Investigation of sweet spot radius of sound reconstruction system based on inverse filtering

Hiraku Okumura, Makoto Otani

INVESTIGATION OF SWEET SPOT RADIUS OF SOUND RECONSTRUCTION SYSTEM BASED ON INVERSE FILTERING

Hiraku Okumura
Yamaha Corporation, Kyoto University
hiraku.okumura@music.yamaha.com

Makoto Otani
Kyoto University
otani@archi.kyoto-u.ac.jp

ABSTRACT

There are several methods aiming at sound field reconstruction a sound field, such as Higher-Order Ambisonics and Boundary Surface Control (BoSC) method. While the BoSC system aims to reconstruct a sound field within a volume surrounded by the boundary surface, some previous studies suggest that a reconstructed area, so-called a sweet spot, would be generated even outside of this controlled area. The authors investigated that the radius of a sweet spot for a BoSC system consisting of a spherical controlling surface and a loudspeaker array placed on a sphere. The results show that the radii of the microphone array do not affect the radii of the sweet spot, whereas the number of microphones could affect it. Furthermore, a simplified implementation only with sound pressure control could affect the radius of sweet spot.

1. INTRODUCTION

Sound field reconstruction techniques are very effective tools for a sound system of live-viewing or acoustical design in architecture. In a live-viewing system, listeners can enjoy highly realistic sound through the system. In addition, the system would allow acoustical designers to evaluate sound fields simulated in architectural spaces before their completion. There are several methods to realize sound field reconstruction, such as Higher-Order Ambisonics [1], Wave Field Synthesis [2], and Boundary Surface Control (BoSC) [3]. It is important to reconstruct a sound field within a broad region in order to allow a listener to look and move around, or to allow multiple listeners to experience the sound field at the same time, whereas reconstruction of sound field in a broad region necessitates a large number of microphones and loudspeakers. Fortunately, it has become easier and less expensive to handle a large number of devices thanks to the progress of computer technologies and network audio technologies, which contributes to a realization of broader reconstruction region, i.e. a broader sweet spot. In the BoSC system, both sound pressure and particle velocity on the boundary surface of a reconstruction region are controlled using inverse filters to reconstruct a sound field. While the BoSC system aims to reconstruct a sound field in a region surrounded by a boundary surface, some studies suggest that a sweet spot would be formed outside of this controlled region [4, 5]. However, the size of sweet spot in sound field reconstruction using inverse filtering is yet to be revealed. In this work, the authors numerically investigated the radius of sweet spot in sound field reconstruction with a spherical controlling surface and a spherical loudspeaker array.

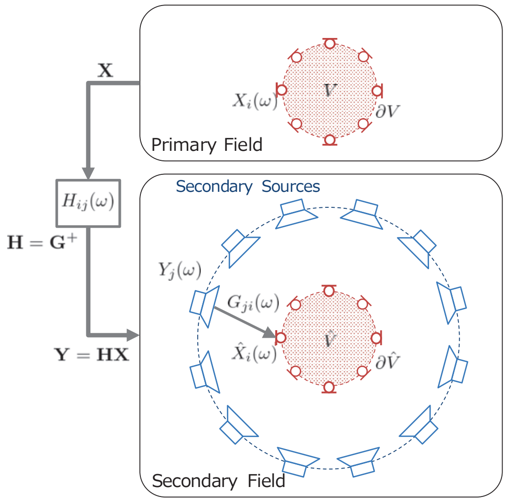

Figure 1. The concept of sound field reproduction using inverse filtering.

2. SOUND FIELD RECONSTRUCTION SYSTEM BASED ON INVERSE FILTERING

In this paper, the principle of boundary surface control [3] is employed as a theory for sound field reconstruction based on inverse filtering . Figure 1 illustrates the concept of sound field reconstruction by using inverse filtering. A sound field in a volume V in the primary field is reconstructed in a volume $\hat{V}$ in the secondary field by reconstructing both sound pressure and particle velocity on the boundary ∂V of the volume V at the corresponding position on the boundary $\partial\hat{V}$ of the volume $\hat{V}$ using secondary sources. It should be noted that particle velocity on the boundary $\partial\hat{V}$ is automatically reconstructed when sound pressure on the boundary $\partial\hat{V}$ is reconstructed, except at the frequencies associated with the internal Dirichlet problem of the volume $\hat{V}$. Therefore, generally, only sound pressures are reconstructed in this type of sound field reconstruction system.

