Experimental Estimation of Turbulence Modification by Inertial Particles at Moderate Re_λ

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We advance a novel method to estimate the carrier-flow dissipation ε_p in the presence of inertial *sub-kolmogorov* particles at moderate Re_λ. Its foundations rely on the unladen flow dissipation calculation using the Rice theorem, and the density of zero crossings of the longitudinal velocity fluctuation $u'(x)$ coming from a LDA device. Our experimental results provide strong evidence, for the first time, regarding the non-negligible effect that *sub-kolmogorov* particles have on the carrier flow energy cascade at $\phi_v = \mathcal{O}(10^{-5})$, and $Re_\lambda \in [200 - 600]$.

Several experimental and numerical studies have aimed at quantifying the impact of inertial particles on turbulent kinetic energy (TKE), and turbulent kinetic energy dissipation (ε) of particle-laden flows. Classical numerical studies have shown that the carrier-phase turbulence remains almost unchanged if the discrete phase volume fraction (ϕ_v) is very small, i.e., $\phi_v \leq 10^{-6}$, and very small particles are present ($D_p < \eta$) [1, 2]. Moreover, recent numerical simulations [3–6] have further explored the consequences of slightly larger concentrations ($\phi_v = \mathcal{O}(10^{-5})$), and have reported enhancement or damping of ε. These observations show the lack of consensus regarding how ε from the carrier phase is affected by the presence of particles at similar ϕ_v. Considering that the former approach (also known as 1-way coupling) has propelled the development of several theoretical models to describe these flows [7–10], a key improvement including the interaction between the two phases (two-way coupling) would be needed if the latter approach did not hold. Indeed, the 1-way coupling hypothesis has not been strictly validated by experiments due to the inherent difficulty [2] to measure the energy dissipation ε with traditional methods, e.g., classical optical techniques (LDA or PTV) at moderate Re_λ. A classical way to model the particle-fluid interaction has been to consider them to behave as point particles [11], and therefore, they follow the equation,

$$\frac{d\mathbf{V}_p}{dt} = -\frac{1}{\tau_p}[\mathbf{V}_p - \mathbf{u}(\mathbf{X}_p, t)] \tag{1}$$

with $\mathbf{V}_p$ the particle velocity and $\mathbf{u}(\mathbf{X}_p, t)$ the carrier's flow velocity evaluated at the particle's location $\mathbf{X}_p$, and τ_p being the particle viscous response time $\tau_p = \rho_p D_p^2/18\nu\rho$. The Eq.(1) is valid given that particle diameters are less than or equal to the Kolmogorov length-scale ($D_p \leq \eta$), i.e., *sub-kolmogorov* particles. The Fourier transform of Eq.(1) yields,

$$\hat{\mathbf{V}_p} = \frac{\hat{\mathbf{U}}}{i\omega\tau_p + 1}. \tag{2}$$

Hence, the particle field velocity is a low-pass filtered version of the carrier phase one, and being the filter a function of the Stokes number ($St = \tau_p/\tau_\eta$, with $\tau_\eta = (\nu/\varepsilon)^{1/2}$ the Kolmogorov time scale of the flow) with a cut-off frequency of $f_c = \tau_p^{-1}/2\pi$, or $f_c\tau_\eta = (2\pi St)^{-1}$.

