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Faisal Islam, Sébastien Joannès, Anthony Bunsell, Lucien Laiarinandrasana

ADAPTATION OF WEIBULL ANALYSIS TO REPRESENT STRENGTH BEHAVIOUR OF BRITTLE FIBRES

Faisal Islam, Sébastien Joannès*, Anthony Bunsell and Lucien Laiarinandrasana

MINES ParisTech, PSL University, Centre des Matériaux (CMAT), CNRS UMR 7633, BP 87 91003 Evry Cedex, France
* Corresponding author (sebastien.joannes@mines-paristech.fr)

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ABSTRACT

Fibre properties are used for estimating the damage and strength behaviour of composite materials and structures, as the mechanical properties of fibre reinforced composites are primarily dependent on those of fibres. Fibre reinforced composites owe their outstanding properties to their constituents, mainly the fibres which reinforce matrices in composites. The accuracy of the predictions made by composite strength models would be highly dependent on the accuracy of the fibre strength distribution which is used as input. Any error in input fibre strength properties would lead to inaccurate model predictions. It has however been highlighted by many studies that there is discrepancy in experimental data even for the same type of fibres. The fibre strength data used to determine fibre strength distributions may be inaccurate. Due to the fibre preselection effect, experimental data is usually incomplete and does not appropriately represent the true fibre strength variation. An adaptation to the Weibull analysis has been presented to predict a more accurate statistical distribution for the representation of the strength behaviour of fibres inside composites. This analysis considers the preselection effect, which is used in modelling the fibre strength behaviour. The described analysis has the potential to significantly improve the accuracy of fibre strength distribution determined using experimentally generated fibre strength data.

1 INTRODUCTION

When fibres are used to reinforce polymeric matrices, it results in significant improvements of their mechanical behaviour. This provides benefits such as light weight, high strength to weight ratio, excellent weathering stabilities and enhanced dimensional stabilities to the resulting composite material. These properties make them ideal to be used for very critical structural applications such as pressure vessels and components for aerospace applications. Due to the widespread use of fibre reinforced composites, several computational models have been developed for predicting the behaviour of composite materials and structures. These models use constituent properties as inputs and predict the properties of composite materials and structures. The reliability of the model predictions depends on the quality of the input data used, mainly that of the properties of fibres, since they are the principal load bearing constituent of the composites. Thus, for failure modelling and to simulate the effective properties of composite products, detailed information about fibre tensile strength distribution is required. Due to the widespread use of fibre reinforced composites, it is very important to determine the characteristics of fibres and matrices and their interaction with each other in order to have a clear understanding of their effect on the composite material and the end-products. Correctly capturing fibre strength statistics and failure kinetics is also crucial to the success of composite stochastic strength models. The accuracy of the predictions made by composite strength models would be highly dependent on the accuracy of the fibre strength distribution which is used as input. Any error in input fibre strength properties would lead to inaccurate model predictions.

In practice, the characterization of fibre properties is known to be very challenging. Their very small cross-sections, morphological variations, or simply the sampling randomness and sample size used for analysis, directly affect determination of fibre properties especially tensile strength. They make the characterization uncertain and may lead to significant variations in results. Apart from the limited precision of measurement tools used, uncertainties could also arise from many other sources such as the inconsistency of the individual performing the measurement, choice of sample size used, sampling error, etc. The representativeness of the experimental fibre tensile strength data set that is used for determining a statistical distribution to characterize the tensile strength behaviour can be a major source of uncertainty in distribution parameters. Implementation of advanced data processing techniques on fibre strength results obtained from experiments may help in drawing attention to unknown limitations in the present system of analysing test results. It may also significantly enhance the reliability of the obtained results and help in extracting more information from the limited resources available.

Technical fibres which are commonly used as reinforcements in composite materials are usually brittle in nature. The stochastic distribution of defects inside them controls their strength. Representing the strength of such fibres thus requires a statistical approach. The most popularly used statistical function for representing strength of brittle fibres is the Weibull distribution. The parameters of this distribution represent the strength variation of fibres and are determined by appropriately fitting experimentally generated fibre strength data with this statistical function. Depending upon the type of fibre strength data, authors usually choose between the generic 3-parameter Weibull distribution, and a specific 2-parameter distribution obtained by fixing the value for one of the parameters. Using either of the two functions is associated with corresponding limitations which have been discussed in this article. It has often been observed that single fibre strength data that is experimentally generated does not conform to the variations of the standard Weibull functions, i.e. there is a misfit between the experimental data points and the Weibull model, on the graphical plot. To overcome this misrepresentation, many authors have proposed using different modified versions of Weibull distributions which have been shown to statistically fit the fibre strength experimental data well. Based on the goodness of fit, conclusions on the physical nature of the fibre strength behaviour have been made. This may have serious consequences as correlation may not necessarily mean causation.

