Origin of the lift effect in segregating granular flows

Lydie Staron*

Sorbonne Université, CNRS, Institut Jean Le Rond d'Alembert, F-75005 Paris, France

Applying the contact dynamics method, we perform two-dimensional discrete numerical simulations of segregating granular flows in the case of single free intruders and bi-disperse granular mixtures. In both configurations, we do not observe any measurable lift force acting on the larger grains which may explain their rising motion. The large force fluctuations they are submitted to reduce to their weight, following the mere action-reaction principle. Hence, the rising dynamics must originate from the properties of the surrounding granular bed itself. We identify the strong asymmetry displayed by granular beds resistance to downward (plunging) and upwards (withdrawing) motion, as reported in details in Hill et al 2005, *Europhys. Lett* 72, 137-143, as the most likely cause. Accordingly, moving an object towards the free surface is about 10 times easier than moving an object toward the rigid bottom. This asymmetry allows for an effective lift effect when large grains are submitted to upward force fluctuations, without being counterweighted by sinking episodes when large grains are submitted to downwards force fluctuations. In addition to gravity, the existence of two different boundary conditions formed by the free surface and the rigid bottom explain this difference of resistance to motion. In this respect, the mechanism allowing size segregation in granular flows is the same as that allowing legged locomotion in sand (Li et al 2013, *Science* 339, 1408-1412).

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I. INTRODUCTION

In their natural occurrence, granular flows rarely exhibit the well defined unique grain size so useful in laboratory experiments or simulations for constraining granular flow behaviour. On the contrary, they usually display a wide range of sizes, that may cover several order of magnitude in extreme cases (as for debris or rock flows) [1–4]. Even sand dunes, one natural granular system closest to its laboratory counterpart, are made of smaller and larger grains [5].

An immediate consequence of the diversity of grains sizes is their sorting: while flowing or being shaken, large grains and smaller grains separate, forming specific patterns and thereby affecting the system's evolution. In natural cases such as rock or debris flows, large grains rise to the free surface where they acquire a larger velocity. They accumulate at the front where they are pushed sideways by the advancing bulk, thus ending up forming levées that confine and channel the flow [6–9]. In geotechnical application, the separation by grain size may undermine the mechanical quality of concrete, or simply of a given soil. Grain size segregation, as the phenomenon is called, is thus a fundamental aspect of granular behaviour. Yet, although a seemingly simple mechanism, and in spite of the effort devoted to it, the mechanical origin of size segregation in granular flows remains elusive.

First attempts at describing size segregation essentially focussed on the probability of large/small grains to migrate in the flow following a percolation-like picture of the phenomena, relying on the geometrical characterisation of the voids opening in a sheared flow as sites that smaller

grains can occupy; the concept of "squeeze expulsion" explains why a large grain at the bottom starts to migrate upwards in the first place [10–15]. A mechanical explanation for grain size segregation was introduced later by Gray and Thornton 2005 [16]. In this model, segregation is understood as resulting from the heterogeneous force transmission typically observed in granular packings of same-size grains, and generalised to polydisperse (many-size) packings [17–19]. Accordingly, pressure partition in the media differs from the classical mixture theory: larger grains sustain a larger part of the mean pressure than prescribed by their volume fraction. This causes them to see larger gravity-induced pressure gradients, and to rise as a result. This model allows for the successful description of gravity-induced segregation patterns by solving shallow-layer equations in a wide range of configurations [9, 20–22]. The mechanical origin assumed (namely non-classical pressure partition) is however difficult to establish [23–26]. It is, besides, uneasy to translate in terms of a Lagrangian description of the dynamics of a given segregated particle. In this perspective, enlightening experiments were performed with the aim of quantifying the forces (namely, lift and drag forces) acting on intruders moving in a granular media [27–30]. In these experiments, an intruder buried in a granular bed at a given depth is submitted to a slow motion (either a slow rotation or a slow drag) while its vertical position is constrained, and the forces exerted on it are measured. All report the existence of a lift force, either dependent on the pressure/depth [27], or independent of it [29, 30]. In all cases however, the intruder has as symmetrical shape, so that the asymmetry necessary to create a lift effect must originate from the granular bed itself. Performing discrete simulations, Guillard et al identified the gravity gradient and a self-screening effect around their rotating object as major ingredient [29, 31]. Recently, similar simulations of intruders constrained in height in a gran-

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*Electronic address: lydie.staron@upmc.fr

TABLE I: Summary of simulations performed.

