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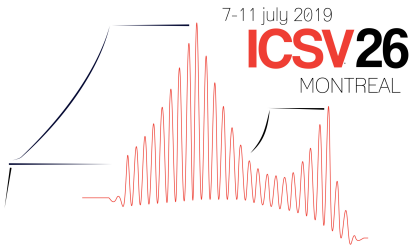
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# POWER BALANCED TIME-VARYING LUMPED PARAMETER MODEL OF A VOCAL TRACT: MODELLING AND SIMULATION

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Voice and speech production greatly relies on the ability of the vocal tract to articulate a wide variety of sounds. This ability is related to the accurate control of the geometry (and its variations in space and time) in order to generate vowels (including diphthongs) and consonants. Some well-known vibro-acoustic models of the vocal tract rely on a discretized geometry, such as concatenated cylinders, the radius of which varies in time to account for the articulation (see e.g. Maeda, Speech Comm. 1:199-229, 1982). We here propose a lumped parameter model of waves in the vocal tract considering the motion of the boundaries. A particular attention is paid to passivity and the well-posedness of the power balance in the context of time-varying geometrical parameters. To this end, the proposed model is recast in the theoretical framework of port-Hamiltonian systems that ensure the power balance. The modularity of this framework is also well-suited to interconnect this model to that of deformable walls (in a power-balanced way). We show the capacities of the model in two time-domain numerical experiments: first for a static configuration (time-invariant geometry), then a dynamic one (time-varying geometries) of a two-cylinder vocal tract.
Keywords: VOCAL TRACT, VOICE MODELING, PORT-HAMILTONIAN SYSTEMS

## 1. Introduction

The vocal tract has been described by four main different approaches: Reflection Type Line Analog models (RTLA); Transmission Line Circuit Analog models (TLCA); Hybrid time frequency models and finite element methods models. Here we are interested in the TLCA approach introduced by Flanagan [1].

Maeda [2] takes this model further introducing a matrix representation that facilitates the simulation. In this model it is assumed that the static pressure and the acoustical volume flow are homogeneous inside every subtract. This approach has been steadily extended: Birkholz et al. [3] added a noise source model

for the synthesis of fricatives; Mokthari et al. [4] and Élie and Laprie [5] improved the matrix formulation in order to connect multiple side branches and bilateral channels.

However all these models implicitly assume that the acoustic waves propagate in a static vocal tract despite the fact that geometry of the vocal tract evolves in order to articulate, e.g., dynamically go from some formantic configuration to another or to produce consonants. Our goal here is to elaborate a simple lumped parameter model that correctly accounts for the power balance with the walls of the waveguide. The port-Hamiltonian systems (pHs) theory provides a valuable framework for this purpose. We hereafter recall its principal features, please report to [6] for an extensive introduction to this theory.

## 2. Port Hamiltonian Systems (pHs)

The pHs framework gives the central role to the power balance in interconnected systems. The rate of the energy $\dot{E}$ stored by some system equals to the dissipated power $P_{diss}$ minus the power exchanged with the outside $P_{ext}$ (positive for power flowing out of the considered system): $\frac{\mathrm{d}E}{\mathrm{d}t} = -P_{diss} - P_{ext}$ The state of an energy-storing system is described by a set of internal variables $\mathbf{X}$ and by the Hamiltonian $\mathrm{H}(\mathbf{X})$, so that $\dot{E} = \dot{\mathbf{X}} \cdot \nabla_{\mathbf{X}}\mathrm{H}$. Similarly, the exchanged power is, without loss of generality, written as the product of the input effort $u$ and its power-dual quantity $y$, i.e., $P_{ext} = u \cdot y$. Ignoring here the dissipation for the sake of conciseness (but with no additional difficulty), the pHs framework considers that the vector of efforts $[\nabla_{\mathbf{X}}\mathrm{H}; u]$ and the vector of flows $[\dot{\mathbf{X}}; y]$ are related by the interconnection matrix $S$:

$$\begin{pmatrix} \dot{\mathbf{X}} \\ y \end{pmatrix} = S \cdot \begin{pmatrix} \nabla_{\mathbf{X}}\mathrm{H} \\ u \end{pmatrix}$$

The power balance is then automatically satisfied provided $S$ is a skew symmetric matrix ($S^\intercal = -S$). A numerical scheme developed in Ref. [7] (implemented in the PyPHS Python library [8]) preserves this property in the discrete time-domain.