The problem does not lie entirely with the statistical distribution but mainly with the data used for determining the distribution. The fibre strength data which is experimentally generated using the single fibre testing process is usually incomplete and does not accurately represent the strength of all the fibres present in composite materials and structures. This is due to the effect of fibre preselection, the causes of which will also be discussed in this article. To address this issue, a generalized Weibull analysis will be presented. This analysis takes into consideration the fibre preselection effect which would be analytically represented in the Weibull distribution. Analysing experimentally generated fibre strength data using the presented methodology can significantly enhance the accuracy of the results obtained and would help in extracting more useful information from the limited resources available. This would therefore also improve the reliability of the predictions made by composite strength models on the behaviour of composite materials and structures.

This work aims at finding an appropriate distribution that could represent the actual fibre strength behaviour accurately. This advanced knowledge may enable the design of new and improved composite materials and products with more accurate information about their strength behaviour.

2 FIBRE STRENGTH AND WEIBULL DISTRIBUTION

The strength of a single fibre cannot be captured in one single average value. Single fibres normally exhibit a distribution of strengths that could be represented by using a probabilistic approach**.** The statistical function that best describes strength behaviour of brittle fibres such as carbon is the Weibull distribution.

The Weibull distribution is considered to be a good representation of the weakest link behaviour of brittle single fibres. The classical Weibull distribution function for fibre strengths is given by Eq. 1.

$$P(\sigma) = 1\text{-}\ \exp\left\{-\left(\frac{L}{L_0}\right)\left(\frac{\sigma-\sigma_u}{\sigma_0}\right)^m\right\} \quad (1)$$

Where, $P(\sigma_f)$ is probability of fibre failure, L being the characteristic gauge length, L_0 the reference gauge length, σ the fibre strength, σ_u the threshold strength, σ_0 the scale parameter and m the shape parameter or Weibull modulus. Eq. 1 is also commonly known as the 3-parameter Weibull distribution. The value for σ_u is usually set to be zero and the resultant is known as a 2-parameter distribution (Eq. 2).

$$P(\sigma) = 1\text{-}\ \exp\left\{-\left(\frac{L}{L_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^m\right\} \quad (2)$$

Certain authors have speculated that multiple strength determining flaw populations may exist inside fibres and have recommended the use of a multimodal Weibull distributions to represent the strength of a fibre population. A power law accelerated Weibull distribution has also been proposed by some authors. It tries to represent the gauge length dependence of fibre strengths by including an additional parameter in the distribution. However, these models are mainly based on curve fitting and do not have a strong physical explanations.

3 EXPERIMENTATION AND RESULTS

T700 carbon fibres are very popularly used in fibre reinforced composites for structural applications and are brittle in nature. Single fibre tensile tests using T700 fibres were conducted to determine the tensile strength of the fibres. The procedure described in ASTM C1557-14 was followed for all tests. The fibre diameters were carefully measured using a laser diffraction system. The failure force was divided by the fibre cross-sectional area to determine the failure stress or strength of the fibre. The test was performed at the recommended crosshead speed of 1 mm/min. A tensile strength data set for 50 fibres was determined at a gauge length of 30 mm.

It is a very common practice to use the 2-parameter distribution to represent single fibre strengths and for deriving conclusions on the physical behaviour of fibres. However, when a sufficiently large single fibre strength data set is represented on a Weibull plot using a 2-parameter distribution, there is a clear misfit between the experimental points and the Weibull model, as shown in Fig. 1. This deviation can partially be explained using the fibre preselection effect, which means that the weak fibres break during the process of fibre extraction and testing. This has also been observed by other researchers. The weak data points are thus not represented in the data set used for analysis. This elimination of weak fibre strength values from the data set causes a deviation in linearity on the Weibull plot, as shown.

