A Bayesian DSGE Model Comparison of the Taylor Rule and Nominal GDP Targeting
Ibrahima Amadou Diallo

To cite this version:
Ibrahima Amadou Diallo. A Bayesian DSGE Model Comparison of the Taylor Rule and Nominal GDP Targeting. 2019. hal-02281971

HAL Id: hal-02281971
https://hal.archives-ouvertes.fr/hal-02281971
Submitted on 9 Sep 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
A Bayesian DSGE Model Comparison of the Taylor Rule and Nominal GDP Targeting

Ibrahima Amadou Diallo *

September 9, 2019

Abstract

This paper performs a comparison of the Taylor Rule and Nominal GDP Targeting by estimating a DSGE model with Bayesian techniques. The first part builds a New Keynesian DSGE model with investment adjustment costs, prices and real wages rigidities, a government sector, and imperfect competition, alongside various shocks. The second part estimates and contrasts the models using Bayesian methods on Euro Area data. The results show that the data strongly prefer the Nominal GDP Targeting Rule over the Taylor Rule. We conduct numerous robustness checks to guarantee the solidity of our results. We also provide impulse response functions evaluation of the two Monetary Policy Rules.

*Centre d’Études et de Recherches sur le Développement International (CERDI), École d’Économie, Université Clermont Auvergne, 26 Avenue Léon-Blum, 63000 Clermont-Ferrand, France, Tel.: (33-4) 73 17 74 08, Fax: (33-4) 73 93 57 07. Author’s contact: zavren@gmail.com. All comments are welcome and, all remaining errors and inaccuracies are mine.
1 Introduction

The idea of Nominal GDP Targeting (NGDPT) was introduced in Economics by Meade (1978) and Tobin (1980). But since the 1990s, it was eclipsed by the Taylor Rule since the introduction of this latter notion by Taylor (1993). Ever since, most of the Central Banks in the Developed and Emerging Countries have implemented the Taylor Rule in one of its many variants as demonstrated by many of the DSGE models that are built in these institutions and the academic literature that followed. However, when the Great Recession of 2008 struck, many voices advocated for alternative Monetary Policy Rules and Practices. Among them, the Blog Posts by Scott Sumner attracted the support of many Economists and the Economic Press. He resuscitated the view point of NGDPT which is summarized in Sumner and Roberts (2018).

Motyovszki (2013) is one of the first works analyzing the concept of NGDPT in a DSGE model framework. He analyzes the issue in the setting of a New Keynesian DSGE model containing three shocks and no zero lower bound. His results show that Nominal GDP Level Targeting gives a steadier real economy than strict inflation targeting at the cost of higher inflation volatility. He also finds that Nominal GDP Level Targeting accomplishes better outcomes in terms of inflation and output gap volatility relative to a flexible Taylor Rule characterized by situations where inflation targets can momentarily be missed. In light of his results, he sums up by arguing that Nominal GDP Level Targeting might deserve to be taken as an alternative tool for Monetary Policy Analysis.

Benchimol and Fourcans (2016) employ a Bayesian method to estimate the Smets and Wouters (2007) DSGE model using nine distinct Monetary Policy Rules on a USA dataset from 1955 to 2015 and with three distinct sub-periods. Their results illustrate the supremacy of the Nominal GDP Level Targeting Rules compared to the Taylor Rules over all considered periods if we take into account only the loss function of the Central Bank. Nevertheless, they discover that the objectives of the Central Bank are not always satisfied by one rule for all the considered periods if we take other criteria into account.

Beckworth and Hendrickson (2016) utilize a New Keynesian DSGE model to suppose that the Central Bank has imperfect information concerning the output gap and consequently have to forecast this variable on the knowledge of past information. They stipulate that the forecast errors made by the Central Bank can possibly cause unexpected
variations of the short-run nominal interest rate that are different from a usual Monetary Policy shock. Their results illustrate that forecast errors made by the Federal Reserve can cause, at most, 13% of the changes in the output gap. Their findings also demonstrate that, in the context of imperfect information, a Nominal GDP Targeting Rule might yield lesser instability in both the output gap and inflation compared to a Taylor Rule.

Garin, Lester and Sims (2016) use a New Keynesian DSGE model with both price and wage rigidity to examine the welfare characteristics of Nominal GDP Targeting. They contrast a Taylor Rule, output gap and inflation targeting, and NGDPT. Output gap targeting is the most suitable rule. NGDPT is almost as good as output gap targeting. NGDPT is characterized by minor welfare losses than inflation targeting and a Taylor Rule. In the presence of supply shocks and when wages are rigid compared to prices, NGDPT beats a Taylor Rule and inflation targeting. If the output gap is observed with errors, NGDPT could leave behind output gap targeting. NGDPT possesses more wanted equilibrium determinacy characteristics than output gap targeting.

Similarly to the works cited above, this paper studies the concept of NGDPT in a New Keynesian DSGE model framework. Compared to previous works on NGDPT, it makes numerous contributions. First, our DSGE model formulation, that is to say the model specification, is different from previous studies on NGDPT. Second, we introduce and estimate real wages rigidities. Third, our model contains a government sector with a slightly different formulation. Fourth, the specification of our Nominal GDP Targeting Rule is different. Fifth, our work is the first to use the newly invented Hamilton Filter, Hamilton (2018)\(^1\), to compute the observable variables of our model. Sixth, this study is the first to perform a Bayesian DSGE Model Comparison of the Taylor Rule and Nominal GDP Targeting. Seventh, our research is the first to conduct a Bayesian DSGE Model Comparison of the Taylor Rule and Nominal GDP Targeting on Euro Area data. Our Bayesian DSGE Model Comparison results attribute a Posterior Model Probability of 0.00 to the Taylor Rule and a Posterior Model Probability of 1.00 to the Nominal GDP Targeting Rule. Our estimation results also give a Bayes Ratio of 1.00 to the Taylor Rule and a Bayes Ratio of $6.17 \times 10^{98}$ to the Nominal GDP Targeting Rule. These results demonstrate that

\(^1\)For the interested reader, I introduce a new Stata User-Written command named “\texttt{hamiltonfilter}” that Calculates the Hamilton Filter for a Single Time Series or for a Panel Dataset. The command is downloadable at: https://ideas.repec.org/c/boc/bocode/s458449.html. Please, see this website for more details.
the Nominal GDP Targeting Rule is overwhelmingly preferred by the Euro Area data than the Taylor Rule. We also ran numerous robustness checks that corroborate these results.

The remaining of the paper is organized in the following manner: the first section presents the theoretical model, the second section exposes the empirical investigations and the last part concludes.
2 Theoretical Model

In this section, we expose the theoretical model and illustrate how the main equations are obtained.

