Al-Biruni and the Mathematical Geography

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Abstract

Abu Raihan Al-Biruni (973 - 1048) is considered one of the intellectual giants of humankind. He was an astronomer, physicist and geographer, that distinguished himself as linguist and historian too. Here we discuss his major contributions to the mathematical geography of Middle Ages. Keywords: History of Science, Medieval Science, Al-Biruni, Mathematical Geography, Geodesy. Author: A.C. Sparavigna, DISAT, Politecnico di Torino, Italy.

Introduction

During the Middle Ages, Khwarezm, a huge region of the western Central Asia, also known as Chorasmia or Khorasam, generated a large number of scientists and scholars. Albumasar, Alfraganus, Al-Farabi [1], Avicenna, Omar Khayyam, Al-Khwarizmi and many others were born there. Across this region, goods moved in long caravans on the Silk Road linking Europe to India and China, and scientific culture moved with them. In this stream of goods and knowledge, we find Abu Raihan Al-Biruni, one of the greatest minds of all time [2].

Al-Biruni was born in 973 AD in the district of Kath, the old capital of Chorasmia, a town situated near the Aral Sea. He had as first teacher an educated Greek. His foster father, Mansur, a member of the royal family and distinguished mathematician and astronomer, introduced him to Euclidean geometry and Ptolemaic astronomy [3].

Youth Al-Biruni soon became a master of geometry and astronomy, such as of several other sciences, including mineralogy, geography and cartography. Being able of giving latitude and longitude of any point of the compass, Al-Biruni used in his geographic works the data that he directly collected during the travels made to escape from wars or in his constant search for a patron. In this manner he became aware that, in many cases, geographic information given by the Muslim astronomers was better than that reported by Ptolemy.

In fact, it was long before Al-Biruni's time that Muslim scholars started their researches on mathematical geography and geodesy. A success obtained by them was the measurement of the meridian arc, measurement made by astronomers of Caliph Al-Mamun. From this measure, Earth's circumference had been deduced and compared to the value given by ancient Greeks. Al-Biruni, eager to find his own value, proposed a new method based on sine trigonometric functions. Using it, he measured the Earth's circumference finding a value quite close to the modern one. As we will see in this paper, Al-Biruni also proposed new methods for measuring longitude and new cartographic projections, some of them reinvented after centuries by modern cartographers. However, before discussing these Al-Biruni's achievements, it is useful to talk shortly of the abovementioned result of Muslim geodesy, that is, the measurement of the arc corresponding to one degree of latitude, ordered by Abbasid Caliph Al-Ma'mun during his reign.

The length of a meridian arc

In geodesy, a meridian arc is the distance between two points with the same longitude. Measurements of arcs at different latitudes allow determining a reference ellipsoid which can be used to define the Figure of the Earth. Assuming our planet as a sphere, it is enough to measure a single arc, from which it is easy to obtain the Earth's circumference. An earliest estimation of it was that given by Eratosthenes (third century BC). After more than a thousand of years, in 9th century, an Abbasid Caliph ordered the first test of this value. Probably in year 820 AD, the astronomers of Caliph Al-Ma'mun determined the equatorial Earth's circumference being 20400 miles. Through the Compendium on the Science of Stars, written by Al-Farghani (Alfraganus), and translated in Latin by John of Seville and Gherardo da Cremona [4], this value of Earth's circumference was well known in Western Europe during the Middle Ages [5,6].

According to Alfraganus, a 'mile' is equal to 4000 'black cubits', which, in the Latin translations, became 'mean cubits'. Carlo Alfonso Nallino, an Italian orientalist, considered the black cubit made of 24 digits and equal, in average, to 0.4933 m; therefore, the 'mile' used by Alfraganus was 1973.2 meters long. In this manner, the meridian arc measured during the reign of Al-Ma'mun, corresponding to one degree and 56 2/3 miles long, was equal to 111.8 km, a very good result indeed when compared to the modern value of 111.3 km [5].

In surveying the Arabic literature reporting about Al-Ma'mun measurement, Nallino found that some authors are giving values of the arc different from that reported by Alfraganus. According to the Ibn Khallikan's Biographical dictionary (Cairo 1229), about Musa b. Shakir, after reading in the ancient books the value of the meridian arc Caliph Al-Ma'mun commissioned the three sons of Musa bin Shakir, the Banu Musa [7], to test this value [6]. The Banu Musa asked where they could find some flat lands suitable for this measurement; the answer was the desert near Senjar in northern Mesopotamia or the lowland of Kufah. They composed a team of expert people and went to the desert of Senjar. They stopped in a place and measured the altitude of North Celestial Pole. They planted a pole in the ground and attached a rope to it. They moved straight northwards on the flat land. When the rope ended, they planted another pole, used another rope, and moved again due North. This operation was repeated until they reached a place where the altitude of North Pole was one degree more than that observed at the starting place. The total length of ropes was 66 2/3 miles. They went back to the first pole and repeated the measurement southwards, until they reached the same total length of ropes. In this place, they found that the altitude of North Pole diminished of one degree. Therefore, they knew the measures were good. Banu Musa came back to the Caliph, who ordered to repeat the measurement in Kufah; so they did and obtained the same result. According to Ibn Khallikan, the length of the arc was 66 2/3 miles. According to Abu'l-feda, Shems Ed-din and several others [4], the Banu Musa had two teams, one moving northwards, that obtained a value of 56 1/2 miles, the other moving southwards, that obtained a value of 56 miles. They used the greater one. It is certain that Alfraganus considered the value obtained by Banu Musa of 56 2/3 miles [4]. This arc, multiplied by 360, gave a Earth's circumference of 20400 miles.