Simulations	Volume fraction of large beads Φ_L	Slope θ	Grain size ratio D/d	Number of independent runs
Intruder	-	20°, 21.5°, 23°	1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5	144
Mixture	0.2, 0.4, 0.6 (± 0.02)	20°, 21.5°, 23°	1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5	72

ular flow led to the interpretation of the lift effect as an equivalent of the Saffman effect in viscous-inertial flows [32].

These intruder experiments are all performed for intruders whose position is constrained in the direction normal to the flow, either attached to a spring or simply a fixed point. Hence they cannot move upward freely in response to the existence of a lift force. On the contrary, motion is forced in the direction of the flow (or equivalently, the flow is forced around them). In segregating flows however, things are very different as the intruder moves vertically in response to the lift. Lift forces may thus relax and therefore may not build up to such high values as observed in [29]. By contrast, there is no (or very little) relative motion in the direction of the flow, as the intruder (or the phase of larger grains) essentially follows the mean velocity profile [32, 37]. At any rate, the direct quantification of lift forces in segregating granular flows seems hardly feasible experimentally, as it would imply measuring the position of, and the resulting force acting on a given entirely free intruder (not to say many intruders). This is where discrete numerical simulations may offer an important contribution, as they give access to all contact forces in a given flow, thus allowing direct computation of the resulting force on intruders, as well as the easy exploration of the experiments parameters (intruder's size, flow velocity, flow composition etc) [32–38]. The quantification of the lift forces acting on large grains in freely segregating granular flows forms the aim of the present paper.

In the following, we present two-dimensional discrete numerical simulations of bi-disperse (two grain sizes) granular flows applying the contact dynamics algorithm [39, 40]. The segregation of a single large grains (namely an intruder), and the segregation of a collection of large grains, are both investigated in term of the average force resultant applied to the large grains. In contrast with [27, 29, 32] for constrained intruders, our simulations do not show the existence of a net lift force. On the contrary, we find that contact forces applied to larger grains exhibit very large fluctuations, but do on average balance the large grains weight following a simple action/reaction principle. Hence the rising motion of large grains must have a different origin than a classical lift force. We argue that this rising motion originates from the strong asymmetry displayed by granular beds resistance to downward (plunging) and upwards (withdrawing) motion, as reported in details in [41–44]. Following these authors, moving an object towards the free surface is about 10 times easier than moving a object toward the rigid bottom. This asymmetry allows for an effective lift effect when large grains are submitted to large

positive (upward) forces without being counterweighted by sinking episodes when large grains are submitted to equivalently large negative (downward) forces.

The numerical techniques are briefly presented in section II. The case of the single large intruder is discussed in section III, while section IV reports the case of bi-disperse granular mixtures. A discussion on the origin of the lift effect follows in section V.

II. CONTACT DYNAMICS SIMULATIONS OF BI-DISPERSE GRANULAR FLOWS

The numerical method applied to simulate the granular flows is the Contact Dynamics (CD) algorithm [17, 39, 40], already applied for segregation problems by the same author in [26, 37]. The basic ingredients of this method are the following. Grains interact at contacts through solid friction and hardcore repulsion. Solid friction imposes that locally, the normal and tangential contact forces satisfy $f_t \leq \mu f_n$, where μ is the coefficient of friction at contact. Moreover, a coefficient of restitution e sets the amount of energy dissipated in collisions. The numerical values of μ and e affect the effective frictional properties of the flow (velocity, angle of repose...) but we do not consider their influence on the segregation process. Their value was set to $\mu = 0.5$ and $e = 0.5$, for all contacts irrespective of the size of the grains involved, and were not varied. The hardcore repulsion ensures that grains at contacts do not overlap beyond an accepted small δ that allows for contact detection. By contrast with Molecular Dynamics (MD) methods which introduce an explicit stiffness to describe the contact rigidity, the hardcore repulsion in the CD method is a non-smooth strict condition. The difference between MD and CD methods is however expected to be virtually null in the flow configuration studied, provided both are used within the range of numerical parameters in which their validity is ensured.

Two dimensional granular beds were simulated, formed of small grains of diameter d , and large grains of diameter D . To prevent the geometrical ordering likely to happen in 2D for strictly mono-sized packings, small grains diameter exhibits a variability of $\simeq 30\%$ in the single intruder configuration (namely $0.044 \leq d \leq 0.06$). The dimensions of the granular bed are $L = 100d$ and $H \in [55d, 70d]$ (depending on the composition), with periodic boundary conditions in the direction of the flow. The basal boundary is made of a row of fixed grains of diameter 0.05 (namely $\simeq d$). The mass density of the intruder(s) is the same as that of the surrounding smaller grains ($\rho = 0.1 \text{ kg.m}^{-2}$).