2.1 Households

The model presumes identical individuals, meaning that they have similar preference parameters. Therefore, we can employ the representative-agent hypothesis within which the analysis is done from the decisions of one agent. The household maximizes the expected value of the present value of his lifetime utility function subject to some constraints and the initial value of capital stock. His optimization program is given by:

\[
\text{Max} \quad E_0 \left( \sum_{t=0}^{\infty} \beta^t a_t \left( \frac{c_t^{1-\theta} - 1 - \zeta l_t^{1+\eta}}{1 - \theta} \right) \right)
\]

Subject to:

\[
c_t + iv_t + 1/2 \phi_k \left( \frac{iv_t}{k_t} - \delta \right)^2 k_t + \frac{B_t}{P_t r_t} = Q_t k_t + W_t l_t + B_{t-1} + D_t - \tau_t
\]

\[
k_{t+1} = (1 - \delta) k_t + x_t iv_t
\]

In the equations above, we have:

\[
\ln (a_t) = \rho_a \ln (a_{t-1}) + \varepsilon_{a,t}
\]

\[
\ln (x_t) = \rho_x \ln (x_{t-1}) + \varepsilon_{x,t}
\]

In expression (1), the household chooses bonds \( B_t \), consumption \( c_t \), investment \( iv_t \), next period physical capital stock \( k_{t+1} \) and labor \( l_t \) to maximize this objective function given the constraints he faces. Equation (2) says that the household gets his income from supplying capital \( Q_t k_t \), supplying labor \( W_t l_t \), bonds holding \( B_{t-1} \) and receiving dividends \( D_t \). His income is expressed in real term by dividing by the price level \( P_t \). To obtain his disposable income, he deducts his previous real income from lump-sum taxes \( \tau_t \). The household uses his disposable income to pay for consumption \( c_t \), investment \( iv_t \), investment adjustment costs \( 1/2 \phi_k \left( \frac{iv_t}{k_t} - \delta \right)^2 k_t \) and new bonds \( \frac{B_t}{P_t r_t} \), all expressed in real terms. The investment adjustment costs function employed in this study has been also

\[\text{The agent lives forever.}\]
utilized by Ireland (2003) and Roehe (2012). Equality (3) is the law of motion of capital stock. Equations (4) and (5) represent the intertemporal preference shock and the shock to the marginal efficiency of investment respectively. In equality (2), $r_t$ is the short-run nominal gross interest rate and in the objective function (1), $E_0(\cdot)$ is the Rational Expectations Operator using all available information. In the expressions above, we have the following conditions for the parameters and the remaining variables: $0 < \beta < 1; 0 < \theta; 0 \leq \eta; 0 < \zeta; 0 < \rho_a < 1; \varepsilon_{a,t} \sim \mathcal{N}(0, \sigma_a^2); 0 < \delta < 1; 0 < \rho_x < 1; \varepsilon_{x,t} \sim \mathcal{N}(0, \sigma_x^2)$.

The First-Order Conditions for the household problem give us the following equations with $\lambda_t$ and $\psi_t$, the Lagrange multipliers of equations (2) and (3) respectively:

First-Order Conditions with respect to $c_t$:

$$\frac{a_t}{c_t^{1-\theta}} - \lambda_t = 0 \quad (6)$$

First-Order Conditions with respect to $l_t$:

$$-a_t \zeta l_t^{\eta} + \frac{\lambda_t W_t}{P_t} = 0 \quad (7)$$

Combining equations (6) and (7), allows us to write the equation for real wages rigidity as it is done in Blanchard and Gali (2007) and Ascari and Rossi (2011) with $0 \leq \gamma < 1$.

$$\frac{W_t}{P_t} = (\frac{W_{t-1}}{P_{t-1}})^\gamma \left( c_t^{1-\theta} l_t^{\eta} \right)^{1-\gamma} \quad (8)$$

First-Order Conditions with respect to $B_t$:

$$-\frac{\lambda_t}{P_t r_t} + E_t \left( \beta \frac{\lambda_{t+1}}{P_{t+1}} \right) = 0 \quad (9)$$

First-Order Conditions with respect to $k_{t+1}$:

$$-\psi_t - E_t \left( \frac{1}{2} \left( \left( \delta^2 P_{t+1} \phi_k - 2 Q_{t+1} \right) k_{t+1}^2 - P_{t+1} \psi_{t+1}^2 \phi_k \right) \beta \lambda_{t+1} \right) - E_t \left( \beta \psi_{t+1} (\delta - 1) \right) = 0 \quad (10)$$

First-Order Conditions with respect to $i_0 t$:
\[
\lambda_t \left( \delta k_t \phi_k - iv_t \phi_k - k_t \right) \frac{1}{k_t} + \psi_t x_t = 0 \quad (11)
\]

2.2 Firms

In this section, we will analyze the decisions of the final good firm and the intermediate goods firms.

2.2.1 Final Good Firm

The profit maximization problem of the final good firm is given by:

\[
\text{Max}_{y(t)} \Pi_t = \int_0^1 (y_t (i)) \frac{\nu - 1}{\nu} \, di - \int_0^1 P_t (i) y_t (i) \, di \quad (12)
\]

In equation (12), \( y_t = \left( \int_0^1 (y_t (i)) \frac{\nu - 1}{\nu} \, di \right)^{\frac{1}{\nu}} \) is the final good, \( y_t (i) \) are the differentiated intermediate goods and \( P_t (i) \) are the prices of the intermediate goods. We have \( 1 < \nu \). Maximizing expression (12) with respect to \( y_t (i) \), doing lots of simplifications and substitutions, we find:

\[
y_t (i) = y_t \left( \frac{P_t (i)}{P_t} \right)^{-\nu} \quad (13)
\]

\[
P_t = \left( \int_0^1 (P_t (i))^{-\nu + 1} \, di \right)^{\frac{1}{-\nu + 1}} \quad (14)
\]

Equation (13) means that the demand for the intermediate good \( i \), \( y_t (i) \), is proportional to the final good \( y_t \) and is a function of its relative price \( \frac{P_t (i)}{P_t} \), where \( \nu \) is the price elasticity of demand. Equation (14) indicates that the final good price \( P_t \) is a constant elasticity of substitution (CES) aggregator function of the prices of the intermediate goods \( P_t (i) \).

2.2.2 Intermediate Goods Firms

The optimization problem of the intermediate goods firms is:

\[
\text{Max}_{\{P_t (i), k_t (i), \lambda_t (i)\}_{i=0}^\infty} \mathbb{E}_0 \left( \sum_{t=0}^{\infty} \beta^t \lambda_t D_t (i) \right) \quad (15)
\]

Subject to:
\[ y_t(i) = (k_t(i))^{\alpha} (z_t d_t(i))^{1-\alpha} \]  \hspace{1cm} (16)

\[ y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\nu} y_t \]  \hspace{1cm} (17)

In the equations above, we have:

\[ \frac{D_t(i)}{P_t} = \frac{P_t(i) y_t(i)}{P_t} - \frac{W_d(i)}{P_t} - \frac{1}{2} \phi_P \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 y_t \]  \hspace{1cm} (18)

\[ \ln (z_t) = (1 - \rho_z) \ln (z) + \rho_z \ln (z_{t-1}) + \epsilon_{z,t} \]  \hspace{1cm} (19)

In expression (15), the intermediate goods firm \( i \) chooses price \( P_t(i) \), capital stock \( k_t(i) \) and labor \( l_t(i) \) to maximize this objective function given the constraints it faces. Equation (18) are the dividends in real terms. In this equality, \( 1/2 \phi_P \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 y_t \) are the quadratic adjustment costs of the nominal price \( P_t(i) \). This concept was introduced by Rotemberg (1982). Equation (16) is the Cobb-Douglas production function of the intermediate goods firm \( i \) and equality (17) represents the demand for the intermediate good \( i \). The technology shock is provided by equation (19). In the expressions above, we have the following conditions for the parameters and the remaining variables: \( 0 < \alpha < 1; \ 0 \leq \phi_P; 0 < z_t; 0 < \rho_z < 1; \epsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2) \).

