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# Tsallis and Kaniadakis Entropic Measures in Stellar Polytropes

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## Abstract

Polytropes are self-gravitating fluid spheres used in astrophysics as crude approximation of more realistic stellar models. They possess equations that have scale parameters linked to mass, energy and entropy. Since Boltzmann distribution yields unphysical results, the use of generalized entropies, such as Tsallis and Kaniadakis entropies, had been proposed. Here we discuss how these entropies are related in polytrope solutions.

**Keywords:** Entropy, Generalized Entropies.

## 1. Introduction

In astrophysics, a polytrope refers to a solution of the Lane–Emden equation. This is an equation which gives the pressure as a function of density [1]. These solutions are modelling self-gravitating fluid spheres that are called “polytropes” too, which are objects used as crude approximation to more realistic stellar models [2]. The solution of the Lane-Emden equation, a dimensionless form of Poisson's equation for the gravitational potential, depends on a parameter which is the polytropic index  $n$ . It is written as  $P=K\rho^{(n+1)/n}$ , where  $P$  is pressure,  $\rho$  is density and  $K$  a constant. If stellar structure is approximated with a polytrope having a given index, then two scaling parameters are needed to express the structure in physical units [3]. The two parameters that we can use are a constant which is related to entropy and the stellar mass. Since Boltzmann distribution yields unphysical results, the Boltzmann entropy had been substituted by a generalized entropy, the Tsallis entropy [4]. Another generalized entropy, the Kaniadakis entropy, had been recently proposed too, in [5]. Here we discuss how these two entropies are related in polytrope solutions, and that the result given in [5] can be easily obtained from [4].

## 2. The entropies

Well-known is the entropy proposed by Claude Shannon in 1948 [6].

He defined the entropy  $H$  of a discrete random variable  $X$ , as the expected value of the information

content:  $H(X) = -\sum_i p_i \log_b p_i$ . The probability of i-event is  $p_i$  and  $b$  is the base of the used logarithm. However, several entropies exist which are generalizing Shannon entropy. Among them we have Tsallis and Kaniadakis entropies [7,8], which are defined, with a corresponding choice of measurement units equal to 1, as follow:

$$(1) \quad T_{\text{Tsallis}} : T = T_q = \frac{1}{q-1} \left( 1 - \sum_i p_i^q \right)$$

$$(2) \quad \text{Kaniadakis } (\kappa \text{ - entropy}) : K_\kappa = -\sum_i \frac{p_i^{1+\kappa} - p_i^{1-\kappa}}{2\kappa}$$

In (1) and (2) we have the entropic indices  $q$  and  $\kappa$ . For its generalized additivity, the Kaniadakis entropy requires another function, defined as follow:

$$\mathfrak{S} = \sum_i (p_i^{1+\kappa} + p_i^{1-\kappa}) / 2$$

A detailed discussion of the generalized additivity of Tsallis and  $\kappa$ -entropy is given in [9]. Tsallis and Kaniadakis entropies are linked:

$$(3) \quad K_\kappa = \frac{T_{1+\kappa} + T_{1-\kappa}}{2},$$

$$\text{where } T(q = 1 + \kappa) = -\frac{1}{\kappa} \sum_i p_i^{1+\kappa} + \frac{1}{\kappa} ; \quad T(q = 1 - \kappa) = \frac{1}{\kappa} \sum_i p_i^{1-\kappa} - \frac{1}{\kappa}$$

Eq.(3) is a simpler form of an expression given in [10,11]. However, besides this relation, because of the generalized additivity possessed by the Kaniadakis entropy, we need also another relation:

$$(4) \quad \mathfrak{S}_\kappa = \frac{\kappa}{2} \left( -T_{1+\kappa} + T_{1-\kappa} + \frac{2}{\kappa} \right)$$

In (3) and (4), we have Kaniadakis functions expressed by Tsallis entropy. As shown in [12], we can also write  $T$  expressed by means of Kaniadakis functions:

$$2K + \frac{2}{\kappa} \mathfrak{S} = T_{1+\kappa} + T_{1-\kappa} + \left( -T_{1+\kappa} + T_{1-\kappa} + \frac{2}{\kappa} \right)$$

$$\text{And then : (5) } K_\kappa + \frac{1}{\kappa} \mathfrak{S}_\kappa = T_{1-\kappa} + \frac{1}{\kappa}$$

Let us have:  $\kappa=1-q$ . From (5) we have immediately the relation between Tsallis and Kaniadakis functions:

$$(6) \quad T_q = K_{1-q} + \frac{\mathfrak{S}_{1-q} - 1}{(1-q)}$$

### 3. With polytropes

The relation (6) between Tsallis and Kaniadakis entropies can be useful in several problems. Here we consider its use in polytropes. In the previous equations, we have  $p_i$  denoting the probability distribution. In Ref.4, it is used letter  $f$  for probability. From now on, we will use this notation. In [4], the distribution from Tsallis entropy is:

$$(7) \quad C_q(f) = \frac{f(1-f^{q-1})}{q-1}$$

After Eq.6, we can write Eq.7 in the following manner:

$$(8) \quad C_q(f) = \frac{f-f^q}{q-1} = \frac{1}{2(q-1)}(f^{2-q} - f^q) - \frac{1}{2(q-1)}(f^{2-q} + f^q) + \frac{f}{q-1}$$

Of course, (7) and (8) are the same equation. As a consequence, Kaniadakis distributions are linked to Tsallis distribution by:

$$(9) \quad t_q(f) = C_q(f) = \frac{(f-f^q)}{q-1} = k_{1-q} - g_{1-q} + \frac{f}{q-1}$$

$$\text{where } K_{1-q} = \sum_i k_{1-q}, \quad \mathfrak{S}_{1-q} = \sum_i g_{1-q}, \quad 1 = \sum_i f$$

From [4], a relation exists between polytrope index and entropic Tsallis index:

$$(10) \quad n = \frac{3}{2} + \frac{1}{q-1}$$

As a special case, for  $q \rightarrow 1$ , we find the isothermal situation. To have Eq.6, as shown in [12], we need  $\kappa=1-q$  or  $\kappa=q-1$ . Then, from (10), considering that we have for the Kaniadakis index,  $-1 < \kappa < 1$ :

$$(11) \quad n = \frac{3}{2} + \frac{1}{\kappa}$$

And in fact, (11) is the relation that we find in [5]. Using then the relation between Tsallis and Kaniadakis entropies and distributions we can easily find results concerning several applications. Polytopes are an example of such a possible approach.

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