Several authors [12–15] starting from Liepmann, have proposed and extended a way to estimate the Taylor microscale (λ) in an unladen flow from the density of zero crossings n_s of the longitudinal velocity fluctuation component $u'(x)$ [13–15] where the temporal measurements are translated into space by means of the Taylor hypothesis. These zero crossings follow a powerlaw function dependent on the ratio between the flow integral lengthscale (L), and the size of a low pass filter η_C (not to be confused with the Kolmogorov length-scale η) applied to the 'raw' signal. Interestingly, the relation $n_s = f(L/\eta_C)$ reaches a plateau above the cut-off length $\eta_C^\star$ for which $n_s \approx$ *constant*, if $\eta_C \geq \eta_C^\star$. More precisely, this plateau occurs when the product of n_s compensated by the average zero crossings distance of the whole signal $\bar{l}$ approaches one, i.e., $n_s\bar{l} \approx 1$, and it provides a criterion to determine whether or not n_s is well resolved. From these observations, and via the Rice theorem, Liepmann [12] proposed $n_s^{-1} = B\lambda$ with B being a constant that accounts for intermittency. Recently, Vassilicos and collaborators [14, 16] have used the latter expression in companion with $\varepsilon = 15\nu u^2/\lambda^2$, and being $u = \langle u'^2\rangle^{1/2}$, to suggest a reliable method to estimate ε, and to ultimately study the effects of the larger scales on the dissipation constant $C_\varepsilon = L\varepsilon/u^3$.

From Eq.(2), if the cut-off frequency f_c is large enough to resolve the dissipation scales, n_s should be recovered regardless particles size distribution. Thus, it is possible to deduce the value of λ from a set of particles velocities. However, in order to the latter argument to hold there should be sufficient particles to sample the fluid flow (see figure2a for criteria). These two observations give credence to extend the mentioned approach to particle laden flows considering that the cut-off wave number $2\pi/\eta_C$, after which Vassilicos and collaborators [14, 15] found a plateau in the density of zero crossings n_s, was at least one order of magnitude larger than the

Kolmogorov length-scale, i.e., a low-pass filtered particle velocity record could still be able to resolve the value of λ.

In this Letter, we apply the method described above to provide the first measurements of two-way coupling between the particles and the carrier flow for active-grid-generated turbulence in a wind tunnel at $Re_\lambda \in [200-600]$ and at liquid fractions ranging from $\phi_v \in [0.5-4.4]\times 10^{-5}$. We describe the method to estimate ε for particle laden flows from records taken by a Phase Doppler Interferometer (PDI) device [17] (a LDA like instrument) by means of extending the single phase approach proposed by Vassilicos and collaborators. We also compute L_p from the re-sampled unidimensional spectrum via $L = \lim_{\kappa\to 0} F_{11}(\kappa)\pi/u^2$ [18], which complies with our low-pass filtering argument (see Eq.(2)) [18]. First, we briefly comment on the experimental setup. Secondly, we succinctly review the signal post-processing. Next, we expose a criterion to discard spurious or poorly sampled signals. Finally, we use our results to show two important consequences of the mechanical coupling between the two phases on this system: first we show that the particles Stokes number is significantly modified compared to the '1-way' coupling approach, as τ_η is strongly affected by the presence of particles. This has important consequences in terms of scalings, for both preferential concentration and settling velocity modification [3, 5, 19]. Then, we show that the particles presence leads to modifications to the dissipation constant C_ε (defined via the relation $\varepsilon = C_\varepsilon u^3/L$). Our results suggest that C_ε evolves with Re_λ, and ϕ_v in a non trivial manner that may suggest changes on the nature of the energy cascade.

The experiment was conducted in the Lespinard wind tunnel, a close-circuit wind tunnel with 75×75 cm^2 square-cross section, and 4m long at LEGI laboratory. High levels of turbulence were generated by means of an active grid [20]. Just downstream of it, a rack of spray nozzles produced polydisperse inertial water droplets [21]. This wind tunnel has been extensively used to study particle-turbulence interaction under homogeneous isotropic turbulent (HIT) conditions (more experimental details are described in [22–24]). The measuring station was located three meters (3m) downstream the active grid, and at the center line of the wind tunnel, where HIT and the classical scalings from K41 theory have been recovered [15]. At this position we set a PDI device (Artium Technologies PDI-200), with an experimental setup, which was almost identical as the one described in Sumbekova [21], with the only difference being two circular holes of ten centimeters (10cm) on each window. The latter was aimed to counteract the water accumulation on the walls (the holes were smaller as possible to reduce the perturbation to the fluid flow), which had an impact on the droplets detection, a problem encountered by Sumbekova [21]. The unladen flow was measured by hot-wire anemometry (HWA) (for details, see [15]), and its characteristics are summarized in Table I. We tried to mimic as closely as possible the horizontal speed U_∞ of the unladen flow, and to vary the volume fraction ϕ_v as much as possible within our experimental limits (see Table I). Each realization consisted of roughly 500×10^3 samples with individual records spanning 10^5 sample points. These records are unevenly sampled due to the very nature of LDV measurements, we treated the records, and resampled them to an average acquisition frequency as explained below.