The First-Order Conditions for the intermediate goods firm \( i \) problem give us the following equations with \( \delta_t \), the Lagrange multiplier of equation (16), after substituting equality (17) in equations (16) and (18) respectively:

First-Order Conditions with respect to \( l_t(i) \):

\[- \frac{\lambda_t W_t}{P_t} + \delta_t (k_t(i))^{\alpha} z_t (z_t d_t(i))^{-\alpha} (1 - \alpha) = 0 \]  \hspace{1cm} (20)

First-Order Conditions with respect to \( k_t(i) \):

\[- \frac{\lambda_t Q_t}{P_t} + \delta_t (k_t(i))^{-1+\alpha} \alpha z_t l_t(i) (z_t d_t(i))^{-\alpha} = 0 \]  \hspace{1cm} (21)

First-Order Conditions with respect to \( P_t(i) \):

\[- \frac{\lambda_t y_t}{P_t P_{t-1}(i)^2} \left( \pi^2 (P_{t-1}(i))^2 (v - 1) \left( \frac{P_t(i)}{P_t} \right)^{-v} - P_t \phi_P (\pi P_{t-1}(i) - P_t(i)) \right) \]

\[ + \delta_t y_t \left( \frac{P_t(i)}{P_t} \right)^{-v} + \mathbb{E}_t \left( \frac{\beta \lambda_{t+1} \phi_P (-\pi P_t(i) + P_{t+1}(i)) y_{t+1} P_{t+1}(i)}{\pi^2 (P_t(i))^3} \right) = 0 \]  \hspace{1cm} (22)
First-Order Conditions with respect to $\vartheta_t$:

$$\left(\frac{P_t(i)}{P_{t-1}}\right)^{-\gamma} y_t + (k_t(i))^\alpha (z_t l_t(i))^{1-\alpha} = 0 \quad (23)$$

### 2.3 Government

The Government (Fiscal Authority) runs a balanced budget in each period. That is, we have:

$$\tau_t = g_t \quad (24)$$

Where $\tau_t$ are the lump-sum taxes and $g_t$ are the General Government final consumption expenditures. The Government consumes in each period, a stochastic share of output. Thus, we have:

$$g_t = \xi_t y_t \quad (25)$$

Where $y_t$ is output and $\xi_t$ is the shock to Government expenditures given by the following equation:

$$\ln (\xi_t) = (1 - \rho \xi) \ln (\xi) + \rho \xi \ln (\xi_{t-1}) + \epsilon_{\xi,t} \quad (26)$$

In this last equation, the following conditions stand for the parameters and the remaining variables: $0 < \xi_t; 0 < \rho \xi < 1; \epsilon_{\xi,t} \sim \mathcal{N} \left(0, \sigma_{\xi}^2 \right)$. Although, there is no Fiscal Authority for the Euro Area as a hole, we use the setting above as an approximate representation of the Stability and Growth Pact (SGP) Agreement in Europe. This assumption of budget equilibrium has also been used by Adolfson, Laséen, Lindé and Villani (2007) in a DSGE model for the Euro Area albeit with a different specification.

### 2.4 Monetary Authority

In this section, we set the two Monetary Policy Rules that we will compare: a modified Taylor Rule, as in Roehe (2012) and the references therein, and a Nominal GDP Level Targeting Rule. The Nominal GDP Level Targeting Rule specification has not been used
The modified Taylor Rule is given by the following equations:

\[
\ln \left( \frac{r_t}{r} \right) = \rho rtr \ln \left( \frac{r_{t-1}}{r} \right) + (1 - \rho rtr) \left( \phi_{ntr} \ln \left( \frac{\pi_t}{\pi} \right) + \phi_{ytr} \ln \left( \frac{y_t}{y} \right) \right) + \ln (\mu_{tr,t}) \tag{27}
\]

\[
\ln (\mu_{tr,t}) = \rho_{\mu tr} \ln (\mu_{tr,t-1}) + \xi_{\mu tr,t} \tag{28}
\]

Equation (27), says that the Central Bank progressively changes the short-run nominal gross interest rate \( r_t \) in reaction to deviations of current gross inflation \( \pi_t \) and output \( y_t \) from their respective steady state amounts. Equality (28) is the Taylor Rule Monetary Policy shock process. In these two equations above, we have the following conditions for the parameters and the remaining variables: \( 0 < \rho_{rtr} < 1; 0 < \phi_{ntr}; 0 \leq \phi_{ytr}; 0 < \rho_{\mu tr} < 1; \xi_{\mu tr,t} \sim N \left( 0, \sigma^2_{\mu tr} \right) \). The Nominal GDP Level Targeting Rule is provided by the following equations:

\[
\ln \left( \frac{r_t}{r} \right) = \rho_{rng} \ln \left( \frac{r_{t-1}}{r} \right) + (1 - \rho_{rng}) \phi_{fng} \ln \left( \frac{F_t}{F} \right) + \ln (\mu_{ng,t}) \tag{29}
\]

\[
\ln (\mu_{ng,t}) = \rho_{\mu ng} \ln (\mu_{ng,t-1}) + \xi_{\mu ng,t} \tag{30}
\]

\[
F_t = P_t y_t \tag{31}
\]

Equation (29), says that the Central Bank progressively changes the short-run nominal gross interest rate \( r_t \) in reaction to deviations of current Nominal GDP \( F_t \) from its steady state amount. Equality (30) is the Nominal GDP Targeting Rule Monetary Policy shock process. Equation (31) defines the Nominal GDP Level \( F_t \) as the product of the GDP Deflator \( P_t \) and Real GDP \( y_t \). In these equations above, we have the following conditions for the parameters and the remaining variables: \( 0 < \rho_{rng} < 1; 0 < \phi_{fng}; 0 < \rho_{\mu ng} < 1; \xi_{\mu ng,t} \sim N \left( 0, \sigma^2_{\mu ng} \right) \).

2.5 Equilibrium Conditions of the Models

To find the equilibrium conditions of the two models, we first invoke the symmetric equilibrium statement in which all Intermediate Goods Firms make similar choices. Then, we apply the market clearing hypothesis on the goods and bond markets. Finally, we express all nominal quantities in real terms by dividing by the appropriate price.
After this, we compute the steady states of the models\(^3\) and log-linearize the models. The log-linearized equations of the two models are given in appendix A for completeness purposes.