Two configurations are considered. In the case of the single intruder, one large grain is buried in a bed of smaller grains inclined at a slope θ . While the flow develops (in steady regime), the intruder rises from its initial position $H/3$ to the free surface (Figure 1). In the case of the granular mixture, small and large grains are initially deposited under gravity in a mixed state (Figure 5), achieved by random positioning of small and large grains prior to deposition. As the flow develops at slope angle θ , the large grains rise in the flow. For all cases, all contact forces, grains position and velocity are computed and known. Simulations were performed varying the grains size ratio D/d from 1.5 to 5, the volume fraction of large grains Φ_L from 0.2 to 0.6 (in the case of mixtures), and the slope from 20° to 23° , as summarized in table I.

III. THE SINGLE LARGE INTRUDER

A single large intruder carried along by a flow of smaller grains forms a specific case of segregation, as it relies only on interactions with a uniform granular matrix, without resorting to cooperative mechanisms with fellow big grains. For this reason, it offers an interesting insight in the nature of the forces exerted by a flow of small grains on bigger objects, as a starting point to understand the dynamics of segregation. In the following, we thus consider a single large grain initially buried in a bed of smaller grains allowed to flow under gravity at an angle θ (for which stationary regime is reached). As the flow starts and reaches the stationary regime, the intruder is left free to move along with the mass of smaller grains, namely none of its degree of freedom is suppressed. Accordingly, its vertical position evolves in time as a response to the forces exerted by the smaller grains on it. Discrete numerical simulations give us access to all contact forces, so that we can accurately follow the resulting force acting on the intruder, and explore the existence of a lift force.

A. Rising dynamics

We consider granular beds of width $L = 100d$, and height $H \simeq 60d$, made of small grains of mean diameter $d = 0.052m$ (uniformly distributed between 0.044 and 0.06) (see Figure 1). A large intruder of diameter D is buried in the granular bed at an initial vertical position $H/3$. The intruder's diameter D is alternatively $D = 1.5d, 2d, 2.5d, 3d, 3.5d, 4d, 4.5d$, and $5d$. The granular bed is tilted at an angle θ for which a stationary flow develops ($\theta = 20^\circ, 21.5^\circ$, and 23°). The duration of the simulations is set to 500 seconds, for which nearly all intruders eventually reach the free surface (but for two cases with $D=1.5d$). For each values of D and θ , 5 to 10 independent runs are performed (for a total of 144 independent simulations).

As a result of the flow, the intruder moves up and down.

Its instantaneous vertical position z_I is recorded in the course of time. Figure 1-c shows the case of an intruder with $D/d = 3.5$: we observe a fluctuating motion which eventually lead to the free surface. In less favorable cases, as shown in Figure 2, the intruder reaching the free surface may be sucked down again in the bulk (here for $D/d = 4$), or its motion may exhibit larger fluctuations which impede the segregation process (for $D/d = 1.5$ for instance). These sinking episodes are reminiscent of the diffusive mechanisms described in [46] and leading to remixing. However, eventually, most intruders are segregated by the flow for the simulation duration considered.

The instantaneous resultant vertical force $f_{z,I}(t)$ resulting from all the intruder's contacts with its neighbours is defined as

$$f_{z,I}(t) = \sum_{\alpha=1}^{n_I^\alpha} \vec{f}_I^\alpha(t) \cdot \vec{z}, \quad (1)$$

where n_I^α is the number of contacts in which the intruder is involved at time t . The example displayed in Figure 1-c ($D/d = 3.5$) shows that $f_{z,I}(t)$ undergoes large fluctuations. We have checked that these force fluctuations do exist for all the simulations performed, and for all size ratio D/d . They can reach above $60\times$ the intruder's weight, and are generally of large amplitude. Their role in the rising dynamics is thus expected to be important.

B. Forces applied to the intruder

We probe the existence of a lift force by simply averaging the instantaneous vertical force on the intruder $f_{z,I}(t)$ over the duration of the simulation:

$$F_I = \frac{1}{N_t} \sum_{t=1}^{N_t} f_{z,I}(t), \quad (2)$$

with $f_{z,I}$ defined in (1). The dependence of F_I (normalized by $m_s g \cos \theta$, $m_s = \rho \pi d^2/4$) with the intruder size D (normalised by d) is shown in Figure 3. For each pair $(D/d, \theta)$, the mean value averaged over all the independent runs is shown (full symbols), as well as individual simulations points (empty symbols). We have moreover included the case $D/d = 1$. For a given value of D/d and θ , the individual simulation points (empty symbol) are scattered, showing that fluctuations during the segregation process are very large, as already observed from Figure 1-c. Yet, averaging over independent runs (full symbols), we essentially observe:

$$F_I \simeq \cos \theta \rho \pi \frac{D^2}{4} g, \quad (3)$$

namely, the averaged contact forces applied by the small grains on the intruder seems to be merely balancing the