## 3. Power balanced model of the vocal tract

We consider the classical representation of the vocal tract as a piecewise cylindrical acoustic tube, i.e., approximating the vocal tract as the concatenation of $N$ sub-tracts each of them having a constant cross section. We first describe the model for a single cylinder as a pHs, before performing the assembly of the full vocal tract, i.e., the interconnection of the $N$ elements.

### 3.1 Single sub-tract

We list our hypothesis about the fluid's behaviour and tract's geometry. Then we are interested in finding the energy stored by the system, the relevant state variables and the associated dynamics. At last we combine these elements to write the full pHs. We treat separately the kinematic and compression behaviour.

**Hypothesis:** We consider an irrotational and compressible flow of an inviscid fluid in $\Omega$, a rectangular block shaped domain with coordinates $x \in (-\ell_0, \ell_0)$, $y \in (0, h(t))$ and $z \in (0, L_0)$ (see Fig. 1). We assume that the fluid fulfills the perfect gas assumption and undergoes an isotropic and adiabatic transformation (reversible process, no heat sources). Finally the volumetric mass is assumed to be homogeneously distributed in each sub-tract.

The local dynamics of such a fluid is given by the conservation of mass, momentum and energy:

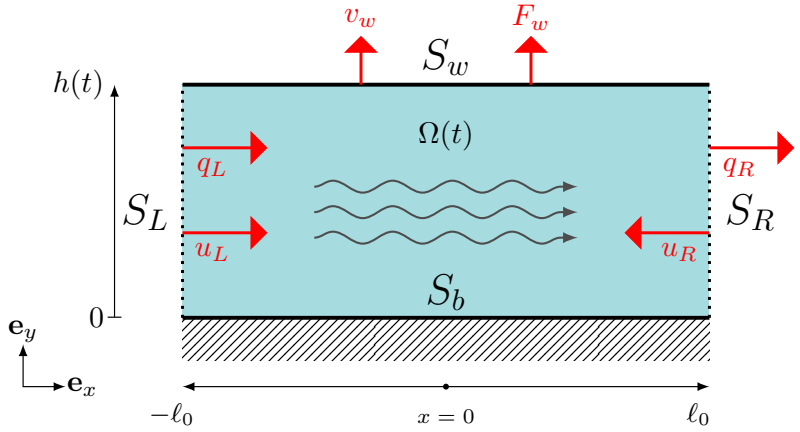



Figure 1: Geometry of one subtract

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0, \quad \frac{\partial \rho \mathbf{v}}{\partial t} + \operatorname{div}(\rho \mathbf{v} \otimes \mathbf{v}) + \operatorname{grad}(p) = 0 \quad \text{and} \quad \rho \frac{\partial}{\partial t}\left(e_m + \frac{1}{2}\mathbf{v}\cdot\mathbf{v}\right) = -\operatorname{div}(p\mathbf{v}) \quad (1)$$

where $\rho$ is the volumetric mass density, $\mathbf{v}$ the (Eulerian) velocity, $p$ the pressure and $e_m$ the specific internal energy.

