High order discretization of seismic waves-problems based upon DG-SE methods

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Abstract

Hybrid meshes comprised of hexahedras and tetrahedras are particularly interesting for representing media with local complex geometrical features like the seabed in offshore applications. We develop a coupled finite element method for solving elasto-acoustic wave equations. It combines Discontinuous Galerkin (DG) finite elements for solving elastodynamics with spectral finite elements (SE) for solving the acoustic wave equation. SE method has demonstrated very good performances in 3D with hexahedral meshes and contributes to reduce the computational burden by having less discrete unknowns than DG. The implementation of the method is performed both in 2D and 3D and it turns out that the coupling contributes to reduce the computational costs significantly: for the same time step and the same elementary mesh size, the CPU time of the coupled method is almost halved when compared to the one of a full DG method.

Keywords: Hybrid meshes, Discontinuous Galerkin method, Spectral Element method, coupling

1 Introduction

We focus on the first-order elasto-dynamic system due to space constraint. We denote by Ω a rectangular domain in 2D or a parallelepiped in 3D. We consider the system of wave equations:

$$\begin{cases} \rho(\mathbf{x})\dfrac{\partial \mathbf{v}}{\partial t}(\mathbf{x},t) &= \nabla \cdot \underline{\underline{\sigma}}(\mathbf{x},t), \\ \\ \dfrac{\partial \underline{\underline{\sigma}}}{\partial t}(\mathbf{x},t) &= \underline{\underline{C}}(\mathbf{x})\underline{\underline{\epsilon}}(\mathbf{v}(\mathbf{x},t)), \end{cases} \tag{1}$$

where ρ is the density, $\underline{\underline{C}}$ the elasticity tensor and $\underline{\underline{\epsilon}}$ the deformation tensor. The space variable is $\mathbf{x} \in \mathbb{R}^d$ (with $d = 2,3$) and $t \geq 0$ is the time variable. The two unknowns are $\mathbf{v}$ the wavespeed and $\underline{\underline{\sigma}}$ the strain tensor. This system can be solved by using DGm (see [1, 2]) or a SEm (see [3–5]). Our objective is to construct a variational formulation resulting from the combination of both approximations. The difficulty of such a coupling is the communication between the two different schemes.

2 Variational Formulation

Offshore geophysical exploration can be represented by a reference domain composed of a layer of water over the ocean bottom (see Fig 1). The computational domain is covered by a hybrid grid composed of hexahedra on the top and tetrahedra within the bottom. Basically, we define two areas: $\Omega_{h,1}$ composed of cartesian cells and $\Omega_{h,2}$ paved with unstructured tetrahedra capable of following the topography of the in-depth site. The transition between both areas is located inside the water. Hence, the interface $\Gamma_{1/2} = \Omega_{h,1} \cap \Omega_{h,2}$ is flat and the two regions communicate with each other through suitable fluxes. The portion $\Omega_{h,1}$ of the mesh can thus be seen as a macro-element of the DG partition. In the following, we use the subscript 1 to designate the fields computed over $\Omega_{h,1}$ while any field with subscript 2 corresponds to a quantity computed over $\Omega_{h,2}$. We introduce the pair (w, ξ) to test the continuous problem (1) and to get a variational formulation set in the whole domain Ω. For the sake of simplicity, we denote by a_j, b_j, c_j and d_j the bilinear forms defined by

$$a_j(v,w) = \int_{\Omega_{h,j}} \rho \partial_t v \cdot w,$$

$$b_j(\underline{\underline{\sigma}}, w) = -\int_{\Omega_{h,j}} \underline{\underline{\sigma}} \cdot \nabla w$$

$$c_j(\underline{\underline{\sigma}}, \underline{\underline{\xi}}) = \int_{\Omega_{h,j}} \partial_t \underline{\underline{\sigma}} : \underline{\underline{\xi}},$$

$$d_j(v, \underline{\underline{\xi}}) = -\int_{\Omega_{h,j}} (\nabla(\underline{\underline{C\xi}})) \cdot v$$

Then the global variational formulation reads as:

$$a_1(v_1, w_1) + a_2(v_2, w_2) = b_1(\underline{\underline{\sigma}}_1, w_1) + b_2(\underline{\underline{\sigma}}_2, w_2) + \sum_{\Gamma \in \Gamma_{int}} \int_\Gamma \{\{\underline{\underline{\sigma}}_2\}\}[[w_2]] \cdot \mathbf{n} + \int_{\Gamma_{1/2}} \{\{\underline{\underline{\sigma}}\}\}[[w]] \cdot \mathbf{n}$$

$$c_1(\underline{\underline{\sigma}}_1, \underline{\underline{\xi}}_1) + c_2(\underline{\underline{\sigma}}_2, \underline{\underline{\xi}}_2) = d_1(v_1, \underline{\underline{\xi}}_1) + d_2(v_2, \underline{\underline{\xi}}_2) + \sum_{\Gamma \in \Gamma_{int}} \int_\Gamma [[\underline{\underline{C}}\underline{\underline{\xi}}_2]]\{\{v_2\}\} \cdot \mathbf{n} + \int_{\Gamma_{1/2}} [[\underline{\underline{C}}\underline{\underline{\xi}}]]\{\{v\}\} \cdot \mathbf{n}$$

where Γ_{int} stands for the set of internal boundaries limiting DG-elements. The shortage of the paper challenges us to omit to speak about external boundary conditions.
At the interface $\Gamma_{1/2}$, we define $\mathbf{n}$ as the unitary normal vector oriented from $\Omega_{h,1}$ to $\Omega_{h,2}$. We can see how the two areas communicate through the different fluxes involving the jump and the mean value respectively defined by:

$$[[w]] = (w_{K_2} - w_{K_1}) \cdot \mathbf{n}, \{\{\underline{\underline{\xi}}\}\} = \frac{1}{2}(\underline{\underline{\xi}}_{K_2} + \underline{\underline{\xi}}_{K_1})$$

K_1 and K_2 are two connected DG-cells and w_{K_j} (resp. $\underline{\underline{\xi}}_{K_j}$) is the value of w (resp. $\underline{\underline{\xi}}$) in K_j. In comparison with the implementation of a full SEm or a full DGm, we have to create new terms corresponding to the handling of $\Gamma_{1/2}$. They are written in terms of integrals mixing DG basis functions with a SE-one.

3 Numerical tests

We consider an exemple of anisotropic elasto-acoustic domain depicted in Figure 1

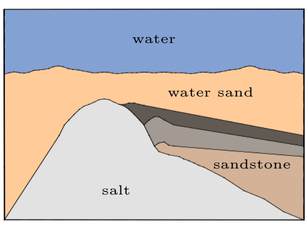

Figure 1: Propagation domain

It is a square 3000 meters paved with 74969 cells composed of 53969 unstructured triangles and 21000 structured quadrangles. The source is a second-order Ricker point source located on the top of the layer of water. Both DGm and SEm have been validated separately in stratified media for which we dispose of analytical solutions. To assess the accuracy of the coupling, we have compared the full DG solution with the DG-SE one at order three and the results are displayed in Table 1. We compare the CPU-time at equal time-step and the relative error between these two solutions and a reference solution computed using DGm at order five. The second column shows that the DG-SEm solution has the same accuracy as the DG one. Then the third column certifies that the coupling allows to reduce the CPU time by a factor of 2. It is worth noting that we have used a global time-step and in the near future, we hope to improve our results by using local-time stepping.

	Relative error(%)	CPU-time(s)
DGm	5e-4	16317
DG_SEm	1e-3	8918

Table 1: DGm vs DG_SEm comparison.

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