FIG. 1: An intruder of diameter D is initially buried in a bed of grains of diameter d (illustration (a)); as the flow develops at slope angle θ as a result of gravity, the intruder rises to the free surface (illustration (b)). The intruder's vertical position, and the vertical force resultant exerted on it, evolve in time as shown in (c). In this example, $D/d = 3$ and $\theta = 23^\circ$. (Color online)

FIG. 2: Vertical position of the intruder z_I (normalised by d) as a function of time for $D/d = 1.5$ for a slope angle $\theta = 23^\circ$, and $D/d = 4$ for a slope angle $\theta = 20^\circ$. (Color online)

intruder's weight, following the action-reaction principle. No additional positive contribution allows us for the identification of a lift force. This means that if a lift force builds up, its intensity is very small compared to the intruder's weight and the force fluctuations. This is not surprising in view of the typical rising dynamics displayed in Figure 1-c. In this example, it takes about 200 seconds for the intruder to reach the free surface, namely to cover a distance of about $50d$, namely a nearly zero acceleration. On the contrary, a sustained measurable lift force would send the intruder very

quickly to the surface, a case never observed in our simulations.

On the other hand, the forces fluctuations seen by the intruder are very important, so that it is reasonable to suppose that they are responsible for the rising dynamics. We may suppose that over short time intervals, when upward/positive force fluctuations become much higher than the typical reaction to the weight of the intruder, the upward motion of the latter is made possible. The accumulation of such upward jumps results in the rising motion of the intruder.

On average however, the vertical force resultant reduces to the intruder's weight. Accordingly, the negative downwards force fluctuations do balance the positive ones. Although we observe the intermittent downward motion of the intruder, its amplitude does not counter-weight the upward motion. In other words, while the upward and downward force resultants on the intruder are symmetrical, their effect in term of motion is not.

We propose that this asymmetry proceeds from the fact that the resistance of a granular bed to an intruder motion is strongly dependent on whether the motion is upward or downward, even at important depths. This asymmetry was evidenced in [41–44], where intruders of different size and shape were alternatively plunged or withdrawn from a granular bed. It shows that for intruders of different shapes (including spherical), the force necessary for plunging the intruder in a granular bed (with no lateral confinement) is one order of magnitude larger than the force necessary for withdrawing it. This can be explained by the asymmetry created by the gravity gradient (as identified by [30]), and above all,

by the different boundary conditions formed by the free surface on the one hand, and the rigid bottom on the other hand (studied in details in [43]). In the context of the present simulation, this asymmetry is enough to account for the rising dynamics of the intruder without the contribution of a net lift force. In this scenario, the agitation, or "temperature", induced by the flow, generates large force fluctuations on the intruder, which is thereby allowed to explore both upward and downward motion in the packing, and meeting much less resistance in the first case.

It would be of great interest to quantify precisely how resistance to motion itself is affected by the intruder's size. The results by [27] report smaller resistance for larger intruders, and a discrepancy between upward and downward motion increasing with the intruder's size, which would imply that larger intruders tend to segregate better.

In a fluid-like picture of granular flows, it could be relevant to use an equivalent buoyant force instead of the simple weight of the intruder [32]. However, this would require the computation of the local solid fraction around the intruder, which depends strongly on the size ratio (through the Voronoi calculation), thereby introducing a geometrical bias in the analysis of our results (while we want the size ratio to remain an independent parameter of our study). Hence, we prefer the straightforward comparison with gravity forces.

It is worthy to note that attempts at varying the time window over which F_I is computed (for instance considering the rising dynamics only and filtering out time spent at the free surface) did not change the results in a significant way. At any rate, it did not help disclosing a different trend with the intruder's size.

Finally, computing the relative velocity in the flow direction between the intruder and the smaller grains at the same height showed the existence of very small lag of fluctuating sign, so that the analysis in term of a viscous-inertial Saffman effect [32] seems not relevant here.