Let $X_i(\omega)(i = 1, \cdots, Q)$ be the components of the angular frequency ω of the signals observed by Q microphones placed on the boundary ∂V in the primary field, and $\hat{X}_i(\omega)$ be the components of the angular frequency ω of the observed signals by Q microphones placed on the boundary $\partial\hat{V}$ in the secondary field. The input signals $Y_j(\omega)(j = 1, \cdots, M)$ to the M loudspeakers in the secondary field are designed to match $\hat{X}_i(\omega)$ to $X_i(\omega)$. $G_{ji}(\omega)$ is the transfer function between the j-th secondary loudspeaker and the i-th microphone in the secondary field. Using matrix representation, the system can be written as,

$$\mathbf{Y} = \mathbf{HX} \tag{1}$$

$$\hat{\mathbf{X}} = \mathbf{GY} = \mathbf{GHX} \tag{2}$$

where,

$$\mathbf{Y} = [Y_1(\omega) \cdots Y_M(\omega)]^T \tag{3}$$

$$\mathbf{X} = [X_1(\omega) \cdots X_Q(\omega)]^T \tag{4}$$

$$\hat{\mathbf{X}} = \left[\hat{X}_1(\omega) \cdots \hat{X}_Q(\omega)\right]^T \tag{5}$$

$$\mathbf{H} = \begin{bmatrix} H_{1,1}(\omega) & \cdots & H_{1,Q}(\omega) \\ \vdots & \ddots & \vdots \\ H_{M,1}(\omega) & \cdots & H_{M,Q}(\omega) \end{bmatrix} \tag{6}$$

$$\mathbf{G} = \begin{bmatrix} G_{1,1}(\omega) & \cdots & G_{1,M}(\omega) \\ \vdots & \ddots & \vdots \\ G_{Q,1}(\omega) & \cdots & G_{Q,M}(\omega) \end{bmatrix}. \tag{7}$$

Finding $\mathbf{H}$ that satisfies $\mathbf{X} = \hat{\mathbf{X}}$ leads to

$$\mathbf{GH} = \mathbf{I}. \tag{8}$$

In this paper, Moore-Penrose generalized inverse matrix $\mathbf{G}^+$ is used as $\mathbf{H}$,

$$\mathbf{H} = \mathbf{G}^+. \tag{9}$$

Kajita et al. [4] reported that a sweet spot could be larger than the boundary of the reconstructed region in the BoSC system. Further, Fazi et al. [5] suggested that sweet spot could be larger than boundary surface in the sound reconstruction based on an integral equation of the first kind.

3. SWEET SPOT

In this paper, the reconstruction performance of the system is evaluated by normalized reconstruction error (NRE) defined as

$$\mathrm{NRE}(\mathbf{x}) = \frac{|\hat{p}(\mathbf{x}) - p(\mathbf{x})|^2}{|p(\mathbf{x})|^2} \times 100\ [\%], \tag{10}$$

where $p(\mathbf{x})$ and $\hat{p}(\mathbf{x})$ are the sound pressure signals at the observation point $\mathbf{x}$ in the primary field and the secondary field respectively.

The sweet spot is defined as the area in which NRE is smaller than $4\ \%$ [6].

Kajita [4] defined the sweet spot as the area in which the S/N ratio is greater than 15 dB. This means that they defined the sweet spot as the region in which NRE is smaller than $3.16\ \%$.

4. NUMERICAL SIMULATIONS

The numerical simulation assuming a free field was performed to investigate the size of the sweet spot in the sound field reconstruction based on inverse filtering. A 122-channels spherical loudspeaker array was assumed as secondary point sources, which located at the vertexes of a geodesic dome of 2.5-meter radius [7]. Further, a Q channels spherical microphone array was assumed as microphone capsules on a spherical surface, which located on a sphere with radius of R_V meters. The positions of microphone capsules were determined by the spherical Fibonacci spiral [7]. The center of the geodesic dome of the secondary sources was set to the origin, which also corresponds to the center of spherical microphone array.