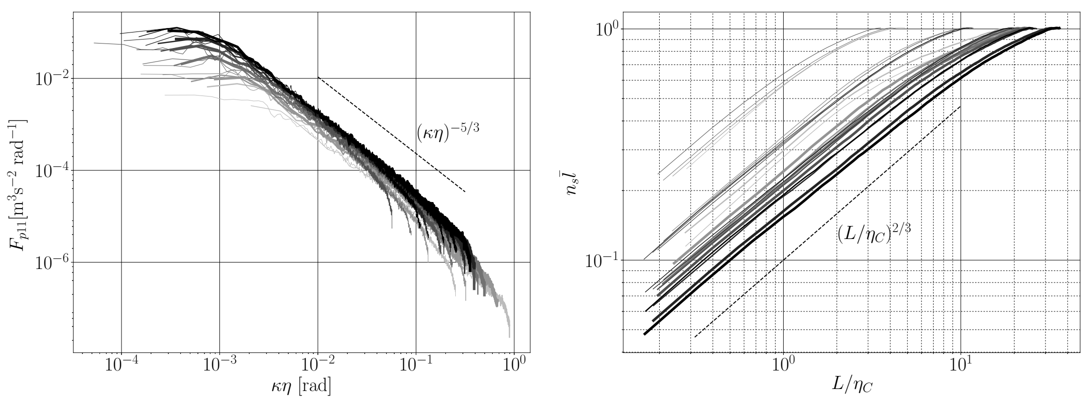
The possibility of estimating the energy spectrum from LDV measurements was early recognized [26], and several methods are available to account for the bias regarding the non-uniform sampled signal recorded by LDV [27]. These methods mainly aim at computing the autocorrelation function (ACF), which via the Wierner-Kinchin theorem, and the Fourier transform yields the energy spectrum. Our approach even though related to these ideas relies on a different argument. We have re-sampled the horizontal velocity particle records using a Piecewise Cubic Hermite Interpolating Polynomial (PCHIP) [28], at frequency equal to the average acquisition frequency, i.e., $\langle f_p\rangle$ =[#`events`]/[`signal length`] (see Table I). The longitudinal spectrum for the droplets signals is shown in Figure 1a, most of the data sets exhibit a powerlaw close to -5/3 over almost two decades. In general, $\langle f_p\rangle$ is not large enough to properly resolve the Kolmogorov length-scale η. The latter however does not invalidate our approach, as we are still able to resolve the density of zero crossings [14, 16, 29], and thereby λ_p from the longitudinal velocity fluctuation u'_p, given that the signals resolution via Taylor hypothesis satisfies the criteria $U_\infty/\langle f_p\rangle = \eta_{pC} \geq \eta^\star_C$ (sufficient particles to sample the flow), and that $f_c\tau_\eta = (2\pi St)^{-1} > 10^{-2}$ (enough resolution to capture the carrier phase temporal fluctuations responsible for λ at our Re_λ values, see Table I). The latter is supported by the exact relation, $\lambda/\eta = 15^{1/4} Re_\lambda^{1/2}$ [18] (see Table I), and the Taylor hypothesis.