C. Focussing on upward force fluctuations

The rising motion results from a succession of upward jumps, presumably occurring when the resulting vertical force on the intruder undergoes a large positive fluctuation. Hence, we focus now on the positive values of $f_{z,I}$:

$$F_I^+ = \frac{\sum_{t=1}^{N_t} \mathcal{H}(f_{z,I}) f_{z,I}}{\sum_{t=1}^{N_t} \mathcal{H}(f_{z,I})}, \quad (4)$$

where $\mathcal{H}$ is the Heaviside function, and the summation is made over all the time steps t of the simulation. We compute F_I^+ for all 96 independent simulations with D/d varying between 1 and 5, and the slope θ alternatively set to 20° , 21.5° and 23° . The results are presented in Figure 4, where $F_I^+/(m_s g \cos \theta)$ is plotted as a function

FIG. 3: Time-averaged vertical force resultant on the intruder F_I (normalised by the projected weight of a small grain $m_s g \cos \theta$) as a function of the normalised intruder's diameter D/d for different slopes. The error bars are showing the variability over independent simulations. We observe that the vertical force exerted on the intruder balances its weight. Inset: same thing for the time-averaged longitudinal force resultant on the intruder (namely the drag).

of the size ratio D/d ($m_s = \rho \pi d^2/4$). We observe that the value of F_I^+ is scattered for larger values of θ , *i.e.* for very dynamical flows (eg $\theta = 23^\circ$). For slower flows ($\theta = 20^\circ$ and $\theta = 21.5^\circ$), F_I^+ follows a clearer trend. In both cases, quadratic fits are acceptable:

$$\frac{F_I^+}{m_s g \cos \theta} \simeq \lambda_I \left(\frac{D}{d} \right)^2 + C, \quad (5)$$

where $\lambda_I \simeq 1.5$ for $\theta = 20^\circ$ and $\theta = 21.5^\circ$, $\lambda_I \simeq 2.5$ for $\theta = 23^\circ$, and $C \simeq 6$ for all slopes investigated.

Essentially, F_I^+ is larger for larger slopes. This coincides indeed with the fact that segregation is more efficient for larger slopes. Larger intruders are pushed upwards with a force increasing with their weight, giving them more power to displace the smaller grains covering them. Smaller intruders, including the grains forming the granular bed, also see large forces pushing them intermittently toward the free surface. But in these cases, segregation is less (or not) efficient, suggesting that the rising motion is counterbalanced by sinking episodes.

IV. THE BI-DISPERSE MIXTURE

A single intruder rises in a granular bed uniform in composition, and the system thus formed is accurately described by the knowledge of the intruder's diameter D and that of the smaller grains forming the granular bed d . In a bi-disperse mixture however, things are very different. Each large grain can be seen as an intruder, yet moving through a matrix of varying composition, and undergoing forces from contacts with both small and large

FIG. 4: Positive contribution F_I^+ of the time-averaged vertical force resultant on the intruder F_I (normalised by the projected weight of a small grain $m_s g \cos \theta$) as a function of the normalised intruder's diameter D/d for different slopes. The dotted lines show quadratic fits.

grains. While it flows, the granular bed changes geometry as each large grain tends to rise at the surface in a highly transient dynamics. Hence, we no longer speak of "intruders", but of the phase of large grains and the phase of small grains, and try to evidence the mechanism that lead to their sorting/separation.

A. Rising dynamics

The systems studied are formed by a mixture of small grains (diameter $d = 0.04$) and large grains (diameter D) initially in a mixed state, as shown in illustration 5. The volume fraction of large grains Φ_L may take the values 0.2, 0.4 or 0.6 (± 0.02), and the large grains diameter is alternatively set to $1.5d$, $2d$, $2.5d$, $3d$, $3.5d$, $4d$, $4.5d$, and $5d$, as in the case of the single intruder (III). The systems thus formed are tilted at an angle θ for which they develop into steady flows ($\theta = 20^\circ$, 21.5° and 23°). As a result, the initially well-mixed phases of large grains and small grains separate, with the larger grains rising at the surface, as shown in Figure 5. A total of 72 independent runs are performed.

The rising dynamics may be described by the position of the centre of mass of the larger grains z_L and its evolution in the course of time (Figure 5-c). As observed elsewhere [15, 24, 34, 37], the segregation is not complete, namely few larger grains remain in the bulk as a result of diffusion and remixing [46].

B. Resulting force on large grains

The instantaneous vertical force resultant seen by each large grain p at a given time t is simply given by the projection of the forces $\vec{f}_p^\alpha$ transmitted at each contact

α involving p :

$$f_{z,p}(t) = \sum_{\alpha=1}^{n_p^\alpha} \vec{f}_p^\alpha(t) \cdot \vec{z}, \quad (6)$$

where n_p^α is the number of contacts in which the grain p is involved. An estimate of the instantaneous mean vertical force seen by large grains p is obtained by averaging $f_{z,p}(t)$ over all the large grains:

$$F_p(t) = \frac{1}{N_L} \sum_{p=1}^{N_L} f_{z,p}(t), \quad (7)$$

N_L being the total number of large grains. Figure 5-d shows an example of the large fluctuations exhibited by F_p in the course of time.