A point source in the primary field was assumed at a random position whose distance to the origin was more than 2.5 meters. Transfer function between a point source $\mathbf{x}$ and a microphone capsule $\mathbf{y}$ is calculable as the free-field Green's function,

$$G(\omega) = \frac{e^{j\frac{\omega}{c}|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|}, \tag{11}$$

where ω is an angular frequency; c is the speed of sound; j is the imaginary unit. In the primary field, sound pressures $\mathbf{X}$ were calculated at the positions of the microphone capsules. In the secondary field, transfer functions $G_{ji}(\omega)(j = 1, \cdots, M, i = 1, \cdots, Q)$ between the $j-$th secondary source and the $i-$th microphone capsule were calculated to obtain $\mathbf{H} = \mathbf{G}^+$. Finally, the input signal $\mathbf{Y}$ to the secondary sources were derived as $\mathbf{Y} = \mathbf{HX}$. In this section, sound pressures were calculated in the frequency domain and its frequency interval was 7.8125 Hz.

To evaluate the sweet spot size, sound pressures were calculated inside V in the primary field and $\hat{V}$ in the secondary field at the grid points of 5.0 cm intervals in x and y-directions in a square with a 2.5 meters sides and centered on the origin.

Figure 2 depicts NRE [%] for the point source at $(-8, 0, 0)$ in the primary field. The group of gray-color points drawn around the origin in Figure 2 represents the microphone capsules, that is a 64-channels microphone array of $R_V = 0.05$ m. The radius of the sweet spot is defined as the minimum distances between the origin and the point where NRE is smaller than $4\ \%$.

Figure 3 demonstrates the radii of the sweet spot for a 64 channels spherical microphone array of radius $R_V = 0.05$ m. The gray lines indicate the radii of the sweet spot of the sound field reconstruction system for point sources placed at the randomly generated positions in the primary field. The thick line represents the average of these radii of sweet spot. In the following sections, the radius of sweet spot is defined as the average of the radii of the sweet spot among 100 randomly generated point sources.

doi:10.25836/sasp.2019.16

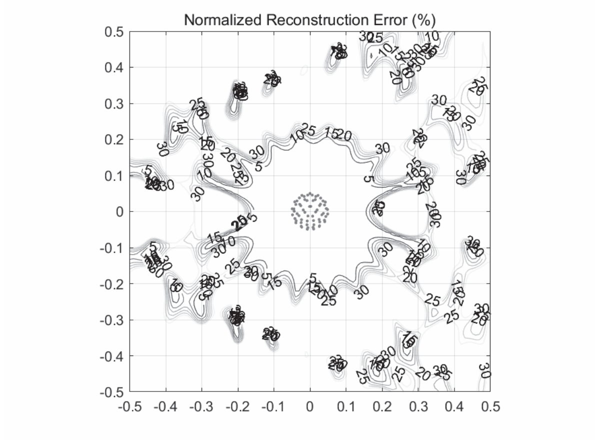

Figure 2. NRE (%) at 2kHz for spherical microphone array of $R_V = 0.05$ m, $Q = 64$.

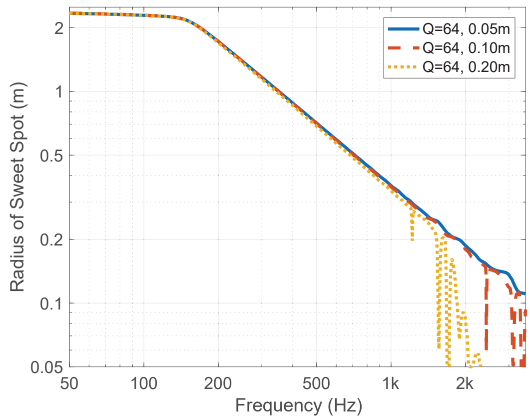

Figure 4. Radii of sweet spot for 64-channel spherical microphone arrays of various R_V.

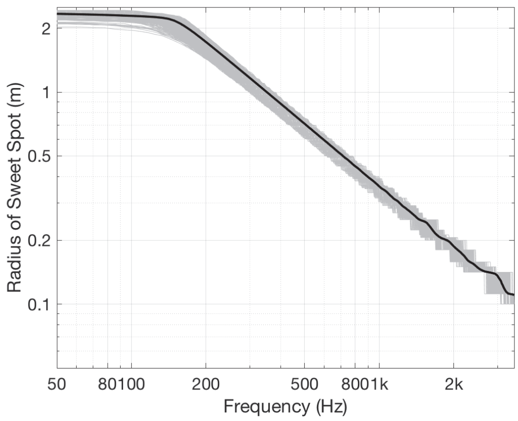

Figure 3. Radii of the sweet spot for the spherical microphone array of $R_V = 0.05$ m, $Q = 64$. The thick line shows the average of these radii.