3 Empirical Investigations

This section presents the estimation methods, the data and variables, and the econometric results.

3.1 Estimation Methods

Following Koop (2003), we will briefly summarize the concepts of Bayesian Estimation and Bayesian Model Comparison. The Bayesian estimation of our DSGE model can be written as:

\[
p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}
\]  

(32)

Where \(\theta\) are the parameters of our DSGE model; \(y\) are the data; \(p(\theta|y)\) is the posterior density; \(p(y|\theta)\) is the likelihood function; \(p(\theta)\) is the prior density and \(p(y)\) is the marginal distribution of \(y\). Since \(p(y)\) is not a function of the parameters \(\theta\), we can express equation (32) as:

\[
p(\theta|y) \propto p(y|\theta)p(\theta)
\]  

(33)

Equation (33) is essential in Bayesian estimation. It says that the posterior distribution of model parameters is proportional to the likelihood function times the prior probability distribution.

For Bayesian Model Comparison, suppose that we want to compare \(r\) models \(M_i\) with parameters \(\theta_i, i = 1, \ldots, r\). Using Bayes’s rule, we can write the posterior model probability as:

\[
p(M_i|y) = \frac{p(y|M_i)p(M_i)}{p(y)}
\]  

(34)

\(^3\)The steady states of the models are available upon request.
Where \( p(M_i | y) \) is the posterior model probability; \( p(y | M_i) \) is the marginal likelihood or the marginal density or the marginal data density; \( p(M_i) \) is the prior model probability and \( p(y) \) is the marginal distribution of \( y \). Given that \( p(y) \) is tedious to compute, we can usually compare two models, \( M_i \) and \( M_j \), by calculating the ratio of their posterior model probabilities which gives us the posterior odds ratio \( PO_{ij} \), defined as:

\[
PO_{ij} = \frac{p(M_i | y)}{p(M_j | y)} = \frac{p(y | M_i)p(M_i)}{p(y | M_j)p(M_j)} \tag{35}
\]

If \( \frac{p(M_i)}{p(M_j)} = 1 \) or \( p(M_i) = p(M_j) \), the posterior odds ratio reduces to the ratio of marginal likelihoods. It then takes a specific designation called the Bayes factor or the Bayes ratio \( BF_{ij} \), as described by the following formula:

\[
BF_{ij} = \frac{p(y | M_i)}{p(y | M_j)} \tag{36}
\]

Bayesian computation of DSGE models involves calculating integrals. In most cases, these integrals do not have a closed-form analytical solution. To solve this problem, we turn to simulation methods like the Markov Chain Monte Carlo (MCMC) Metropolis-Hastings (MH) Algorithm to get random draws from the posterior distribution. To perform the Bayesian computations of our DSGE models, we employ the software Dynare as described in Adjemian, Bastani, Juillard, Karamé, Maih, Mihoubi, Perendia, Pfeifer, Ratto and Villemot (2011).

### 3.2 Data and Variables

For our estimations and models comparison exercises, we employ seasonally adjusted quarterly Euro Area data from 1987Q1 to 2007Q4. We focus on the Great Moderation Period in the Euro Area to avoid complications arising from the other high volatility periods of the business cycle. All the data come from the Area-Wide Model (AWM) database (Fagan, Henry and Mestre (2005)). The parameters that characterize the steady-sates of the two models are calibrated and taken from the literature. The remaining parameters are estimated with their prior values also taken from the literature. Following the tradition of DSGE model estimation in Dynare, the priors for the estimated parameters
are selected according to the subsequent guidelines: gamma distribution for parameters that are required to be positive; inverse gamma distribution for the standard deviation of the shocks; beta distribution for parameters that must be between 0 and 1, and normal distribution for all the remaining parameters. We have five observed endogenous variables: real GDP, real investment, inflation, short-term interest rate and real wages. Real GDP and real investment are computed in per capita values. All variables are logged using the natural logarithm and filtered utilizing the Hamilton Filter (Hamilton (2018)). Hamilton (2018) gives the criticisms of the Hodrick-Prescott Filter and explains why the Hamilton Filter is a superior alternative. The use of the Hamilton Filter is one of the main contributions of the current paper because it has not been employed before in all the literature of DSGE modeling. The issue of stochastic singularity is circumvented in our case because the number of observed variables is equal to the number of structural shocks in our DSGE models.

3.3 Bayesian Estimation Results

In this part, we will present the main estimation results, the Bayesian DSGE model comparison results, the impulse response functions study and the robustness analysis.

3.3.1 Main Estimation Results

Figures 1 and 2 show the multivariate convergence diagnostic of the MH sampling algorithm for the Taylor Rule and the Nominal GDP Targeting Rule respectively. The results are synthesized in three graphics panels, where each panel exhibits a particular convergence measurement and containing two different lines representing the results within and between chains. These measurements are associated to the investigation of the parameters first central moments (indicated by interval), the parameters second central moments (indicated by m2) and the parameters third central moments (indicated by m3). In each of the three graphics panels, in order to obtain good results, the two lines ought to stabilize horizontally and must be near to each other. For the two figures representing the two Monetary Policy Rules, we observe that general convergence is

\footnote{Univariate convergence diagnostics are available upon request, but are not reported due to space constraints.}
accomplished, for all the three moments under examination, both within and between chains. But we notice that the MH sampling algorithm for the Nominal GDP Targeting Rule have converged more than the Taylor Rule. This, because the Taylor Rule takes longer to converge than the Nominal GDP Targeting Rule.

Figures 3, 4 and 5 display the priors and posteriors for the parameters of the Taylor Rule while Figures 6 and 7 exhibit the priors and posteriors for the parameters of the Nominal GDP Targeting Rule. In each of these graphics, the green vertical line designates the posterior mode resulting from the maximization of the posterior kernel. The black line specifies the posterior distribution whereas the grey line represents the prior probability density function. For the two Monetary Policy Rules, except few parameters, we observe that for most of the parameters, the mode resulting from maximization coincides with the mode of the posterior distribution obtained from the MH algorithm. For the majority of parameters, we notice that the shapes of the prior and posterior distributions are not excessively distant from each other, for the two Monetary Policy Rules. For these latter Rules, we see that the outcomes are not exclusively prior driven, because the prior and posterior distributions are practically different for most parameters. This put forward that the observed data do offer supplementary information in updating the prior information in our Bayesian estimations. The patterns of the posterior distributions are nearly normal, conforming with the asymptotic properties of Bayesian estimation.

