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Abstract

We investigate the decoherence of maximally mixed multi-partite cluster states of a physical system of n qubits. We introduce the disconnected cluster states for an ensemble of non interacting qubits and we give the corresponding separable density matrices. The maximally entangled cluster states can be generated from disconnected cluster states, by applying the entanglig operators between the pair of qubits. When exposed to a local noisy interaction with the environment, the multi-qubit system evolves from its initial pure maximally entangled cluster state to a decohered mixed state. The decohered mixed states for multipartite systems are explicitly expressed and their bipartite entanglement is characterized by using the genuine multipartite negativity as a quantifier of entanglement.

Conclusion

Entangled cluster states are obtained by a dynamical evolution of multi-qubit system by using an Hamiltonian of Ising type. Cluster states are of great interest in quantum computing and are suitable in various tasks of quantum computation. Decohered mixed states are derived via a local noisy evolution of the entangled multi-qubit system in contact with environment. More precisely, under the influence of the decoherence, the multi-qubit system loses entanglement and evolves from it's initial maximally entangled cluster state to a decohered mixed state. More precisely we have given explicit expressions of decohered mixed states derived from n-qubit entangled cluster states. The separability for decohered mixed states is addressed with the concept of the genuine multipartite negativity.

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I. Maximally mixed multi-qubit cluster states

A cluster state is defined as following $\mid \phi_{\vec{0}} >= \left[\mid +, +, ..., + >= \frac{1}{\sqrt{2}}(\mid 0 > + \mid 1 >) \right]^{\otimes n}$ with the corresponding density matrices $\sigma_{\vec{0}} =\mid \phi_{\vec{0}} >< \phi_{\vec{0}} \mid = \left[\frac{1}{2} \sum_{k_i,k'_i}^{1} \mid k_i >< k'_i \mid \right]^{\otimes n}$. We generate the entangled cluster states $\mid \Phi_{\vec{0}} >$ by applying the entangling operators G_{ij} on each pair of qubits as follows

$$|\Phi_{\vec{0}}\rangle = \prod_i (G_{ij})^{q_{ij}} \, |+\rangle^{\otimes n}; \quad \varrho_{\vec{0}} = |\Phi_{\vec{0}}\rangle\langle\Phi_{\vec{0}}| \tag{1}$$

The operators G_{ij} stand for the controlled phase entangling gates given by

$$G_{ij} = |0\rangle\langle 0|_i \otimes |0\rangle\langle 0|_j + |0\rangle\langle 0|_i \otimes |1\rangle\langle 1|_j + |1\rangle\langle 1|_i \otimes |0\rangle\langle 0|_j - |1\rangle\langle 1|_i \otimes |1\rangle\langle 1|_j. \tag{2}$$

In the following, we consider the splitting of the entire system into two subsystems; one subsystem A_1 containing only one qubit $A_1 = \{k_i\}$ and the other $A_2 = \{k_1, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n\}$ containing the remaining $(n-1)$ qubits. The density matrix associated to the subsystem $A_1 = \{k_i\}$ is obtained by tracing out the qubits in A_2

$$\rho_{A_1} = \frac{1}{d^n} \sum_{k_1,\ldots,k_{i-1},k_{i+1},\ldots,k_n,k'_i} (-1)^{k_1 q_{1i}(k_i - k'_i)} \ldots (-1)^{k_{i-1} q_{(i-1)i}(k_i - k'_i)}$$

$$(-1)^{k_{i+1} q_{(i+1)i}(k_i - k'_i)} \ldots (-1)^{k_n q_{ni}(k_i - k'_i)} \mid k_i\rangle\langle k'_i|$$

Therefore the reduced density matrix to the subset A_1 is totally mixed, $\rho_{A_1} = \frac{1}{d^{|A_1|}} I_{d^{|A_1|}}$ if and only if the vector $(q_{1i}, q_{2i}, ...q_{(i-1)i} q_{(i+1)i}, ..., q_{ni}) \neq 0$; that is the arbitrary qubit $\{k_i\}$ is maximally entangled with the set of qubits $\{k_1, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n\}$.