Averaging over the whole duration of the simulation, we compute the average vertical force resultant seen by each element of the phase of large grain during the whole segregation process:

$$F_L = \frac{1}{N_t} \frac{1}{N_L} \sum_{t=1}^{N_t} \sum_{p=1}^{N_L} f_{z,p}(t) = \frac{1}{N_t} \sum_{t=1}^{N_t} F_p(t), \quad (8)$$

where N_t is the number of simulation time steps. We compute F_L for all 72 simulations with different composition Φ_L , different slope angle θ , and different size ratio D/d . The dependence of F_L (normalised by $m_s g \cos \theta$, $m_s = \rho \pi d^2 / 4$) with the size ratio D/d is shown in Figure 6. We exactly recover

$$F_L = \cos \theta \rho \pi \frac{D^2}{4} g, \quad (9)$$

namely, on average, contact forces exactly balance the weight of the large grains. As for single intruders, we do not measure any net lift force.

On the other hand, the force fluctuations larger grains are submitted to are very important, as visible from Figure 5-d. As for single intruders, this suggests that the rising dynamics results from these large force fluctuations coupled with the asymmetry of the resistance to upwards motion (toward the free surface) and downward motion (toward the bottom) exhibited by granular beds [41–44]. Accordingly, large positive force fluctuations induce upward jumps toward the free surface, without being counter weighted by "sinking episodes" when negative forces fluctuations come into play.

C. Focussing on positive force fluctuations

Analysing the rising motion of large grains is made difficult by the fact that grains do not move in a synchronised way, and while some move up, other may sink down. Nevertheless, we can suppose that the rising dynamics can be understood from the analysis of the positive contribution of the mean vertical force $F_p(t)$ seen by the phase of large grains. Hence, we compute:

$$F_L^+ = \frac{\sum_{t=1}^{N_t} \mathcal{H}(F_p) F_p}{\sum_{t=1}^{N_t} \mathcal{H}(F_p)}, \quad (10)$$

FIG. 5: Example of a mixture of smaller grains and larger grains such that $D/d = 3$ and $\Phi_L \simeq 0.4$, (a) in the initial state and (b) after segregation occurred due to flow under gravity at slope ($\theta = 21.5^\circ$); the position of the center of mass of the larger grains z_L is shown in the course of time in (c) (normalised by the flow thickness H). The dashed line shows an exponential fit. In (d), the instantaneous mean vertical force seen by large grains $F_p(t)$ is shown for all time steps (dotted line) and averaged over 25 time steps ($\langle F_p(t) \rangle$ full line); the dashed lines shows the weight of a large grain. (Color online)

FIG. 6: Mean time-averaged vertical force resultant on any large grain F_L (normalised by the projected weight of a small grain $m_s g \cos \theta$) as a function of the normalised intruder's diameter D/d for different slopes. We observe that on average the vertical force exerted on the large grains exactly balances their weight.

where $\mathcal{H}$ is the Heaviside function, and the summation is made over all the time steps t of the simulation. The value of F_L^+ is computed for all simulations with different

TABLE II: Parameters for scaling (11).

Slope angle	λ	a	b	$c = 2 - (a + b)$
20°	5.89	0.98	0.89	0.134
21.5°	7.28	0.89	0.98	0.124
23°	8.92	0.72	1.20	0.078

composition Φ_L , different slope angle θ , and different size ratio D/d . From their analysis, a non-trivial dependence between F_L^+ and the size ratio D/d emerges. We observe the following form (Figure 7):

$$\frac{F_L^+}{m_s g} \simeq \lambda \left(\frac{D}{d} \right)^a \cdot \left(\frac{\langle d \rangle}{d} \right)^b, \quad (11)$$

where $\langle d \rangle = \Phi_L D + (1 - \Phi_L)d$ is the mean grain diameter for the mixture. The proportionality coefficient λ and the exponents a and b vary with the slope θ , and their value is summarized in table II. The positive influence of the slope on segregation efficiency is visible in the value λ .

The scaling (11) can be rearranged into a buoyancy-

FIG. 7: Positive contribution of the vertical force resultant on the phase of large grains F_L^+ (normalised by the projected weight of a small grain $m_s g \cos \theta$) as a function of $(D/d)^a (\langle d \rangle / d)^b$, where a and b varie with the slope θ (see text for values). Dashed lines show linear fits.

