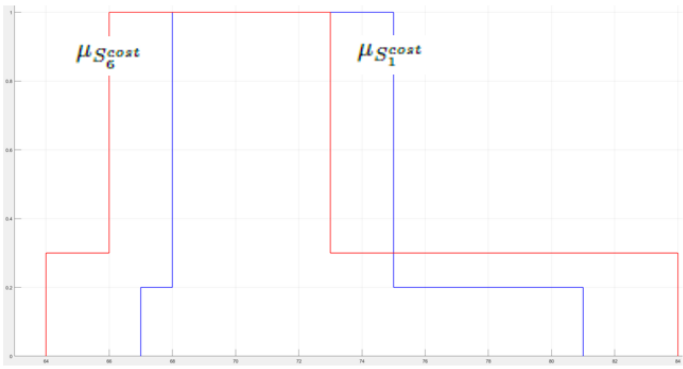

Fig. 2. Modeling of the values of a decision criterion

6.1 Examples of the modeling of the values of the decision criteria

In this section, the modeling of the evaluation of two technical bid solutions S_1 and S_2 with regard to the decision criterion *Cost* is presented. For each solution, the possible values of the cost are given in the Table 2. For S_1: the evaluation domain are $[a,d]_{cost} = [64,84]$ and the most possible values (or the estimation values) are $[b,c]_{cost} = [66,73]$; the confidence in the estimation values is $\rho_{cost} = 0{,}7$. For S_2: the evaluation domain are $[a,d]_{cost} = [67,81]$ and the most possible values (or the estimation values) are $[b,c]_{cost} = [68,75]$; the confidence in the estimation values is $\rho_{cost} = 0{,}8$. Based on this, the possibility to have a value out of the estimation values [b,c], noted e, is computed using the equation 6 in section 3. Thus : $e_{S_1}^{cost} = 0,3$ and $e_{S_2}^{cost} = 0,2$. The possibility distributions representing the evaluation of the two technical bid solutions S_1 and S_2 with regard to the criterion *Cost* are presented in the Fig. 2.

		Sj								
		S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
Si	**S1**	IND	WPD	SPD	SPD	CD	WPD	CD	WPD	SPD
	S2	NR	IND	WPD	WPD	CD	WPD	CD	WPD	WPD
	S3	NR	NR	IND	NR	CD	NR	CD	NR	NR
	S4	NR	NR	SPD	IND	CD	NR	CD	NR	WPD
	S5	NR	NR	NR	NR	IND	NR	IND	NR	NR
	S6	NR	NR	SPD	SPD	CD	IND	CD	WPD	SPD
	S7	NR	NR	NR	NR	IND	NR	IND	NR	NR
	S8	NR	NR	SPD	WPD	CD	NR	CD	IND	SPD
	S9	NR	NR	WPD	NR	CD	NR	CD	NR	IND
Possibilistic dominance degree for the cost										

		Sj								
		S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
Si	**S1**	IND	NR	NR	NR	IND	NR	WPD	NR	NR
	S2	CD	IND	SPD	SPD	CD	WPD	CD	WPD	SPD
	S3	SPD	NR	IND	NR	SPD	NR	CD	NR	WPD
	S4	CD	NR	SPD	IND	CD	NR	CD	IND	CD
	S5	IND	NR	NR	NR	IND	NR	SPD	NR	NR
	S6	CD	NR	SPD	WPD	CD	IND	CD	WPD	CD
	S7	NR	NR	NR	NR	NR	NR	IND	NR	NR
	S8	SPD	NR	SPD	IND	SPD	NR	CD	IND	SPD
	S9	SPD	NR	NR	NR	SPD	NR	CD	NR	IND
Possibilistic dominance degree for the duration										

Fig. 3. Possibilistic dominance degrees of S_i over S_j

6.2 Computation of the possibilistic dominance degrees

The possibilistic dominance degrees are computed using the function Algorithm 1 presented in section 5. They are presented in Fig. 3. The numerical values corresponding to the possibilistic dominance degrees, computed using the Table 1, are presented in Fig. 4.