The figure 1b shows the normalized zero crossings density $n_s\bar{l}$, where $\bar{l}$ has been independently obtained by averaging distances between consecutive zero-crossings of the unfiltered signal [14]. In order to apply the aforementioned criterion to our data, Vassilicos and collaborators [14, 29] have reported that the parameter $A = \eta_C/\eta = \mathcal{O}(10)$ for large Re_λ, which quantifies the filter cut-off length η_C (defined as the intersection between a 2/3 power law fitted to the data, and the horizontal line $n_s\bar{l} \approx 1$, see figure 1b) to the Kolmogorov length-scale. The latter value of A has been also recovered at our facility using unladen flow measurements [15].

The figure 2a illustrates an expected result; if the average frequency of events is too small, we are unable to properly resolve λ_p, which is defined as the equivalent Taylor length-scale from the particles record. Taking all the records for which $A < 40$ (it has been shown that A is a function of Re_λ [14]) the multi-phase flow dissipation

TABLE I: Parameters of the unladen phase (measured by HWA) at the measuring station 3m downstream the active grid. L was computed following [25]. The notation p refers to particle values measured by the PDI device. $\phi_v \approx Q_W/Q_A$. Where Q_W, and Q_A are the volumetric flux of water, and air respectively. D_p is the droplet diameter.$\langle f_p \rangle$ is the average droplet acquisition frequency.

Re_λ	U_∞[ms^{-1}]	$\langle f_p \rangle$[kHz]	u/U_∞	u_p/u	$\phi_v \times 10^{-5}$	D_p[μm]	L[m]	ε[m^{2}s^{-3}]	$\lambda[m]$	η [μm]
232	2	[0.35,1.32,1.37]	0.1273	[0.93, 0.95,1.21]	[0.9, 3.0, 4.4]	[10-200]	0.0570	0.0777	0.0136	457
321	3	[0.39,0.96,1.86,1.85]	0.1343	[0.98, 1.01, 0.99, 1.04]	[0.6, 1.0, 2.0, 3.0]	[10-200]	0.0721	0.2577	0.0119	338
404	4	[0.42,1.08,1.94,2.33]	0.1405	[0.98, 1.01, 1.04, 1.05]	[0.4, 0.7, 1.5, 2.2]	[10-200]	0.0845	0.6058	0.0108	273
503	5	[0.39,1.23,2.23,2.64]	0.1476	[0.97, 1.02, 1.02, 1.05]	[0.4, 0.6, 1.2, 1.8]	[10-200]	0.0980	1.1667	0.0102	231
601	6	[1.33,2.21,2.96,3.74]	0.1541	[0.99, 1.01, 1.00, 1.02]	[0.5, 1.0, 1.5, 2.0]	[10-200]	0.1110	2.1116	0.0098	200
648	7	[1.41,2.38,3.17,4.47]	0.1578	[1.01, 1.01, 1.01, 1.02]	[0.4, 0.8, 1.3, 1.7]	[10-200]	0.1158	3.3862	0.0090	178



(a) (b)

FIG. 1: a)Longitudinal energy density spectra F_{p11}. The darker the color the larger the mean velocity U_∞. b)Normalized zero crossings density $n_s\bar{l}$ against the normalized filter size L/η_C. The thicker the line the larger the liquid fraction ϕ_v (see Table I).