FIG. 8: From Hill et al (2005): Rescaled plots of the plunging forces, F_+ , and the withdrawal forces, F_- for horizontal rods in beds of monodisperse glass beads.

like force:

$$F_L^+ = \lambda \left(\frac{\langle d \rangle}{D} \right)^b \cdot \left(\frac{d}{D} \right)^c \times \rho \pi \frac{D^2}{4} g, \quad (12)$$

where b and $c = 2 - (a + b)$ are respectively of the order of 1 and 0.1 (see table II for exact values). The pre-factor formed by $(\langle d \rangle / D)^b \cdot (d / D)^c$ exhibits an explicit dependence on the composition through the mean grain diameter $\langle d \rangle = \Phi_L D + (1 - \Phi_L) d$. Accordingly, for a given grain size ratio, higher volume fractions of large grains favour large positive force fluctuations, hence presumably segregation. This holds at least in the range of volume fractions investigated; for larger values of ϕ_L however, the scaling (12) is likely to break down, when smaller grains do no longer form a continuum but are trapped in the matrix of large grains.

V. DISCUSSION

Applying the contact dynamics method, we have performed discrete numerical simulations of segregating

FIG. 9: From Schröter et al (2007): Forces measured during a full cycle of insertion and withdrawal of an intruder in a granular bed at a volume fraction $\phi = 0.602$.

granular flows in the case of single free intruders, and in the case of bi-disperse granular mixtures. In both configurations, while segregation occurs, we did not observe any measurable lift force acting on the larger grains. On the contrary, we observe that the large force fluctuations they are submitted to reduce to their weight, following the mere action-reaction principle.

Experiments consisting of plunging an intruder in a static granular bed or withdrawing it from an initially buried state report a strong asymmetry between the forces necessary to accomplish these two motions. In Hill et al (2005) [41], forces necessary to withdraw a large intruder are about one order of magnitude smaller than the forces necessary to bury it (see Figure 8). These results holds for different intruder sizes and shapes, different burying depths, as well as different container's width. This asymmetry was later corroborated in Schröter et al (2007) [42] for rods (see Figure 9), and in Martinez 2013 [43] for spheres and rods, also reporting withdrawal forces at least 10 times smaller than plunging forces. In Li et al (2013) [44], similar results are reported and used to explain locomotion in sands.

This asymmetry is not surprising, and reflects the difference of boundary conditions at the top and bottom of the granular container: while plunging requires pushing aside and rearranging grains whose motion will be opposed by a rigid wall, withdrawing motion is easily accommodated by the freely deforming free surface. It is however interesting that this effect persists at large depth, implying that the boundary condition formed by the free surface is felt throughout the system. As suggested by [41], we may suppose the existence of a cut-off depth at which the difference of forces between plunging and withdrawing will vanish, in a very large container. Meanwhile, such (symmetrical) regime has not been observed yet.

Hence, any body buried in a granular flow and submitted to force fluctuations, will meet a different resistance when subjected to upward or downward momentum, and should logically rise as a result. This seems enough

to explain the origin of segregation in granular flows. We can try to quantify this effect using a very simple model, based on a frictional representation of the resistance to motion. This can be justified by the fact that the upward motion of the intruder is slow; in Figure 1-c for instance, the upward velocity is about $50d/100$, namely $\simeq 2.10^{-2}\sqrt{gD}$, so that inertial effects can be neglected. We suppose that a given intruder is alternatively submitted to force fluctuations $F^\uparrow$ and $F^\downarrow$ respectively in the upward and downward direction, such that $F^\uparrow \simeq -F^\downarrow$, and large compared to the weight of the intruder (as is the case in Figure 1). These forces are transmitted during an unknown time lapse Δt , so that the energy gained by the intruder during these events is $(F^\uparrow \Delta t)^2/m_I \simeq (F^\downarrow \Delta t)^2/m_I$, where m_I is the mass of the intruder. We suppose that this energy is dissipated by the work of resisting friction forces exerted by the surrounding granular bed on the intruder while it moves over the distance $\Delta z^\uparrow$ (respectively $\Delta z^\downarrow$) as a result of $F^\uparrow$ (respectively $F^\downarrow$). If the intruder is buried at a depth $(H - z_I)$ under the free surface, the resisting force takes the form $\mu_e \rho g (H - z_I)^\alpha D$, where both the effective coefficient of friction μ_e and the exponent α depend on whether the motion is upward or downward [41, 45]. For the sake of simplicity, we first assume $\alpha = 1$. Following [41–44], we consider two distinct values $\mu_e^\uparrow$ and $\mu_e^\downarrow$ for upward and downward motion respectively, such that $\mu_e^\uparrow < \mu_e^\downarrow$. Equating the energy dissipated by the friction forces $\mu_e^\uparrow \rho g (H - z_I) D \Delta z^\uparrow$ and $-\mu_e^\downarrow \rho g (H - z_I) D \Delta z^\downarrow$ with $(F^\uparrow \Delta t)^2/m_I \simeq (F^\downarrow \Delta t)^2/m_I$, we simply obtain $\Delta z^\uparrow/\Delta z^\downarrow = -\mu_e^\downarrow/\mu_e^\uparrow$. Since the ratio $\mu_e^\downarrow/\mu_e^\uparrow$ is of the order of 10, we immediately see how upward motion is favoured, and how an intruder would rapidly end up reaching the free surface. The result is less spectacular if instead of considering the simplified form for the resisting forces $\mu_e \rho g (H - z_I) D$, we adopt the full dependence given in [41] for spheres, namely $\mu_e^\uparrow \rho g (H - z_I)^{1.8} D^{-0.8}$ for upward motion with $\mu_e^\uparrow = 1.2$, and $\mu_e^\downarrow \rho g (H - z_I)^{1.2} D^{-0.2}$ for downward motion, with $\mu_e^\downarrow = 15$. In this case, for an intruder buried at a depth 10 times its diameter under the free surface, we find $\Delta z^\uparrow/\Delta z^\downarrow \simeq 2.46$, which seems more realistic, and would definitely lead to segregation too.

