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**Abstract**

The Multi-period Electric Vehicle Routing Problem (MP-E-VRP) consists on designing routes to be performed by a fleet of electric vehicles (EVs) to serve a set of customers over a planning horizon of several periods. EVs are charged at the depot at any time, subject to the charging infrastructure capacity constraints (e.g., number of available chargers, power grid constraints, duration of the charging operations). Due to the impact of charging and routing practices on EVs battery aging, degradation costs are associated with charging operations and routes. The MP-E-VRP integrates EV routing and depot charging scheduling, and has coupling constraints between days. These features make the MP-E-VRP a complex problem to solve. In this talk we present a two-phase matheuristic for the MP-E-VRP. In the first phase it builds a pool of routes via a set of randomized route-first cluster-second heuristics. Routes are then improved by solving a traveling salesman problem. In the second phase, the algorithm uses the routes stored in the pool to assemble a solution to the MP-E-VRP. We discuss computational experiments carried out on small-size instances.

## 1 Introduction

We define the Multi-period Electric Vehicle Routing Problem (MP-E-VRP) as follows. Let $\mathcal{P} = \{0, \ldots, P_{max} - 1\}$ be the set of periods within the planning horizon. Let $\mathcal{I}$ be the set of visit nodes and 0 a node representing the depot. Each node $i \in \mathcal{I}$ represents a customer with a service time $v_i > 0$ and a period $p_i \in \mathcal{P}$ in which the service must take place. A finite set of homogeneous electric vehicles (EVs) denoted by $\mathcal{K}$ is available to visit the nodes in $\mathcal{I}$. Each EV $k \in \mathcal{K}$ has a battery capacity $Q$ (in kWh). Travelling from one location $i$ (the depot or a visit node) to another location $j$ incurs a driving time $t_{ij} \geq 0$ (in hours) and an energy consumption $e_{ij} \geq 0$ (in kWh). The triangular inequality holds for driving times and energy consumptions. At period $p \in \mathcal{P}$, an EV cannot leave the depot before $E_p$ or return after $L_p$ (with $L_p > E_p$). In other words, time interval $[E_p, L_p]$ corresponds to the opening hours of the depot for period $p \in \mathcal{P}$.

At the beginning of the planning horizon each EV $k \in \mathcal{K}$ has an energy $kWh_0^k$ (in kWh). The EVs can be fully or partially recharged at the depot at any time. There is no possibility to recharge an EV outside of the depot. The EVs can only perform one route per period and they are allowed to be recharged only once between two routes. Multiple chargers are available at the depot. Each charger is associated with a charging mode (e.g., slow, fast, moderate) belonging to set $\mathcal{M}$. The depot has $A_m$ available chargers for each charging mode $m \in \mathcal{M}$ (usually, but not necessarily, $\sum_{m \in \mathcal{M}} A_m < |\mathcal{K}|$). During a charging operation, a charger of mode $m$ consumes a power $pw_m$ (in kW) from the grid. An EV can be recharged using any available charger, but at any time the power grid capacity $PW_{max}$ must not be exceeded. Moreover, it is forbidden to switch the charging mode while charging an EV. Charging operations are also non-preemptive (i.e. they cannot be interrupted and resumed later). Each charging mode is associated with a piecewise linear charging function $g_m(\Delta)$ and a fixed cost per charge $u_m$.

Routing and charging decisions impact the lifespan of EV batteries. To take this into account, a cumulative wear cost is incurred when performing routes and charging operations. This cost is modeled as a piecewise linear function that maps the state of charge (SOC) of the battery $\sigma$ to the sum of the wear cost associated to charge the EV to every SOC from 0% to $\sigma$ or discharge it from every SOC $\sigma$ to 0%. The SOC of a battery is the percentage of available battery capacity. The cumulative wear cost function is derived from the model proposed by [2].

Feasible solutions to the MP-E-VRP satisfy the following conditions: 1) each node $i \in \mathcal{I}$ is visited by a single EV in period $p_i$, 2) each route starts and ends at the depot, 3) each EV can perform one route per period, 4) when an EV starts a route, it must have enough energy to complete it (no mid-route charging is allowed), 5) no more than $|\mathcal{K}|$ EVs are used on a single period, 6) EVs visit nodes in $\mathcal{I}$ in period $p_i \in \mathcal{P}$ only during time window $[E_{p_i}, L_{p_i}]$, 7) the power consumed by all chargers at any time does not exceed the maximum power $PW_{max}$, and 8) no more than $A_m$ EVs simultaneously charge at the depot using mode $m \in \mathcal{M}$.

The objective of the MP-E-VRP is to determine for each period a set of routes visiting all the customers and to schedule the charging operations of the EVs while minimizing the cost of battery degradation. This cost is defined as the sum of the fixed costs of all charging operations and the cumulative wear costs associated with all charging operations and routes.

We also define a related problem that takes as inputs a set of pre-generated routes. The problem that consists on selecting a subset of routes from the pool, assigning EVs to them and scheduling charging operations while satisfying conditions 1)-8) is referred to as the Multi-period Electric Vehicle Route Assignment and Charge Scheduling Problem (MP-E-VRACSP).