Table 1 gives the Bayesian estimation results for the Taylor Rule. The first column of the table gives the parameters, the second big column exhibits the information on the prior distribution: type, mean and standard deviation. The third big column provides the information on the posterior distribution: mode, standard deviation, mean and 90% highest posterior density (HPD) interval. The bottom of the table exposes information about additional statistics. We performed 100000 draws to ensure convergence of the MH Algorithm. We obtained a Log Marginal Data Density of 499.949. The Acceptance Rate per chain are also reasonable because they both do not exceed 50% and are not too small. In addition, we checked that the rank condition is verified. All coefficients are statistically significant, are in the intervals in which they were supposed to be as set in the theoretical model, are plausible and moreover keep their expected signs. Most of the parameters are estimated with a high degree of precision because the standard deviations of their
posterior distributions are very small. The output elasticity of capital $\alpha$ is greater than the value of 0.333 typically used in calibrated DSGE models. This suggests that $\alpha$ is bigger in the Euro Area. The implied Frisch intertemporal elasticity of substitution in labor supply $1/\eta = 2.031$ is greater than 1. Real wages are mildly rigid in the Euro Area according to the Taylor Rule model as illustrated by the value of $\gamma$. The implied constant intertemporal elasticity of substitution in consumption $1/\theta = 0.405$ is smaller than 1 and is closer to the value employed in the Economic Growth literature. The value of the price adjustment cost parameter $\phi_P$ suggests that nominal prices are rigid in the Euro Area and this result is near to what have been found in the DSGE models literature. Turning to Monetary Policy parameters, we observe that the smoothing parameter of the nominal interest rate $\rho_{rtr}$ is very small. We notice that the European Central Bank respond aggressively to an increase in output and inflation relative to their steady state values as suggested by the quantities of $\phi_{ytr}$ and $\phi_{ntr}$ respectively. The value of the investment adjustment costs parameter $\phi_k$ indicates that there are investment adjustment costs in the Euro Area. The autocorrelation parameters for the shock processes of the Monetary Policy Rule shock $\rho_{\mu tr}$, the intertemporal preference shock $\rho_a$, the shock to Government expenditures share $\rho_\xi$ and the technology shock $\rho_z$ are all very high, illustrating that the corresponding shock processes are very persistent. Contrarily, the shock process to the marginal efficiency of investment is not persistent as shown by the value of $\rho_x$. The previous results for the autocorrelation parameters also demonstrate a sign of the nonexistence of unit roots in these processes. Finishing with the estimated standard deviations, we observe that none of the shocks processes are too volatile.

Table 2 gives the Bayesian estimation results for the Nominal GDP Targeting Rule. The first column of the table gives the parameters, the second big column exhibits the information on the prior distribution: type, mean and standard deviation. The third big column provides the information on the posterior distribution: mode, standard deviation, mean and 90% highest posterior density (HPD) interval. The bottom of the table exposes information about additional statistics. We performed 100000 draws to ensure convergence of the MH Algorithm. We obtained a Log Marginal Data Density of 643.176. The Acceptance Rate per chain are also reasonable because they both do not exceed 50% and are not too small. In addition, we checked that the rank condition is
verified. All coefficients are statistically significant, are in the intervals in which they were supposed to be as set in the theoretical model, are plausible and moreover keep their expected signs. Most of the parameters are estimated with a high degree of precision because the standard deviations of their posterior distributions are very small. The output elasticity of capital $\alpha$ is greater than the value of 0.333 typically used in calibrated DSGE models. This suggests that $\alpha$ is bigger in the Euro Area. The implied Frisch intertemporal elasticity of substitution in labor supply $1/\eta = 2.134$ is greater than 1. Real wages are rigid in the Euro Area according to the Nominal GDP Targeting Rule model as illustrated by the value of $\gamma$. The implied constant intertemporal elasticity of substitution in consumption $1/\theta = 0.398$ is smaller than 1 and is closer to the value employed in the Economic Growth literature. The value of the price adjustment cost parameter $\phi_P$ suggests that nominal prices are rigid in the Euro Area and this result is near to what have been found in the DSGE models literature. Turning to Monetary Policy parameters, we observe that the smoothing parameter of the nominal interest rate $\rho_{\text{mg}}$ is very small. We notice that the European Central Bank respond aggressively to an increase in Nominal GDP relative to its steady state value as suggested by the quantity of $\phi_{\text{mg}}$. The value of the investment adjustment costs parameter $\phi_k$ indicates that there are investment adjustment costs in the Euro Area. The autocorrelation parameters for the shock processes of the Monetary Policy Rule shock $\rho_{\text{mg}}$, the intertemporal preference shock $\rho_\eta$, the shock to Government expenditures share $\rho_\xi$ and the technology shock $\rho_z$ are all very high, illustrating that the corresponding shock processes are very persistent. Contrarily, the shock process to the marginal efficiency of investment is not persistent as shown by the value of $\rho_x$. The previous results for the autocorrelation parameters also demonstrate a sign of the nonexistence of unit roots in these processes. Finishing with the estimated standard deviations, we observe that none of the shocks processes are too volatile.

### 3.3.2 Bayesian DSGE Model Comparison Results

Table 3 gives the Bayesian model comparison results with equal prior probability distribution to the two monetary policy rules models. The first column shows information on the following statistics: Priors, Log Marginal Density, Bayes Ratio and Posterior Model Probability. The second column displays the statistics for the Taylor Rule while the third
column exhibits the statistics for the Nominal GDP Targeting Rule. In this table, we have equal priors for the two competing models. The log marginal density or the log marginal likelihood or the log marginal data density for the Nominal GDP Targeting Rule is larger than that of the Taylor Rule. The Bayes ratio or the Bayes factor demonstrates that there is extreme evidence for the Nominal GDP Targeting Rule. The posterior model probability for the Nominal GDP Targeting Rule is 1.000 and the posterior model probability for the Taylor Rule is 0.000. This illustrates that the Nominal GDP Targeting Rule have more chance of occurring than the Taylor Rule. All these statistics previously examined go in the same direction. They all show that the Nominal GDP Targeting Rule is overwhelmingly preferred by the Euro Area data than the Taylor Rule.

Table 4 gives the Bayesian model comparison results with a bigger prior probability attributed to the Taylor Rule than the Nominal GDP Targeting Rule. The first column shows information on the following statistics: Priors, Log Marginal Density, Bayes Ratio and Posterior Model Probability. The second column displays the statistics for the Taylor Rule while the third column exhibits the statistics for the Nominal GDP Targeting Rule. In this table, we have a bigger prior probability attributed to the Taylor Rule model than the Nominal GDP Targeting Rule model. The log marginal density or the log marginal likelihood or the log marginal data density for the Nominal GDP Targeting Rule is larger than that of the Taylor Rule. The Bayes ratio or the Bayes factor demonstrates that there is extreme evidence for the Nominal GDP Targeting Rule. The posterior model probability for the Nominal GDP Targeting Rule is 1.000 and the posterior model probability for the Taylor Rule is 0.000. This illustrates that the Nominal GDP Targeting Rule have more chance of occurring than the Taylor Rule. All these statistics previously examined go in the same direction. They all show that the Nominal GDP Targeting Rule is overwhelmingly preferred by the Euro Area data than the Taylor Rule.