The boundary of $\Omega$ is denoted by $\partial\Omega = S_L \cup S_R \cup S_w \cup S_b$ where: $S_L$ and $S_R$ are the left and right open surfaces respectively located at $x = -\ell_0$ and $x = \ell_0$; $S_w$ is an impermeable wall moving with normal velocity $\mathbf{v} = v_w\,\mathbf{e}_y$ located at $y = h(t)$; $S_b$ is a motionless wall at $y = 0$. The total volume is given by $V(t) = 2\ell_0 L_0 h(t) = S_w h(t)$ and its dynamics by $\dot{V}(t) = S_w \dot{h}(t) = S_w v_w$. The mass of fluid contained in $\Omega$ is denoted by $m = \rho V$.

The energy balance obtained by integration of Eq. (1) over $\Omega(t)$ displays the power exchanged through the open boundaries $S_L$ and $S_R$ expressed as the product of the mass flow rate ($q_{S_L}$ and $q_{S_R}$) and their power-dual quantities ($u_L$ and $u_R$, sometimes called *specific enthalpy*) :

$$P_{ext,S_L} = y_L u_L \quad \text{where} \quad y_L = -q_{S_L} = \iint_{S_L} \rho\mathbf{v}\cdot\mathbf{e}_x \,\mathrm{d}S \text{ and } u_L = \left\langle e_m + \frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \frac{p}{\rho}\right\rangle_{S_L} \quad (2)$$

$$P_{ext,S_R} = y_R u_R \quad \text{where} \quad y_R = +q_{S_R} = \iint_{S_R} \rho\mathbf{v}\cdot\mathbf{e}_x \,\mathrm{d}S \text{ and } u_R = \left\langle e_m + \frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \frac{p}{\rho}\right\rangle_{S_R}. \quad (3)$$

In order to ease the future interconnection of sub-tracts, we choose to use the upstream incoming $q_L$ and the downstream outgoing $q_R$ mass flow rates. The power exchanged with the moving wall is the product of the wall velocity $v_w$ and the net force $F_w$ of the wall on the fluid.

**Kinematics:** we consider the following velocity field

$$\mathbf{v} = \begin{pmatrix} v_x(x,t) \\ v_y(y,t) \end{pmatrix} = \begin{pmatrix} v_{mx}(t) - \alpha(t)x/\ell_0 \\ v_w(t)y/h(t) \end{pmatrix} \quad (4)$$

generalizing the kinematics proposed in Ref. [9]. It accounts for a uniform axial velocity ($v_{mx}(t)$), an axial gradient ($\alpha(t)$) and the linear gradient of the transverse velocity between the base and the moving wall ($v_w(t)$). The time evolution of the mass density $\rho(t)$ is then given by $\dot{\rho} = \rho\,(\alpha/\ell_0 - v_w/h)$.

The inertial (kinetic) energy is:

$$E_i(t) = \frac{1}{2}\iiint_\Omega \rho\mathbf{v}\cdot\mathbf{v}\,\mathrm{d}\Omega = \frac{1}{2}m\left(v_{mx}^2 + \frac{\alpha^2}{3} + \frac{v_w^2}{3}\right) = \frac{\Pi_x^2}{2m} + \frac{3\Pi_y^2}{2m} + \frac{3\Pi_c^2}{2m} = H_i(\Pi_x, \Pi_y, \Pi_c, V, \rho) \quad (5)$$

where we introduced the three corresponding momenta $\Pi_x = m v_{mx}$, $\Pi_y = m v_w/3$ and $\Pi_c = m\alpha/3$. Their dynamics are obtained using the principle of virtual power with the virtual velocities $\tilde{\mathbf{v}} = \mathbf{e}_x$, $\tilde{\mathbf{v}} = x\,\mathbf{e}_x$ and $\tilde{\mathbf{v}} = y\,\mathbf{e}_y$, respectively. Without calculation details for the sake of brevity, their time evolution are given by:

$$\dot{\Pi}_x = \frac{m}{2\ell_0}\left(2\alpha v_{mx} + u_L - u_R\right), \quad \dot{\Pi}_y = F_w + S_w\langle\, p\,\rangle_\Omega + \frac{m}{3\ell_0} v_w \alpha, \quad \dot{\Pi}_c = \frac{2m}{3\ell_0}\alpha^2 + F_L - F_R - S_w\langle\, p\,\rangle_\Omega. \tag{6}$$