Our results show that the positive (upward) force resultant on intruders (or on the phase of large grains in the case of bi-disperse mixture) depends linearly on the intruder's volume, which coincide effectively with a more efficient segregation for larger grains. The results by [27] also report smaller resistance for larger intruders, and a discrepancy between upward and downward motion increasing with the intruder's size, which would alone explain why larger intruder's segregate better.

The works by Hill et al (2005) [41], Shröter et al (2007) [42], Martinez 2013 [43] and Li et al (2013) [44] were all considering static granular beds. Generalisation to granular flows hence requires additional work. However, there are no reason to suppose that the asymmetry to upward and downward motion does not hold in the case of dense flows. In this case, the granular temperature induced by the shear is expected to decrease the

overall resistance to motion, as reported in [47]. But the geometrical asymmetry formed by the boundary condition remains, so that the induced asymmetry on upward and downward resistance to bottom should not be suppressed.

In [43], plunging and withdrawing experiments were carried out while adding a weighting lid at the surface of the granular bed. The effect of this weighting lid is to impede grain rearrangements at the free surface. Accordingly, both withdrawing forces and plunging forces are greatly increased by its presence, yet the anisotropy of the resistance to motion in the upward and downward directions is preserved. Hence, segregation occurring in confined settings such as rotating drums, as that studied by Golick et al (2009) [48], do not contradict the line of argument developed in this paper. In [48], it is shown that increasing the pressure on the top lid results in a slower segregation process, which fits the observation by Martinez (2013) of an increasing resistance to motion in the granular packing.

In a different manner, the larger grains which are first segregated in a flowing bi-disperse flow form a lid at the top of the mixture (see Figure 5). Since the density of large and small grains is the same, this lid is not weighting. But because it involves larger grains and thus a lesser contact density (simply for geometrical reasons), we may suppose that it forms a less tractable free surface, thereby increasing the resistance to upward motion. If that was the case, this could partly explain why segregation saturates, leaving larger grains behind in the flow bulk. Rather than remixing, it could simply be that upward motion is increasingly difficult because of the lid of larger grains already covering the free surface. This would also account for the fact that segregation in three dimensions is much more efficient than is two dimensions, since grains reaching the free surface are often redirected in a different area of the flow (forming levées for instance), and are not given the opportunity to form a lid.

Segregation processes are often described in terms of the smaller grains having more chances to fill in the gap opening in the flow due to shear deformation [11]. This is indeed what one sees when watching the progress of a large grain in a flow of smaller ones: space opening in the wake of the large grain and closing behind it so it seems squeezed out. We explain this mechanism by the fact that the large grain, submitted to large force fluctuations, is allowed to cut its way through the matrix of smaller grains in the upward direction, thus leaving an empty space behind, while the equivalent in the downward direction is not true. More than the gravity, the existence of two different boundary conditions formed by the free surface and the rigid bottom explain this difference of resistance to motion. In this respect, the mechanism allowing the rising dynamics of larger grains in granular flows is the same than that allowing legged locomotion in sand [44], and bears little resemblance with its hydrodynamical counterpart.

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