Table 5 gives the Bayesian model comparison results with a bigger prior probability attributed to the Nominal GDP Targeting Rule than the Taylor Rule. The first column shows information on the following statistics: Priors, Log Marginal Density, Bayes Ratio and Posterior Model Probability. The second column displays the statistics for the Taylor Rule while the third column exhibits the statistics for the Nominal GDP Targeting Rule. In this table, we have a bigger prior probability attributed to the Nominal GDP Targeting
Rule model than the Taylor Rule model. The log marginal density or the log marginal likelihood or the log marginal data density for the Nominal GDP Targeting Rule is larger than that of the Taylor Rule. The Bayes ratio or the Bayes factor demonstrates that there is extreme evidence for the Nominal GDP Targeting Rule. The posterior model probability for the Nominal GDP Targeting Rule is 1.000 and the posterior model probability for the Taylor Rule is 0.000. This illustrates that the Nominal GDP Targeting Rule have more chance of occurring than the Taylor Rule. All these statistics previously examined go in the same direction. They all show that the Nominal GDP Targeting Rule is overwhelmingly preferred by the Euro Area data than the Taylor Rule.

### 3.3.3 Impulse Response Functions Study

Figure 8 provides the impulse response functions to Monetary Policy Shocks where the estimated parameters are updated to the posterior mean. The red line shows the x-axis, the green dashed line represents the Taylor Rule and the blue solid line designates the Nominal GDP Targeting Rule. We only exhibit the impact on the following variables of interest: output, consumption, investment, labor, interest rate, inflation, real wage, government spending and the shock processes for the two Monetary Policy Rules. A positive temporary shock to Monetary Policy cause the Taylor Rule shock process to increase more than that of the Nominal GDP Targeting Rule shock process. This is why the impact on all the remaining variables of interest is larger for the Taylor Rule than the Nominal GDP Targeting Rule. A positive Monetary Policy shock engenders a fall in all the remaining variables of interest for both Rules. We observe that the effects of the two Monetary Policy Rules go in the same directions, although the impact of the Taylor Rule is more pronounced than that of the Nominal GDP Targeting Rule as previously mentioned. This illustrates that our specification of the Nominal GDP Targeting Rule is not wrong because our specification is capable of giving similar results as the already well established Taylor Rule. All the variables of interest return to their steady state equilibrium after some time for the two Monetary Policy Rules, strengthening the statement given by the rank and the Blanchard-Kahn conditions that the models are definitely stable.
3.3.4 Robustness Analysis

Table 6 gives the Robustness of the Bayesian model comparison results with equal prior probability distribution to the two monetary policy rules models. To obtain these results, we have changed the prior probability distribution of the parameter $\phi_{\text{ng}}$ from a Gamma distribution to a Normal distribution. As in the main results tables, here also we observe that all the statistics go in the same direction. They all show that the Nominal GDP Targeting Rule is overwhelmingly preferred by the Euro Area data than the Taylor Rule.

Tables 7 and 8 provides the Robustness of the Bayesian model comparison results with a bigger prior probability attributed to the Taylor Rule than the Nominal GDP Targeting Rule and a bigger prior probability attributed to the Nominal GDP Targeting Rule than the Taylor Rule respectively. In all the previous Bayesian model comparison results tables, the calculations were based on the Laplace approximation. In tables 7 and 8, the computations are based on the Modified Harmonic Mean Estimator. As in the main results tables, here also we observe that all the statistics go in the same direction for tables 7 and 8. They all show that the Nominal GDP Targeting Rule is overwhelmingly preferred by the Euro Area data than the Taylor Rule.
4 Conclusion

In this paper, we carry out a comparison of the Taylor Rule and Nominal GDP Targeting by estimating a DSGE model with Bayesian techniques. The theoretical part builds a New Keynesian DSGE model with investment adjustment costs, prices and real wages rigidities, a government sector, and imperfect competition, alongside various shocks. The empirical part estimates and contrasts the models using Bayesian methods on Euro Area data. Our Bayesian DSGE Model Comparison results attribute a Posterior Model Probability of 0.00 to the Taylor Rule and a Posterior Model Probability of 1.00 to the Nominal GDP Targeting Rule. Our estimation results also give a Bayes Ratio of 1.00 to the Taylor Rule and a Bayes Ratio of $6.17 \times 10^9$ to the Nominal GDP Targeting Rule. These results demonstrate that the Nominal GDP Targeting Rule is overwhelmingly preferred by the Euro Area data than the Taylor Rule. We also ran numerous robustness checks that corroborate these results.

Though the results found were informative, some extensions could be made. First, we compare only two models instead of many models. Second, it would be good to extend our study using USA data in addition to Euro Area data. These avenues of research are left for our future studies.

From economic policy perspectives, the results illustrate that Nominal GDP Targeting is strongly supported by the data. Hence, it represents a viable and solid alternative to the Taylor Rule and it should be considered by Central Bankers around the World.
References

URL: https://ideas.repec.org/p/cpm/dynare/001.html


Ireland, P. N.: 2003, Endogenous Money or Sticky Prices?, Journal of Monetary Economics 50(8), 1623–1648.


Sumner, S. and Roberts, E.: 2018, The Promise of Nominal GDP Targeting, Mercatus Center, MERCATUS POLICY PRIMER.


A The Log-Linearized Models Equations

Log-linearization is a mean of transforming the stochastic dynamic nonlinear equations to stochastic dynamic linear equations. This makes the calculations and interpretations easier. A hat over a variable approximately indicates percentage deviation of the variable around the steady-state.

The log-linearized equations of the two models are given by:

\[
\dot{x}_t = \rho_x \dot{x}_{t-1} + \varepsilon_{x,t} \\
\dot{a}_t = \rho_a \dot{a}_{t-1} + \varepsilon_{a,t} \\
\dot{z}_t = \rho_z \dot{z}_{t-1} + \varepsilon_{z,t} \\
\dot{\mu}_{ng,t} = \rho_{\mu ng} \dot{\mu}_{ng,t-1} + \varepsilon_{\mu ng,t} \\
\dot{\mu}_{tr,t} = \rho_{\mu tr} \dot{\mu}_{tr,t-1} + \varepsilon_{\mu tr,t} \\
\dot{\xi}_t = \rho_\xi \dot{\xi}_{t-1} + \varepsilon_{\xi,t} \\
\dot{\lambda}_t = -\theta \dot{c}_t + \dot{a}_t \\
\dot{\omega}_t = (\eta \gamma + \eta) \dot{l}_t + (\gamma \theta + \theta) \dot{c}_t + \gamma \dot{\omega}_{t-1} - \dot{\lambda}_t + \dot{\pi}_t + \mathbb{E}_t(\dot{\lambda}_{t+1}) - \mathbb{E}_t(\dot{\pi}_{t+1}) = 0 \\
\frac{\mathbb{E}_t(\dot{\lambda}_{t+1})(1 + (-1 + \delta) \beta)}{\beta} + \delta^2 \phi_k \mathbb{E}_t(\dot{\nu}_{t+1}) - \delta^2 \phi_k \dot{k}_{t+1} \\
- (-1 + \delta) \mathbb{E}_t(\dot{\psi}_{t+1}) + \frac{\mathbb{E}_t(\dot{q}_{t+1})(1 + (-1 + \delta) \beta)}{\beta} - \frac{\dot{\psi}_t}{\beta} = 0 \\
- \delta \dot{\psi}_t \phi_k + \delta \phi_k \dot{k}_t + \dot{\psi}_t x + x \dot{x}_t - \dot{\lambda}_t = 0 \\
y \dot{\gamma}_t = k \delta \dot{\nu}_t + c \dot{c}_t + g \dot{g}_t \\
\dot{g}_t = \dot{\xi}_t + \dot{\gamma}_t \\
\dot{k}_{t+1} = (1 - \delta) \dot{k}_t + x \delta \dot{\nu}_t + x \delta \dot{x}_t \\
d \dot{d}_t = -k k_d q - k q \dot{q}_t - l \dot{w} + l \dot{w} \dot{w}_t + y \dot{\gamma}_t \\
\hat{\mu}_{ng} = \mu_{ng} \hat{\mu}_{ng} \hat{\mu}_{ng,t-1} + \varepsilon_{\mu ng,t} \\
\hat{\mu}_{tr} = \mu_{tr} \hat{\mu}_{tr} \hat{\mu}_{tr,t-1} + \varepsilon_{\mu tr,t} \\
\hat{\xi} = \hat{\xi} \hat{\xi}_{t-1} + \varepsilon_{\xi,t} \\
\hat{\lambda} = -\theta \hat{c}_t + \hat{a}_t \\
\hat{\omega} = (\eta \gamma + \eta) \hat{l}_t + (\gamma \theta + \theta) \hat{c}_t + \gamma \hat{\omega}_{t-1} - \hat{\lambda}_t + \hat{\pi}_t + \mathbb{E}_t(\hat{\lambda}_{t+1}) - \mathbb{E}_t(\hat{\pi}_{t+1}) = 0 \\
\frac{\mathbb{E}_t(\hat{\lambda}_{t+1})(1 + (-1 + \delta) \beta)}{\beta} + \delta^2 \phi_k \mathbb{E}_t(\hat{\nu}_{t+1}) - \delta^2 \phi_k \hat{k}_{t+1} \\
- (-1 + \delta) \mathbb{E}_t(\hat{\psi}_{t+1}) + \frac{\mathbb{E}_t(\hat{q}_{t+1})(1 + (-1 + \delta) \beta)}{\beta} - \frac{\hat{\psi}_t}{\beta} = 0 \\
- \delta \hat{\psi}_t \phi_k + \delta \phi_k \hat{k}_t + \hat{\psi}_t x + x \hat{x}_t - \hat{\lambda}_t = 0 \\
y \hat{\gamma}_t = k \delta \hat{\nu}_t + c \hat{c}_t + g \hat{g}_t \\
\hat{g}_t = \hat{\xi}_t + \hat{\gamma}_t \\
\hat{k}_{t+1} = (1 - \delta) \hat{k}_t + x \delta \hat{\nu}_t + x \delta \hat{x}_t \\
d \hat{d}_t = -k k_d q - k q \hat{q}_t - l \hat{l}_w + l \hat{l}_w \hat{l}_w + y \hat{\gamma}_t
\[
- \hat{\delta}_t - \hat{y}_t + \hat{l}_t + \omega_t + \hat{\lambda}_t = 0 \\
\hat{\delta}_t + \hat{y}_t - \hat{\lambda}_t - \hat{k}_t = 0 \\
- \phi_p \hat{x}_t - \hat{\lambda}_t (\nu - 1) + (\nu - 1) \hat{\delta}_t + \beta \phi_p \mathbb{E}_t (\hat{x}_{t+1}) = 0 \\
\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) \hat{l}_t + (1 - \alpha) \hat{z}_t \\
\hat{r}_t = -\phi_{gtr} (\rho_{rtr} - 1) \hat{y}_t - \phi_{ntr} (\rho_{ntr} - 1) \hat{x}_t + \rho_{ntr} \hat{r}_{t-1} + \hat{\mu}_{tr,t} \\
\hat{r}_t = \hat{y}_t - \hat{l}_t \\
\hat{r}_t = -\left(1 + \rho_{ng}\right) \left( \hat{y}_t + \hat{P}_t \right) \phi_{ng} + \rho_{ng} \hat{r}_{t-1} + \hat{\mu}_{ng,t} \\
\hat{P}_t = \hat{P}_{t-1} + \hat{r}_t
\]

**B Graphics and Tables of Results**

Figure 1: Multivariate Convergence Diagnostic: Taylor Rule
Figure 2: Multivariate Convergence Diagnostic: Nominal GDP Targeting Rule

Figure 3: Priors and Posteriors for the Parameters: Taylor Rule, Part 1
Figure 4: Priors and Posteriors for the Parameters: Taylor Rule, Part 2

Figure 5: Priors and Posteriors for the Parameters: Taylor Rule, Part 3
Figure 6: Priors and Posteriors for the Parameters: Nominal GDP Targeting Rule, Part 1

Figure 7: Priors and Posteriors for the Parameters: Nominal GDP Targeting Rule, Part 2
**Table 1: Bayesian Estimation Results: Taylor Rule**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type Mean S.D. Mode S.D. Mean HPD Interval</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Beta 0.300 0.050</td>
<td>0.441 0.032 0.429 0.379 0.480</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Normal 0.300 0.090</td>
<td>0.544 0.122 0.492 0.321 0.677</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Beta 0.300 0.050</td>
<td>0.425 0.055 0.425 0.341 0.528</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Normal 2.000 0.200</td>
<td>2.451 0.181 2.466 2.172 2.776</td>
</tr>
<tr>
<td>$\phi_T$</td>
<td>Gamma 50.000 10.000</td>
<td>56.496 4.007 71.920 65.659 79.107</td>
</tr>
<tr>
<td>$\phi_{TR}$</td>
<td>Normal 0.125 0.200</td>
<td>1.370 0.058 1.320 1.237 1.397</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>Normal 1.300 0.300</td>
<td>3.208 0.049 3.134 3.067 3.208</td>
</tr>
<tr>
<td>$\phi_{rT}$</td>
<td>Gamma 4.000 1.000</td>
<td>2.472 0.604 2.524 1.544 3.505</td>
</tr>
<tr>
<td>$\rho_{\mu T}$</td>
<td>Beta 0.600 0.150</td>
<td>0.786 0.036 0.784 0.727 0.844</td>
</tr>
<tr>
<td>$\rho_{aT}$</td>
<td>Beta 0.750 0.150</td>
<td>0.878 0.048 0.845 0.771 0.922</td>
</tr>
<tr>
<td>$\rho_{rT}$</td>
<td>Beta 0.750 0.150</td>
<td>0.057 0.017 0.061 0.032 0.090</td>
</tr>
<tr>
<td>$\rho_{zT}$</td>
<td>Beta 0.750 0.150</td>
<td>0.130 0.052 0.153 0.071 0.236</td>
</tr>
<tr>
<td>$\rho_{\xi T}$</td>
<td>Beta 0.850 0.050</td>
<td>0.803 0.058 0.792 0.704 0.874</td>
</tr>
<tr>
<td>$\sigma_{x}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.501 0.043 0.513 0.441 0.582</td>
</tr>
<tr>
<td>$\sigma_{a}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.235 0.025 0.249 0.205 0.289</td>
</tr>
<tr>
<td>$\sigma_{z}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.013 0.002 0.013 0.010 0.017</td>
</tr>
<tr>
<td>$\sigma_{\mu T}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.159 0.014 0.165 0.141 0.186</td>
</tr>
<tr>
<td>$\sigma_{\xi T}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.028 0.003 0.028 0.024 0.033</td>
</tr>
</tbody>
</table>