**Compressibility:** we now express the internal energy stored by the fluid under compression. Under the hypothesis of isotropic and adiabatic transformation of the fluid from an initial state (atmospheric pressure $P_0$, mass density $\rho_0$ and temperature $T_0$), the specific internal energy $e_m$ is given by Joule's law, function of the sole mass density $\rho$: $e_m(\rho) = c_v T_0\left({}^{\rho^{\gamma-1}}/_{\rho_0^\gamma}\right)$ where $c_v$ is the specific heat capacity at constant volume and $\gamma$ the heat capacity ratio, so that the internal energy stored on $\Omega$ is

$$E_c(t) = \iiint_\Omega \rho e_m \,\mathrm{d}\Omega = m e_m = V c_v \rho_0 T_0 \left(\frac{\rho}{\rho_0}\right)^\gamma = \frac{P_0 V}{\gamma - 1}\left(\frac{\rho}{\rho_0}\right)^\gamma = H_c(V,\rho) \tag{7}$$

according to the ideal gas law. Here, we consider a weak form of the definition of the thermodynamic pressure: $\langle\, p\,\rangle_\Omega = \partial_\rho e_m(\rho) = \left(\rho\partial_\rho H_c - H_c\right)/V$.

**First port Hamiltonian representation of the sub-tract** The state of the fluid is finally described by the state vector $\mathbf{X} = \left(\Pi_x, \Pi_c, \Pi_y, V, \rho\right)^T$, the Hamiltonian is the sum of the inertial and the internal energies:

$$\mathrm{H}_1(\mathbf{X}) = H_i(\mathbf{X}) + H_c(V,\rho) \tag{8}$$

$$\text{with} \quad H_i(\mathbf{X}) = \frac{\Pi_x^2}{2\rho V} + \frac{3\Pi_y^2}{2\rho V} + \frac{3\Pi_c^2}{2\rho V} \quad \text{and} \quad H_c(V,\rho) = \frac{P_0 V}{\gamma - 1}\left(\frac{\rho}{\rho_0}\right)^\gamma \tag{9}$$

$$\text{with gradient } \nabla_{\mathbf{X}}\mathrm{H} = \left(\frac{\Pi_x}{\rho V}, \quad \frac{3\Pi_c}{\rho V}, \quad \frac{3\Pi_y}{\rho V}, \quad \frac{-H_i + H_c}{V}, \quad \frac{-H_i + H_c}{\rho} + \rho V \partial_\rho e_m\right)^\intercal \tag{10}$$

The first component of the gradient regenerates $v_{mx}$, the second regenerates $\alpha$ and the third $v_w$. We have access to the pressure by combining the last two components: $< p >_\Omega = {}^{\rho}/_{V}\partial_\rho \mathrm{H}_1 - \partial_V \mathrm{H}_1$. After identification of the inputs and of the components of the gradient in the previous equations, we can write the full port Hamiltonian system:

$$\left(\begin{array}{c} \dot{\Pi}_x \\ \dot{\Pi}_c \\ \hline \dot{\Pi}_y \\ \hline \dot{V} \\ \dot{\rho} \\ \hline\hline y_L \\ y_R \\ -v_w \end{array}\right) = \left(\begin{array}{cc|c|cc||cc|} 0 & {}^{\Pi_x}/_{\ell_0} & 0 & 0 & 0 & {}^{m}/_{2\ell_0} & {}^{-m}/_{2\ell_0} & \\ {}^{-\Pi_x}/_{\ell_0} & 0 & {}^{-\Pi_y}/_{\ell_0} & 0 & {}^{-\rho}/_{\ell_0} & {}^{m}/_{2\ell_0} & {}^{m}/_{2\ell_0} & \\ \hline 0 & {}^{\Pi_y}/_{\ell_0} & & -S_w & {}^{\rho S_w}/_{V} & & & 1 \\ \hline 0 & 0 & S_w & & & & & \\ 0 & {}^{\rho}/_{\ell_0} & {}^{-\rho S_w}/_{V} & & & & & \\ \hline\hline {}^{-m}/_{2\ell_0} & {}^{-m}/_{2\ell_0} & & & & & & \\ {}^{m}/_{2\ell_0} & {}^{-m}/_{2\ell_0} & & & & & & \\ \hline & & -1 & & & & & \end{array}\right) \left(\begin{array}{c} \partial_{\Pi_x}\mathrm{H} \\ \partial_{\Pi_c}\mathrm{H} \\ \hline \partial_{\Pi_y}\mathrm{H} \\ \hline \partial_V \mathrm{H} \\ \partial_\rho \mathrm{H} \\ \hline\hline u_L \\ u_R \\ F_w \end{array}\right) \tag{11}$$

## 3.2 Connection of two adjacent sub-tracts

**Recast into a symmetric pHs representation** The previous model considers specific enthalpies and mass flow rates as inputs and outputs, respectively, for the two open boundaries of the sub-tract $S_L$ and $S_R$. The adjacent sub-tracts expect exactly the same inputs and outputs which causes a realisability issue. The general handling of such situation is described in Ref. [10], but it can be tackled more easily if the outputs ($y_L$ and $y_R$) are directly contained in the gradient of the Hamiltonian. This means that we may introduce equivalent internal variables $X_L$ and $X_R$ such that $y_L = \partial_{X_L}\mathrm{H}$ and $y_R = \partial_{X_R}\mathrm{H}$. For the assumed kinematics, these variables are $X_L = \ell_0\left(v_{mx} + {}^{\alpha}/_{3}\right)$ and $X_R = \ell_0\left(v_{mx} - {}^{\alpha}/_{3}\right)$.

This particular expressions should be interpreted in the sense of the quadrature rule accounting for the given degrees of freedom and the respective basis functions. The new vector of internal variables is $\mathbf{X}_2 = [X_L\ X_R\ \Pi_y\ V\ \rho]^\intercal$, while the Hamiltonian, its gradient and the interconnection matrix become:

$$\mathrm{H}_2(\mathbf{X}_2) = \frac{\rho V}{2\ell_0^2}\left(X_L^2 + X_R^2 - X_L X_R\right) + \frac{3\Pi_y^2}{2\rho V} + V\frac{P_0}{\gamma-1}\left(\frac{\rho}{\rho_0}\right)^\gamma \tag{12}$$

$$\nabla_{\mathbf{X}_2}\mathrm{H}_2 = \left(q_L,\quad q_R,\quad v_w,\quad \frac{\rho}{2}\left(v_{mx}^2 + \frac{\alpha^2}{3} - \frac{v_w^2}{3}\right) + \frac{P_0}{\gamma-1}\left(\frac{\rho}{\rho_0}\right)^\gamma,\quad \frac{V}{2}\left(v_{mx}^2 + \frac{\alpha^2}{3} - \frac{v_w^2}{3}\right) + \frac{\gamma P_0 V}{\rho(\gamma-1)}\left(\frac{\rho}{\rho_0}\right)^\gamma\right)^\intercal \tag{13}$$