could be estimated via the expressions $n_s^{-1}|_{\eta^\star{}_{pC}} = B\lambda_p$, where we took $B = 1.2\pi$ as the one found for the unladen flow [15], and $\varepsilon_p = 15\nu u_p^2/\lambda_p^2$, with $u_p = \langle u_p'^2 \rangle^{1/2}$. Figure 2b illustrates $St/St_0 = (\varepsilon_p/\varepsilon)^{1/2}$. This estimate is remarkably accurate considering the rather simple algorithm followed. Moreover, the results confirm that at fixed concentration $\phi_v \approx 2 \times 10^{-5}$ (see the largest markers, and inset in figure 2b), the flow becomes less dissipative [6] at increasing Re_λ. On the contrary, for increasing concentration values ϕ_v at fixed Re_λ; ε could be dampened or enhanced [3, 5] by the particles presence. These observations therefore invalidate any approach that does not include the effects of 'two-way' coupling on the carrier flow turbulence at similar concentrations values $\phi_v = \mathcal{O}(10^{-5})$. Furthermore, Poelma et al. [30] proposed the parameter $\Phi_{St} = 6\pi^{-1}\phi_v(\eta/D_p)^3 St$ to quantify $\varepsilon_p/\varepsilon$, and reporting that for $\Phi_{St} > 0.003$ this ratio was larger than one (being this relationship linear). If we take the Sauter diameter ($D_{23} \approx 60\mu$m; $St_{32} \in [1-5]$) as representative of our droplets distribution [23], we recover values of $\Phi_{St} \in [0.001 - 0.035]$, which is in agreement with the results reported in figure 2b, despite the fact that these authors did their experiments at $Re_\lambda \approx 30$, and $\phi_v = \mathcal{O}(10^{-3})$. The previous observations are crucial considering that ε has a strong role enhancing the particle settling velocity in particle laden flows [8, 19, 31, 32].

Next, we investigate if the energy cascade is being affected by the presence of droplets. Considering that analogous Kolmogorov K41 scalings $\varepsilon = C_\varepsilon u^3/L$ (with C_ε constant for fixed boundary conditions) [33] are applicable to our particle laden flow, i.e., $\varepsilon_p = C_\varepsilon^p u_p^3/L_p$. The integral length-scale for our particle laden datasets was computed by $L = \lim_{\kappa\to 0} F_{11}(\kappa)\pi/u^2$ [18]. Being the most reasonable method to compute this quantity, given that our records do not possess the characteristics required to apply alternative procedures [25, 34]. Hence, an expected discrepancy with a factor close to two was found with respect to the unladen datasets [15], which ultimately does not change the functionality of ε.