Draws: 100000.000
Log D. D.: 499.949
Acc. Rate Ch. 1: 31.1%
Acc. Rate Ch. 2: 30.2%

**Table 2: Bayesian Estimation Results: Nominal GDP Targeting Rule**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type Mean S.D. Mode S.D. Mean HPD Interval</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Beta 0.300 0.050</td>
<td>0.495 0.032 0.495 0.443 0.548</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Normal 0.300 0.090</td>
<td>0.476 0.082 0.469 0.333 0.605</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Beta 0.300 0.050</td>
<td>0.620 0.038 0.590 0.536 0.643</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Normal 2.000 0.200</td>
<td>2.519 0.175 2.514 2.216 2.790</td>
</tr>
<tr>
<td>$\phi_T$</td>
<td>Gamma 50.000 10.000</td>
<td>99.486 13.107 100.609 78.053 121.429</td>
</tr>
<tr>
<td>$\phi_{Ng}$</td>
<td>Gamma 2.000 0.250</td>
<td>4.022 0.137 3.855 3.660 4.022</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>Gamma 4.000 1.000</td>
<td>3.140 0.854 3.434 2.805 4.837</td>
</tr>
<tr>
<td>$\rho_{Ng}$</td>
<td>Beta 0.600 0.150</td>
<td>0.937 0.026 0.928 0.886 0.971</td>
</tr>
<tr>
<td>$\rho_k$</td>
<td>Beta 0.750 0.150</td>
<td>0.807 0.041 0.797 0.732 0.864</td>
</tr>
<tr>
<td>$\rho_{Ng}$</td>
<td>Beta 0.750 0.150</td>
<td>0.072 0.027 0.080 0.035 0.123</td>
</tr>
<tr>
<td>$\rho_k$</td>
<td>Beta 0.750 0.150</td>
<td>0.093 0.043 0.113 0.045 0.181</td>
</tr>
<tr>
<td>$\rho_{\xi}$</td>
<td>Beta 0.850 0.050</td>
<td>0.812 0.045 0.808 0.735 0.883</td>
</tr>
<tr>
<td>$\rho_{z}$</td>
<td>Beta 0.750 0.150</td>
<td>0.972 0.019 0.962 0.933 0.992</td>
</tr>
<tr>
<td>$\sigma_{x}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.473 0.038 0.485 0.425 0.547</td>
</tr>
<tr>
<td>$\sigma_{a}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.206 0.022 0.220 0.183 0.254</td>
</tr>
<tr>
<td>$\sigma_{z}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.020 0.002 0.021 0.017 0.025</td>
</tr>
<tr>
<td>$\sigma_{\mu T}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.141 0.011 0.145 0.125 0.162</td>
</tr>
<tr>
<td>$\sigma_{\xi T}$</td>
<td>Inverse Gamma 0.010 0.500</td>
<td>0.032 0.003 0.032 0.027 0.037</td>
</tr>
</tbody>
</table>

Draws: 100000.000
Log D. D.: 643.176
Acc. Rate Ch. 1: 30.3%
Acc. Rate Ch. 2: 30.3%
### Table 3: Bayesian Model Comparison: Equal Prior Probability Distribution

<table>
<thead>
<tr>
<th>Model</th>
<th>Taylor Rule</th>
<th>Nominal GDP Targeting Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>415.963</td>
<td>643.437</td>
</tr>
<tr>
<td>Bayes Ratio</td>
<td>1.000</td>
<td>6.175E+98</td>
</tr>
<tr>
<td>Posterior Model Probability</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: Based on Laplace approximation

### Table 4: Bayesian Model Comparison: Bigger Prior to the Taylor Rule

<table>
<thead>
<tr>
<th>Model</th>
<th>Taylor Rule</th>
<th>Nominal GDP Targeting Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
<td>0.700</td>
<td>0.300</td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>415.963</td>
<td>643.437</td>
</tr>
<tr>
<td>Bayes Ratio</td>
<td>1.000</td>
<td>2.646E+98</td>
</tr>
<tr>
<td>Posterior Model Probability</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: Based on Laplace approximation

### Table 5: Bayesian Model Comparison: Bigger Prior to the Nominal GDP Targeting Rule

<table>
<thead>
<tr>
<th>Model</th>
<th>Taylor Rule</th>
<th>Nominal GDP Targeting Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
<td>0.300</td>
<td>0.700</td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>415.963</td>
<td>643.437</td>
</tr>
<tr>
<td>Bayes Ratio</td>
<td>1.000</td>
<td>1.441E+99</td>
</tr>
<tr>
<td>Posterior Model Probability</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: Based on Laplace approximation

### Table 6: Bayesian Model Comparison: Robustness, Equal Prior Probability Distribution

<table>
<thead>
<tr>
<th>Model</th>
<th>Taylor Rule</th>
<th>Nominal GDP Targeting Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>415.963</td>
<td>620.809</td>
</tr>
<tr>
<td>Bayes Ratio</td>
<td>1.000</td>
<td>9.196E+88</td>
</tr>
<tr>
<td>Posterior Model Probability</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: Based on Laplace approximation
Figure 8: Impulse Response Functions of Monetary Policy Shocks

Table 7: Bayesian Model Comparison: Robustness, Bigger Prior to the Taylor Rule

<table>
<thead>
<tr>
<th>Model</th>
<th>Taylor Rule</th>
<th>Nominal GDP Targeting Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
<td>0.700</td>
<td>0.300</td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>499.949</td>
<td>643.176</td>
</tr>
<tr>
<td>Bayes Ratio</td>
<td>1.000</td>
<td>6.835E+61</td>
</tr>
<tr>
<td>Posterior Model Probability</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: Based on Modified Harmonic Mean Estimator

Table 8: Bayesian Model Comparison: Robustness, Bigger Prior to the Nominal GDP Targeting Rule

<table>
<thead>
<tr>
<th>Model</th>
<th>Taylor Rule</th>
<th>Nominal GDP Targeting Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
<td>0.300</td>
<td>0.700</td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>499.949</td>
<td>643.176</td>
</tr>
<tr>
<td>Bayes Ratio</td>
<td>1.000</td>
<td>3.721E+62</td>
</tr>
<tr>
<td>Posterior Model Probability</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: Based on Modified Harmonic Mean Estimator