$$\left(\begin{array}{c} \dot{X}_L \\ \dot{X}_R \\ \hline \dot{\Pi}_y \\ \hline \dot{V} \\ \dot{\rho} \\ \hline\hline y_L \\ y_R \\ \hline -v_w \end{array}\right) = \left(\begin{array}{cc|c|cc||cc|c} & & -\Pi_y/m & 0 & -1/V & 1 & 0 & \\ & & \Pi_y/m & 0 & 1/V & 0 & -1 & \\ \hline \Pi_y/m & -\Pi_y/m & & -S_w & \rho S_w/V & & & 1 \\ \hline 0 & 0 & S_w & & & & & \\ 1/V & -1/V & -\rho S_w/V & & & & & \\ \hline\hline -1 & 0 & & & & & & \\ 0 & 1 & & & & & & \\ \hline & & -1 & & & & & \end{array}\right) \left(\begin{array}{c} \partial_{X_L}\mathrm{H}_2 \\ \partial_{X_R}\mathrm{H}_2 \\ \hline \partial_{\Pi_y}\mathrm{H}_2 \\ \hline \partial_V \mathrm{H}_2 \\ \partial_\rho \mathrm{H}_2 \\ \hline\hline u_L \\ u_R \\ \hline F_w \end{array}\right) \tag{14}$$

**Connection of two adjacent sub-tracts** Let us denote $A$ and $B$ two adjacent sub-tracts (with Hamiltonians $\mathrm{H}_A(\mathbf{X}_A)$ and $\mathrm{H}_B(\mathbf{X}_B)$ respectively) with the right surface $S_{RA}$ of $A$ connected to the left surface $S_{LB}$ of $B$. At that interface (denoted $S_{AB}$ hereafter), the mass flowrate is assumed to be conserved: $q_{RA} = q_{LB}$. In the classical acoustic waveguide theory, due to the continuity of the mass density, the continuity of the volume flowrate is generally considered.

The fact that the efforts $q_{RA} = \partial_{X_{RA}}\mathrm{H}_A$ and $q_{LB} = \partial_{X_L^B}\mathrm{H}_B$ are equal implies that the corresponding variables $X_{RA}$ and $X_{LB}$ can be merged into a new junction variable $X_{AB} = X_{RA} + X_{LB}$ using the procedure detailed in Ref [10]. The state vector, the Hamiltonian and its gradient become

$$\begin{aligned} \mathbf{X}_{AB} =& \left(X_{LA}, X_{AB}, X_{RB}, \Pi_{y_A}, \Pi_{y_B}, V_A, V_B, \rho_A, \rho_B\right)^\intercal; \\ \mathrm{H}_{AB} =& H_{cA}(\rho_A, V_A) + H_{cB}(\rho_B, V_B) + \frac{3\Pi_{y_A}^2}{2m_A} + \frac{3\Pi_{y_B}^2}{2m_B} \\ &+ \frac{3m_A}{8\ell_{0A}^2}X_{LA}^2 + \frac{3m_B}{8\ell_{0B}^2}X_{RB}^2 + \frac{1}{2}\frac{\left(X_{AB} - (X_{LA} + X_{RB})/2\right)^2}{\ell_{0A}^2/m_A + \ell_{0B}^2/m_B} \end{aligned} \tag{15}$$

$$\nabla_{\mathbf{X}_{AB}}\mathrm{H}_{AB} = \left(\ q_{LA},\ q_{RA} = q_{LB},\ q_{RB},\ v_{wA}, v_{wB},\ \partial_{V_A}\mathrm{H}_{2_A},\ \partial_{V_B}\mathrm{H}_{2_B},\ \partial_{\rho_A}\mathrm{H}_{2_A},\ \partial_{\rho_B}\mathrm{H}_{2_B}\right)^\intercal$$