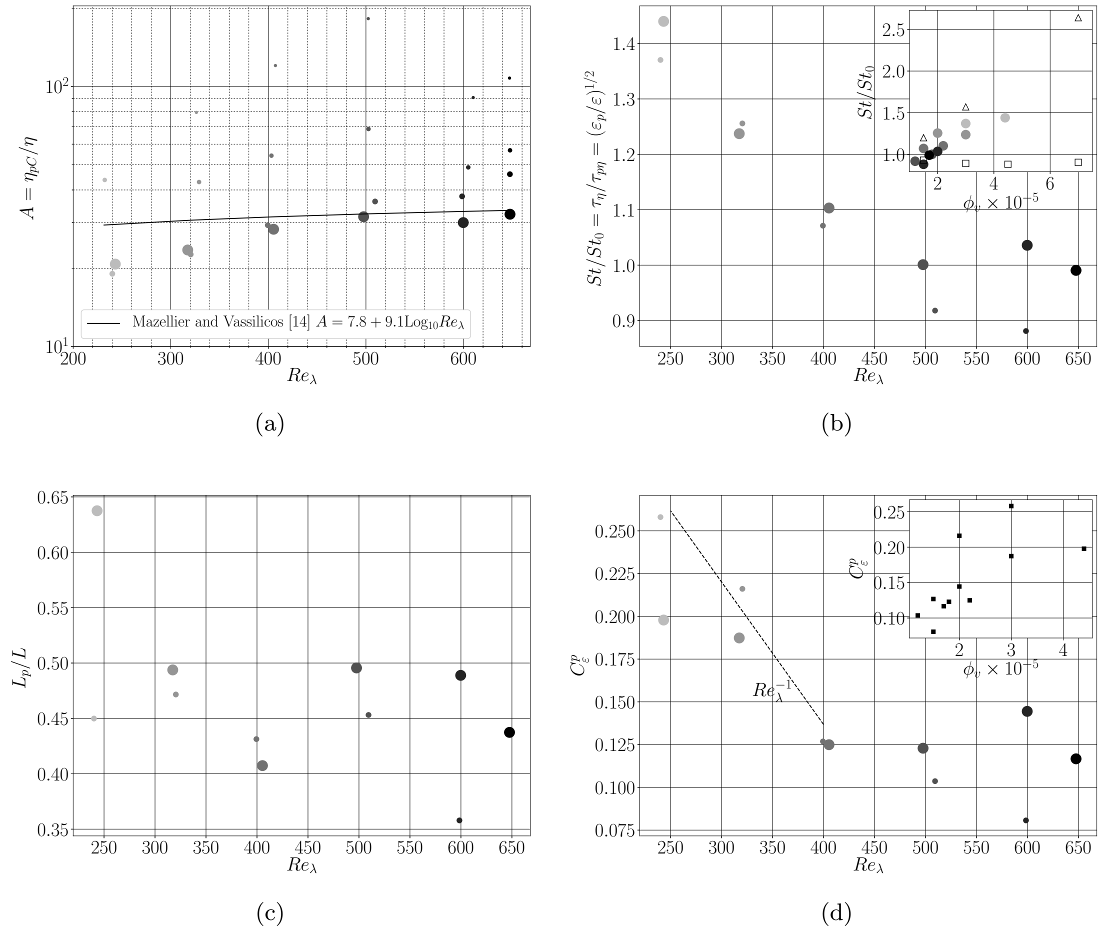


FIG. 2: Several parameters plotted against the unladen Re_λ. In all figures, the larger the marker size, the larger the concentration, and the darker its color the larger the Re_λ (see Table I). Figures b-d cover the datasets from figure 2a with the condition $A < 40$, $\phi_v \in [1.2 - 4.4] \times 10^{-5}$a) $A = \eta_{pC}/\eta$. Our values of A are in good agreement with previously reported unladen flows values [14, 15, 29], i.e., $A = \mathcal{O}(10) \approx 7.8 + 9.1\mathrm{Log}_{10}Re_\lambda$ [14]. b) Stokes number modification $St/St_0 = (\varepsilon_p/\varepsilon)^{1/2}$. In the inset, ($\triangle$) represents the results of Dejoan and Monchaux [5] for $V_T/u = 1$ (in their notation V_T is the particle terminal velocity), and ($\square$) illustrate the results of Bosse et al.[3]. c) Ratio of integral length-scales L_p/L, a factor close to 2 with respect to the unladen flow[15] is due to the method [25] to estimate L_p. d) $C_\varepsilon^p = \varepsilon_p L_p/u_p^3$. The inset shows C_ε^p vs ϕ_v.

After applying the latter method to the spectra found in figure 1a (which is unaffected by the low-pass filtering effect of the particles c.f. Eq.(2)), the value of C_ε^p can be estimated. Figure 2d reveals that at large Re_λ (computed from unladen phase), C_ε^p reaches a 'plateau' as expected [33], there is, however, a transition region where C_ε^p is inversely proportional to Re_λ^{-1}, i.e., $C_\varepsilon^p = f(Re_\lambda^{-1})$. This behavior was previously reported by Valente and Vassilicos [33, 35] studying the unladen cascade in grid experiments at similar Re_λ. Our results suggest for the first time that the carrier phase cascade might be altered by the presence of particle at similar low concentrations ϕ_v. A conjecture that if proven correct critically undermines '1-way' coupling approach, widely used in theoretical models and numerical simulations [36] at $Re_\lambda = \mathcal{O}(100)$. The inset in figure 2d further illustrates the entanglement between Re_λ, and ϕ_v. That given, our results might be biased by the very nature of our approximation via Eq.(2), as the particle do not completely sample the whole carrier flow velocity field: this error however seems to be small at the light of the data here presented.

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