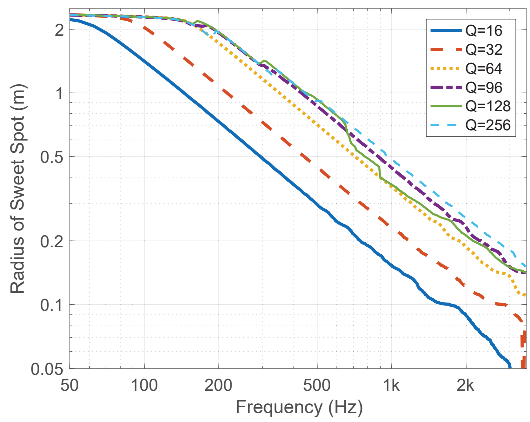

Figure 5. Radii of sweet spot for the sperical microphone array of $R_V = 0.05$ m with various Q.

4.1 Effects of radius and number of microphone capsules on sweet spot radius

Figure 4 illustrates radii of the sweet spot for 64-channel spherical microphone arrays of $R_V = 0.05$ m, 0.10 m and 0.20 m. The radii of the sweet spot do not differ among the three radii of the microphone array at frequencies below 1.1 kHz.

When $R_V = 0.1$ m and 0.2 m, the radii of the sweet spot drop suddenly at around 2.4 kHz and 1.2 kHz respectively. The minimum microphone intervals of the spherical microphone array of $R_V = 0.1$ m and 0.2 m are 0.0386 m and 0.0772 m, respectively. These lengths correspond to a quarter of the wavelength λ for 2.4 kHz ($\lambda = 0.0354$ m) and 1.2 kHz ($\lambda = 0.0773$ m). Therefore, these frequencies could be regarded as the upper limit frequencies for the sound reconstruction system using the spherical microphone array of $R_V = 0.1$ m and 0.2 m.

Figure 5 depicts the radii of the sweet spot for the spherical microphone array of radius $R_V = 0.05$ m with number of the microphone capsules $Q = 16, 32, 64, 96, 128$ and 256. Generally, the radius of the sweet spot increases as the number of the microphone capsules Q increases. However, the radii of the sweet spot do not increase prominently when $Q > 96$. In this study, the number of secondary sources is 122, and the system defined as equation (8) is overdetermined when $Q = 128$ and 256. Therefore, the sound reconstruction system does not have an exact solution and this is considered to be the reason why the radii of the sweet spot do not increase when $Q > 96$. Thus, when the interval between microphone capsules of the spherical microphone array is greater than a quarter of the wavelength, the radius of the sweet spot depends on the number of microphone capsules of the spherical microphone array and does not depend on its radius [8].

Fazi [5] formulated a sound field reproduction system that reconstructs only the sound pressure on the boundary with an integral equation of the first kind, and suggested the correspondence of the Higher-Order Ambisonics (HOA) by expressing it using spherical harmonics. As described above, usually only sound pressures are reconstructed in the BoSC system. This corresponds to the method proposed by Fazi [5]. It is known that the size of the sweet spot of HOA depends on the number of divisions of the surface of the spherical microphone array

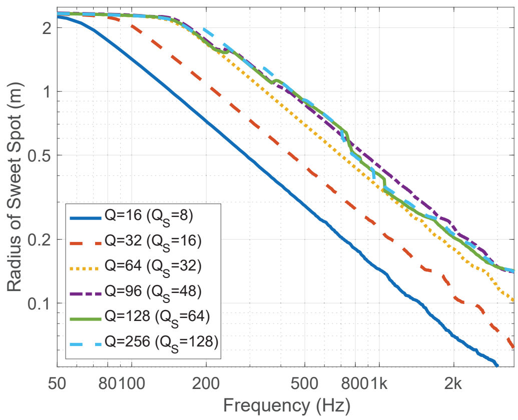

Figure 6. Radii of sweet spot for double-layered spherical microphone array of $R_V = 0.05$ m with a variety of Q.