$$\left(\begin{array}{c} \dot{X}_{LA} \\ \dot{X}_{AB} \\ \dot{X}_{RB} \\ \hline \dot{\Pi}_{yA} \\ \dot{\Pi}_{yB} \\ \hline \dot{V}_A \\ \dot{V}_B \\ \hline \dot{\rho}_A \\ \dot{\rho}_B \\ \hline\hline -q_{LA} \\ q_{RB} \\ \hline -v_{wA} \\ -v_{wB} \end{array}\right) = \left(\begin{array}{ccc|cc|cc|cc||cc|cc} & & & -\frac{\Pi_{y_A}}{m_A} & 0 & & & \frac{-1}{V_A} & 0 & 1 & 0 & & \\ & & & \frac{\Pi_{y_A}}{m_A} & -\frac{\Pi_{y_B}}{m_B} & & & \frac{1}{V_A} & -\frac{1}{V_B} & 0 & 0 & & \\ & & & 0 & \frac{\Pi_{y_B}}{m_B} & & & 0 & \frac{1}{V_B} & 0 & -1 & & \\ \hline \frac{\Pi_{y_A}}{m_A} & -\frac{\Pi_{y_A}}{m_A} & 0 & & & -S_{wA} & 0 & \frac{\rho_A S_{wA}}{V_A} & 0 & & & 1 & 0 \\ 0 & \frac{\Pi_{y_B}}{m_B} & -\frac{\Pi_{y_B}}{m_B} & & & 0 & -S_{wB} & 0 & \frac{\rho_B S_{wB}}{V_B} & & & 0 & 1 \\ \hline & & & S_{wA} & 0 & & & & & & & & \\ & & & 0 & S_{wB} & & & & & & & & \\ \hline \frac{1}{V_A} & -\frac{1}{V_A} & 0 & -\frac{\rho_A S_{wA}}{V_A} & 0 & & & & & & & & \\ 0 & \frac{1}{V_B} & -\frac{1}{V_B} & 0 & -\frac{\rho_B S_{wB}}{V_B} & & & & & & & & \\ \hline\hline -1 & 0 & 0 & & & & & & & & & & \\ 0 & 0 & 1 & & & & & & & & & & \\ \hline & & & -1 & 0 & & & & & & & & \\ & & & 0 & -1 & & & & & & & & \end{array}\right) \left(\begin{array}{c} \partial_{X_{LA}}\mathrm{H}_{AB} \\ \partial_{X_{AB}}\mathrm{H}_{AB} \\ \partial_{X_{RB}}\mathrm{H}_{AB} \\ \hline \partial_{\Pi_{y_A}}\mathrm{H}_{AB} \\ \partial_{\Pi_{y_B}}\mathrm{H}_{AB} \\ \hline \partial_{V_A}\mathrm{H}_{AB} \\ \partial_{V_B}\mathrm{H}_{AB} \\ \hline \partial_{\rho_A}\mathrm{H}_{AB} \\ \partial_{\rho_B}\mathrm{H}_{AB} \\ \hline\hline u_L \\ u_R \\ \hline F_{wA} \\ F_{wB} \end{array}\right)$$

This can be generalized to the assembly of several sub-tracts. However the introduction of the junction

variables and the calculation of the total Hamiltonian is not detailed here for the sake of brevity.

### 3.3 Full vocal tract

The full vocal tract is finally obtained by adding a lumped parameter model of elastic wall (see Fig. 2). This also enables the control of the boundaries by means of the velocities of the tissues ($v_{mi}$) instead of the net forces of the wall ($F_{wi}$ in the inputs of the model described in Sec. 3.2). The resulting model is then implemented using the facilities offered by the PyPHS library [8] to connect pHs models and simulate using a power-balanced numerical scheme.