## 2 Matheuristic for the MP-E-VRP

To tackle the MP-E-VRP we present a matheuristic based on the multi-space sampling heuristic (MSH) [3]. MSH has two phases: sampling and assembling. In the sampling phase the algorithm uses a set of randomized traveling salesman problem (TSP) heuristics to obtain a set $N$ of TSP-like tours. Then, MSH extracts every feasible route that can be obtained without altering the order of the customers of each TSP tour, following the route-first cluster-second principle [1, 4]. MSH uses these routes to build a set $\Omega \subset \mathcal{R}$, where $\mathcal{R}$ is the set of all feasible routes. In the assembling phase MSH finds a solution $s$, from the set of all feasible solutions to the problem $\mathcal{S}$, by solving a set partitioning formulation on set $\Omega$.

Algorithm 1 describes the general structure of our matheuristic. The procedure starts by entering the sampling phase (lines 1-1) for each period $p \leq |\mathcal{P}| - 1$. At each iteration $n \leq N$, the algorithm selects a sampling heuristic from a set $\mathcal{T}$ (line 1) and uses it to build a TSP tour $tsp_p$ visiting the customers that must be serviced in that period ($i \in \mathcal{I} : p_i = p$). Then, the algorithm uses a tour splitting procedure (known as `split`) to retrieve a set of routes added to the set of routes for period $p$, $\Omega'_p \in \mathcal{R}$. Routes in $\Omega'_p$ respect a maximum duration, related with the opening hours of the depot ($L_p - E_p$), and the maximum energy consumption, $Q$. Then, the algorithm calls procedure `custSeqs`($\mathcal{R}_p$) - line 1. This procedure retrieves all the unique sets of customers present in set $\Omega'_p$ and generates the set of customer sequences $\mathcal{CS}_p$. Then the algorithm invokes procedure `tsp`($\mathcal{CS}_p$) - line 1. The latter solves a TSP for each set of customers in $\mathcal{CS}_p$ and stores in $\Omega_p$ the resulting routes. Routes in $\Omega_p$ are then stored in $\Omega$. In the assembly phase (line 1), the algorithm invokes a procedure called `mpevracsp` to solve a MP-E-VRACSP by selecting routes from $\Omega$ to find a feasible MP-E-VRP solution. We propose a continuous-time mixed integer linear programming (MILP) formulation of the MP-E-VRACSP. This formulation uses arc-based tracking variables for time and energy. More details on the algorithm implementation and the MILP formulation will be discussed in the talk.

## 3 Computational experiments

We implemented our matheuristic in Java (jre V.1.8.0) and used Gurobi Optimizer (version 8.0.1) to solve the MILP formulation of the MP-E-VRACSP (Algorithm 1, line 1). We compare the results of out matheuristic with results obtaining from solving a MILP formulation of the MP-E-VRP. We set a time limit of 3 hours. We generated 100 small size instances of the MP-E-VRP. We consider 3 periods of 8 hours. We generated instances with 5, 10, or 15 customers in each period. Customers locations

**Algorithm 1** Matheuristic: general structure

**function** MATHEURISTIC($\mathcal{P}, \mathcal{I}, \mathcal{K}, \mathcal{M}, \mathcal{T}, N$)<br>
    $\Omega \leftarrow \emptyset$<br>
    $n \leftarrow 1$<br>
    **for** $p = 0$ to $p = |\mathcal{P}| - 1$ **do**<br>
        $\Omega'_p \leftarrow \emptyset$<br>
        **while** $n \leq N$ **do**<br>
            **for** $l = 1$ to $l = |\mathcal{T}|$ **do**<br>
                $h \leftarrow \mathcal{T}_l$<br>
                $tsp_p \leftarrow h(\mathcal{I})$<br>
                $\Omega'_p \leftarrow \Omega'_p \cup \texttt{split}(\mathcal{I}, tsp_p)$<br>
                $n \leftarrow n + 1$<br>
            **end for**<br>
        **end while**<br>
        $\mathcal{CS}_p \leftarrow \texttt{custSeqs}(\Omega'_p)$<br>
        $\Omega_p \leftarrow \texttt{tsp}(\mathcal{CS}_p)$<br>
        $\Omega \leftarrow \Omega \cup \Omega_p$<br>
    **end for**<br>
    $\sigma \leftarrow \texttt{mpevracsp}(\Omega, \mathcal{P}, \mathcal{I}, \mathcal{K}, \mathcal{M})$<br>
    **return** $\sigma$<br>
**end function**

are randomly generated in a geographic space of approximately 45×45 km. This represents an urban operation in which 16 kWh-EVs can perform routes without need for mid-route charging. For each number of customers per period we generated 5 sets of customer locations. Service time is fixed to 0.75 hours. We assume the EVs have an energy consumption rate of 0.125 kWh/km. The initial energy for all EVs is fixed to 8 kWh or 12.8 kWh. For instances with 5 customers per period we considered 2 EVs, for the rest of the instances, 4 or 5 EVs. We included two types of charging mode: $slow$ (with a power of 6 kW and a fixed cost per charge of \$1.22) and $moderate$ (with a power of 11 kW and a fixed cost per charge of \$1.42). For $slow$ mode, there is the same number of available chargers as for EVs, while for $moderate$ mode there is only one charger available. We considered instances with only one charging mode, $slow$, and instances with two charging modes, $slow$ and $moderate$. The power grid capacity is adjusted depending on the number of EVs, so that all chargers of both charging modes cannot be used at the same time. We fixed parameter $N = 1000$.

For the 5 customers per period - instances our matheuristic was able to find the optimal solutions. For the 10 customers per period - instances our method reported average improvements of 0.2% with respect to the solutions delivered by the solver running the MP-E-VRP. For the 15 customers per period - instances, the improvement is of 4.45%.

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