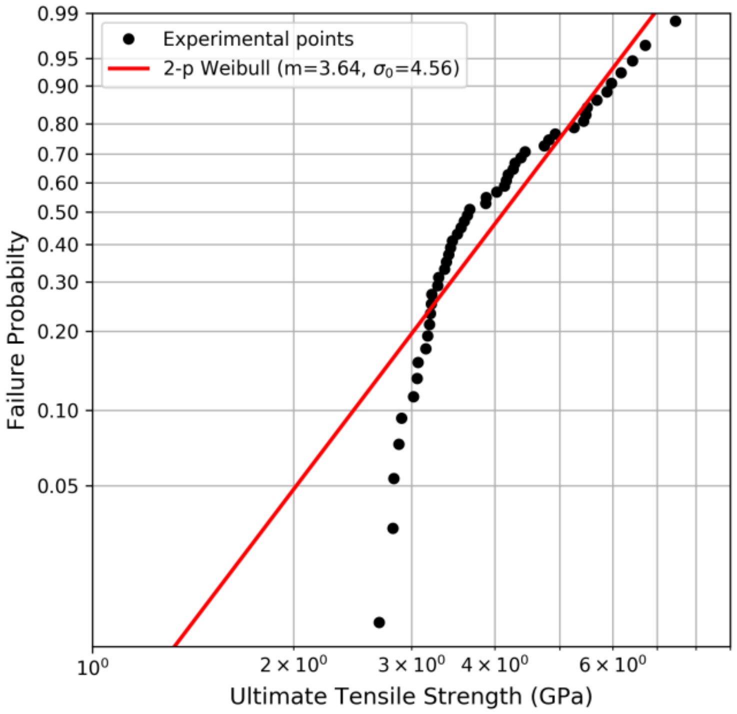

Figure. 1. Weibull plot with 50 fibre strength data points for T700 carbon fibres (Determined at Dia-Stron Ltd.).

The 2-parameter Weibull distribution is unable to capture this non-linear behaviour accurately. It may be tempting to use the 3-parameter Weibull distribution (Eq. 1) as it would be able to present a distribution that would fit the experimental data points very well. However, this distribution would assume that the minimum strength value in the experimental data set represents the weakest fibre in the population which is an incorrect assumption. It is not desirable to only obtain the best fit distribution but the objective is to find a representation of the actual strength behaviour of the fibres.

3 DISCUSSIONS

Using the theory of conditional probability, a generalized Weibull distribution can be derived. This distribution is capable of representing the preselected fibre strength data obtained from experiments. The generalized equation is given by Eq. 3.

$$P(\sigma) = 1\text{-}\exp\left\{-\left(\frac{L}{L_0}\right)\left(\frac{(\sigma-(1-\omega)\sigma_r)^m-(\sigma_r-(1-\omega)\sigma_r)^m}{\sigma_0{}^m}\right)\right\} \quad (3)$$

Where, σ_r is a truncation limit and ω is a constant. The quantities,σ_r, ω and the threshold strength σ_u are related by the empirical relationship given by Eq. 4. The value of ω can be determined by a user based on the observed behaviour of fibre strengths.

$$\sigma_u = (1-\omega)\sigma_r \quad (4)$$

For certain specific conditions, Eq. 3 simplifies to return the classical 2 and 3-parameter Weibull distributions. (Eqs. 1 and 2). The generalized Weibull distribution contemplates the incompleteness of the experimental data set and can further be used to determine an accurate representation of the actual fibre strength behaviour, otherwise not possible using the classical distributions.

4 CONCLUSIONS

The inadequacy of the popular statistical distributions for representing fibre strength has been highlighted and the experimental data set is analyzed statistically. This is due to the problems in experimental data set and not necessarily due to the inadequacy of the statistical distribution used.. Experimentally generated fibre tensile strength data sets are not always reliable representatives for the strength of the fibres inside a fibre bundle, composite material or a composite structure. Weaker fibres are unable to be tested due to the effect of fibre preselection. They are thus not represented in the experimentally determined data set. This results in a truncation in the obtained fibre strength data which also propagated into the resulting strength distribution. Analyzing this incomplete data with the standard Weibull functions may result in incorrect conclusions about the physical nature of the strength behavior of fibres. This error would also propagate into the predictions made by composite strength models which use the fibre strength information as input.

It is shown that inclusion of a truncation parameter in the Weibull distribution could help in estimating the strength behavior of a given fibre population. Inclusion of a user defined constant ω allows the experimental results for different types of fibres to be addressed. By varying ω, the effect of the fibre strength distribution on the composite strength models can also be simulated. Ideally, this distribution may also have been obtained if the entire fibre strength spectrum had been experimentally accessible. However, since that is practically impossible, the presented truncated Weibull analysis can be used to represent the actual strength behavior of fibres inside composite materials.

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