II. Decoherence of Maximally entangled cluster states

We introduce a positive trace-preserving map denoted $\mathcal{G}_{\vec{m}}$ that maps the maximally entangled state $\varrho_{\vec{0}}$ to the density matrix $\varrho_{\vec{m}}$ labeled by the sets of parameters $\{\vec{m} \equiv (m_1, m_2, \cdots, m_n); m_i = 0, 1\}$ by $\varrho_{(m_1,\ldots,m_n)} \equiv \varrho_{\vec{m}} = (\otimes_{i=1}^n Z_i^{m_i}) \varrho_{\vec{0}} (\otimes_{i=1}^n Z_i^{m_i})^\dagger$. Now we encode the noisy evolution of the entangled system by an noisy operator denoted $\mathcal{G}^{\vec{p}} \equiv \mathcal{G}^{(p_1,\ldots,p_n)}$ that will maps the entangled phase state $\varrho_{\vec{0}}$ to a decohered state denoted $\varrho_{\vec{m}}^{\vec{p}}$ and given by $\varrho_m^{\vec{p}} \equiv \mathcal{G}^{\vec{p}}(\varrho_{\vec{0}}) = \Pi_{i=1}^n \Lambda_i^{p_i}(\varrho_{\vec{0}})$. with $\Lambda_i^{p_i}(\varrho_{\vec{0}}) = (1-p_i)\,\varrho_{\vec{0}} + p_i Z_i \varrho_{\vec{0}} (Z_i)^\dagger$ and $0 \leq p_i \leq 1/2$. Consequently, the decohered mixed state denoted $\varrho_n^{\vec{p}}$ can be cast in the compact form

$$\varrho_n^{\vec{p}} = \sum_{r_1,\cdots,r_n} \prod_{i=1}^n (1-p_i)^{(1-r_i)} \, (p_i)^{r_i} \, \varrho_{(r_1,\cdots,r_n)}, \quad n_i = 0, 1. \tag{3}$$

In the case of entangled cluster states when ($q_{12} = q_{23} = ... = q_{(n-1)n} = 1$), given by $\varrho_{\vec{0}} = \frac{1}{2^n} \sum_{\substack{k_1,\ldots,k_n \\ k'_1,\ldots,k'_n}} (-1)^{(\sum_i (k_i k_{i+1} - k'_i k'_{i+1}))} |k_1, \ldots, k_n\rangle\langle k'_1, \ldots, k'_n|$. We encode the noisy evolution of the multi-qubit system by a positive map $\mathcal{G}^{(p_s,p_{s+1})}$ as follows $\mathcal{G}^{(p_s,p_{s+1})}(\varrho_{\vec{0}}) = \Lambda_s^{p_s} \Lambda_{s+1}^{p_{s+1}}(\varrho_{\vec{0}})$. The multipartite decohered mixed state $\varrho_n^{(p_s,p_{s+1})}$ takes the form

$$\varrho_n^{(p_s,p_{s+1})} = G_{s(s+1)} \Big[\sum_{r_s,r_{s+1}=0}^{1} \prod_{i=s}^{s+1} (1 - \frac{p_i}{2})^{(1-r_i)} \, (\frac{p_i}{2})^{r_i} \prod_{i=1}^{s-1} G_{j(j+1)} \, Z_s^{r_s} \underbrace{|+,..,+\rangle\langle +,..,+|}_{s} (Z_s^{r_s})^\dagger \, (\prod_{j=1}^{s-1} G_{j(j+1)})^\dagger$$

$$\otimes \prod_{j \geq s+1} G_{j(j+1)} \, Z_{s+1}^{r_{s+1}} \underbrace{|+,..,+\rangle\langle +,..,+|}_{n-s} (Z_{s+1}^{r_{s+1}})^\dagger \, (\prod_{j \geq s+1} G_{j(j+1)})^\dagger \Big] G_{s(s+1)}^\dagger$$

In order to measure the bipartite entanglement of the multi-qubit decohered mixed state $\varrho_n^{(p_s,p_{s+1})}$ we use the concept of genuine multiparities negativity (GMN) $\tilde{N}(\varrho_n^{(p_s,p_{s+1})})$ introduced via the optimization problem $\tilde{N}(\varrho_n^{(p_s,p_{s+1})}) = -min \ tr \ \left(\varrho_n^{(p_s,p_{s+1})} \mathcal{W} \right)$, where the entanglement witness $\mathcal{W}$ is a observable fully decomposable able do detect the presence of entanglement in the decohered mixed state $\varrho_n^{(p_s,p_{s+1})}$. In this work, we define the genuine multiparticle negativity (GMN) as twice its standard expression given by $\tilde{N}(\varrho_n^{(p_s,p_{s+1})}) = 2 \, max\{0, F(\varrho_n^{(p_s,p_{s+1})}, \rho_{\vec{0}}) - \frac{1}{2}\}$. Then genuine multiparticle negativity (GMN) of the decohered mixed state $\varrho_n^{(p_s,p_{s+1})}$ is obtained as

$$\tilde{N}(\varrho_n^{(p_s,p_{s+1})}) = -2p_s - 2p_{s+1} + 2p_s p_{s+1} + 1 \tag{4}$$

From this equation it follows that if the condition$(-2p_s - 2p_{s+1} + 2p_s p_{s+1} + 1 > 0)$ is fulfilled then the two boundary qubits s and $(s+1)$ are entangled. Otherwise the bipartition $A_1|A_2$ loose entanglement if we have

$$-2p_s - 2p_{s+1} + 2p_s p_{s+1} + 1 \leq 0 \tag{5}$$