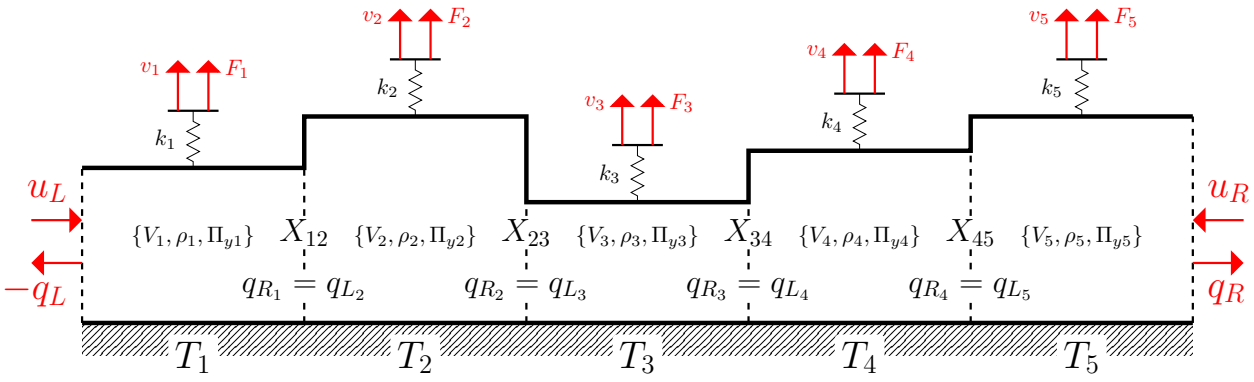



Figure 2: Full system: ports are denoted with red font and arrows

## 4. Conclusion

This paper introduces a new model of the vocal tract accounting for the controlled motion of the surrounding tissues. It classically considers the concatenation of several cylindrical sub-tracts, but carefully models the energy transfers both between sub-tracts and with the deformable tissues. In fact, the usual acoustical approximation deriving from the linearization of the fluid motion equations fails to correctly account for non-infinitesimal motion of walls and for long-term power transfers. In this short-length paper, the model is derived in terms of the thermodynamical and fluid dynamics framework, and a further work will consist in the reformulation in order to introduce variables that are closer to the classical acoustical ones, i.e., deviations from the rest state with or without the linearization that is usually performed in the linear acoustics theory.

This model is also a first step towards more realistic models of the vocal tract that could incorporate more physical phenomena such as visco-thermal effects and turbulence. We will aim at simplifying its use in simulation by adapting existing articulatory models.

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## REFERENCES


1. Flanagan, J. L., Ishizaka, K. and Shipley, K. L. Synthesis of Speech From a Dynamic Model of the Vocal Cords and Vocal Tract, *Bell System Technical Journal*, **54** (3), 485–506, (1975).

2. Maeda, S. A digital simulation method of the vocal-tract system, *Speech communication*, **1** (3-4), 199–229, (1982).

3. Birkholz, P., Jackèl, D. and Kröger, B. J. Simulation of losses due to turbulence in the time-varying vocal system, *IEEE Transactions on Audio, Speech and Language Processing*, **15** (4), 1218–1226, (2007).

4. Mokhtari, P., Takemoto, H. and Kitamura, T. Single-matrix formulation of a time domain acoustic model of the vocal tract with side branches, *Speech Communication*, **50** (3), 179–190, (2008).

5. Elie, B. and Laprie, Y. Extension of the single-matrix formulation of the vocal tract: Consideration of bilateral channels and connection of self-oscillating models of the vocal folds with a glottal chink, *Speech Communication*, **82**, 85–96, (2016).

6. Schaft, A. J. v. d. and Maschke, B. M. Hamiltonian formulation of distributed-parameter systems with boundary energy flow, *Journal of Geometry and Physics*, **42** (1–2), 166–194, (2002).

7. Falaize, A., *Modélisation, simulation, génération de code et correction de systèmes multi-physiques audios: Approche par réseau de composants et formulation Hamiltonienne à Ports*, Ph.D. thesis, Université Pierre et Marie Curie - Paris VI, (2016).

8. *PyPHS: Passive modeling and simulation in python*. Software available at `https://afalaize.github.io/pyphs/` (mainly developed by A. Falaize), last accessed: 2019-04-31.

9. Lopes, N. and Hélie, T. Energy Balanced Model of a Jet Interacting With a Brass Player's Lip, *Acta Acustica united with Acustica*, **102** (1), 141–154, (2015).

10. Najnudel, J., Hélie, T., Boutin, H., Roze, D., Maniguet, T. and Stéphane, V. Analog circuits and Port-Hamiltonian realizability issues: a resolution method for simulations via equivalent components., *Audio Engineering Society Convention*, New York, United States, Oct., (2018).