Based on this, the overall dominance degree are computed using the equation 15. They are presented in Fig. 5.

6.3 Computation of the outranking flows and the complete ranking

Based on the overall dominance degrees, the positive, negative and net outranking flows of each technical bid solution are computed using the equation 17, 18 and 5,

		Sj								
		S1	S2	S3	S4	S5	S6	S7	S8	S9
Si	S1	0	0,33	0,66	0,66	1	0,33	1	0,33	0,66
	S2	-	0	0,33	0,33	1	0,33	1	0,33	0,33
	S3	-	-	0	-	1	-	1	-	-
	S4	-	-	0,66	0	1	-	1	-	0,33
	S5	-	-	-	-	0	-	0	-	-
	S6	-	-	0,66	0,66	1	0	1	0,33	0,66
	S7	-	-	-	-	0	-	0	-	-
	S8	-	-	0,66	0,33	1	-	1	0	0,66
	S9	-	-	0,33	-	1	-	1	-	0
Numerical values of the possibilistic dominance degree for the cost										

		Sj								
		S1	S2	S3	S4	S5	S6	S7	S8	S9
Si	S1	0	-	-	-	0	-	0,33	-	-
	S2	1	0	0,66	0,66	1	0,33	1	0,33	0,66
	S3	0,66	-	0	-	0,66	-	1	-	0,33
	S4	1	-	0,66	0	1	-	1	0	1
	S5	0	-	-	-	0	-	0,66	-	-
	S6	1	-	0,66	0,33	1	0	1	0,33	1
	S7	-	-	-	-	-	-	0	-	-
	S8	0,66	-	0,66	0	0,66	-	1	0	0,66
	S9	0,66	-	-	-	0,66	-	1	-	0
Numerical values of the possibilistic dominance degree for the duration										

Fig. 4. Numeriacl values of possibilistic dominance degrees

		Sj								
		S1	S2	S3	S4	S5	S6	S7	S8	S9
Si	S1	0	0,25	0,5	0,5	0,75	0,25	0,83	0,25	0,5
	S2	0,25	0	0,41	0,41	1	0,33	1	0,33	0,41
	S3	0,17	0	0	0	0,92	0	1	0	0,08
	S4	0,25	0	0,66	0	1	0	1	0	0,5
	S5	0	0	0	0	0	0	0,17	0	0
	S6	0,25	0	0,66	0,58	1	0	1	0,33	0,75
	S7	0	0	0	0	0	0	0	0	0
	S8	0,17	0	0,66	0,25	0,92	0	1	0	0,66
	S9	0,17	0	0,25	0	0,92	0	1	0	0
Numerical values of the possibilistic dominance degree for the cost										

Fig. 5. Overall dominance degrees

respectively. They are presented in Table 3. The complete ranking of the solutions are computed based upon the net outranking flows ϕ. The more the net outranking flow of a solution is higher, the more it is preferred to the other solutions. The complete ranking of the nine solutions is presented in Fig. 6. The most interesting design solution is the solution S_6.

Table 3. The outranking flows

Solution	ϕ^+	ϕ^-	ϕ
S1	0,476	0,156	0,321
S2	0,518	0,031	0,488
S3	0,270	0,392	-0,122
S4	0,426	0,217	0,209
S5	0,021	0,812	-0,791
S6	0,570	0,072	0,498
S7	0,000	0,875	-0,875
S8	0,456	0,113	0,343
S9	0,291	0,362	-0,071

Fig. 6. Complete ranking of the solutions

7. CONCLUSION

In this article, we have proposed a multi-criteria decision support approach based on possibility theory and PROMETHEE II ranking method. The application of the proposed approach has shown that it allows to effectively rank several potential design solutions while taking into account imprecision and uncertainty related to the evaluation of the design solutions. In the context of an engineering design process, this approach is very useful, especially in the early phases.

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