(corresponding to the number of microphones). Therefore, the size of sweet spot in a sound field reconstruction that reconstructs only the sound pressure on the boundary can be regarded similar to one in HOA, which corresponds well with the above results in the current paper.

4.2 Effects of double-layered microphone arrays on the radii of sweet spots

Based on the BoSC principle, in order to reconstruct both the sound pressure and the particle velocity on the boundary, a double-layered microphone array has been proposed with the microphone capsules arranged at positions with offset both inward and outward from the boundary [3]. Assuming that the boundary surface is discretized by Q_S control points, a double-layered microphone array uses twice as many microphones as a single-layer microphone array that is used to reconstruct only the sound pressure on the boundary. Thus, $Q = 2Q_S$ for a double-layered microphone array whereas $Q = Q_S$ for a single-layer one.

Figure 6 shows the radii of the sweet spot for a spherical double-layered microphone array of radius $R_V = 0.05$ m with $Q = 16, 32, 64, 96, 128$ and 256, where the offset of inner and outer layers of the microphone capsule from the boundary is 2.5 mm. It is observed that the radii of the sweet spot in the sound field reconstruction using a double-layered spherical microphone array are very similar to that using a single-layer spherical microphone array in Figure 5. However, considering the number of divisions of the surface of the spherical microphone array, $Q_S = Q/2 = 8, 16, 32, 48, 64,$ and 128, it appears that for the same number of divisions of the surface of the spherical microphone array, the size of the sweet spot of the sound field reconstruction system can be made greater by using a double-layered spherical microphone array. This indicates that a microphone array implemented using a less-dense array of microphone capsules can yield a larger sweet spot.

5. CONCLUSION

The size of the sweet spot in a sound reconstruction system using inverse filtering was investigated.

A sweet spot was generated not only inside but also outside of the controlled region by reconstructing sound pressures on the boundary of the volume. The numerical results revealed that, when the interval between microphone capsules of the spherical microphone array is greater than a quarter of the wavelength, the size of sweet spot in the sound field reconstruction depends on the number of microphone capsules in the spherical microphone array, and it does not depend on the radius of the spherical microphone array. Furthermore, the results also show that the size of sweet spot in the sound field reconstruction can be expanded by using a double-layered spherical microphone array to reconstruct both sound pressures and particle velocities on the boundary of the volume. This suggests that a larger sweet spot can be generated even when using a sparse (less dense) microphone array.

6. REFERENCES

[1] J. S. Bamford and J. Vanderkooy, “Ambisonic sound for us.,” in *Proc. of Audio Eng. Soc. Conv.*, (New York, U.S.A.), 1995.

[2] A. J. Berkhout, “Ambisonic sound for us.,” *Journal of Audio Eng. Soc.*, vol. 36, no. 12, pp. 977–995, 1988.

[3] S. Ise, “A principle of sound field control based on the kirchhof-helmholtz integral equation and the theory of inverse systems.,” *Acoustica - Acta Acoustica*, vol. 85, pp. 78–87, 1999.

[4] Y. Kajita, K. Miura, Y. Watanabe, and S. Ise, “A study of relationship between microphone-positions and bosc-system reproduced-area using sound field estimation-method with lu decomposition.,” in *Proc. of Research Meeting of the Acoustical Society fo Japan (in Japanese)*, (Tokyo, Japan), pp. 493–494, 2018.

[5] F. M. Fazi, P. A. Nelson, J. E. N. Christensen, and J. Seo, “Surround system based on three dimensional sound field reconstruction.,” in *Proc. of Audio Eng. Soc.*, (San Francisco, USA), 2008.

[6] D. B. Ward and T. D. Abhayapala, “Reproduction of a plane-wave sound field using an array of loudspeakers.,” *IEEE Trans. on Speech and Audio Processing*, vol. 9, no. 6, pp. 697–707, 2001.

[7] S. Kaneko, T. Suenaga, and H. Okumura, “Development of a 64-channel spherical microphone array and a 122-channel loudspeaker array system for 3d sound field capturing and reproduction technology research.,” in *Proc. of Audio Eng. Soc. Conv.*, (Milan, Italy), 2018.

[8] H. Okumura and M. Otani, “Investigation of the sweet spot of sound field reproduction using inverse filtering.,” in *Proc. of the Architectual Acoustics Research Meeting (in Japanese)*, (Sapporo, Japan), 2018.

doi:10.25836/sasp.2019.16