Of course, Al-Biruni, who dealt with Earth in many of his works [3,8], knew these measures of meridian arcs. We can imagine that he had some concerns about a method that required stretching several ropes on a quite long path. Instead of using measurements of lengths, he proposed to measure angles. In his method, Al-Biruni used the sine law, as we can see in the Figure 1. First, he determined the elevation of a hill, and then he climbed the hill and measured the angle of the dip of horizon [9].

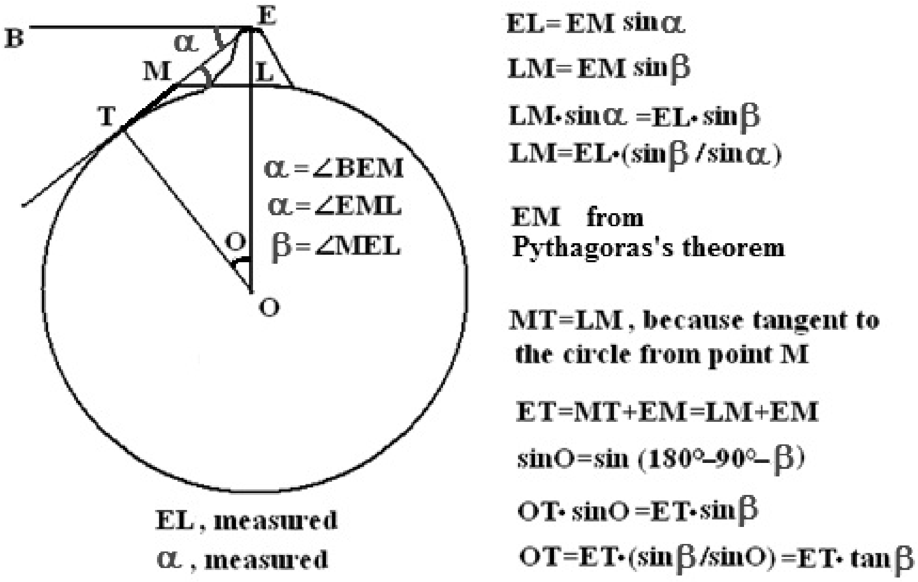


Figure 1 - Al-Biruni's method to measure Earth's radius [9].

The result obtained by Al-Biruni had been discussed in [2]. Let us just tell that, if the Arabic mile used by Al-Biruni was equal to 1.225947 English miles [10], his value of the Earth's radius was different of only 2% from the mean radius of curvature of the reference ellipsoid at the latitude of measurements [10] (Al-Biruni made measurements when he was at the Fort of Nandana in Punjab [11]).

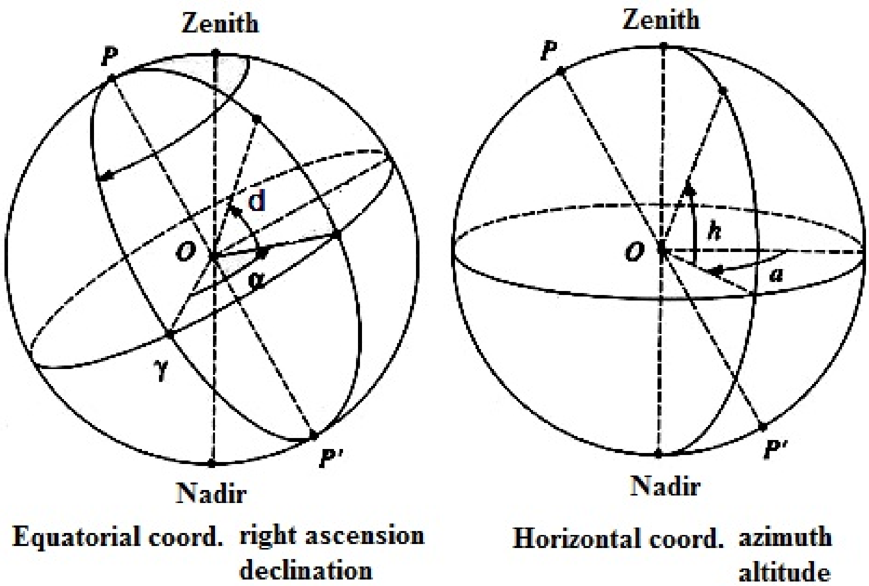


Figure 2 – Equatorial and horizontal coordinates.

Measuring latitude and longitude

Muslim scholars used the Earth's radius to calculate distance and direction to Mecca from any given point on Earth: this determined the Qibla, or Muslim direction of prayer. Therefore, Muslim mathematicians developed the spherical trigonometry which was necessary for these calculations. Al-Biruni worked on the determination of Qibla too, with his 'Demarcation of the Coordinates of Cities' [12]. In fact, once we know latitude and longitude of Mecca and of the place where a prayer is, these data can be applied to a spherical triangle, and the angle from prayer's meridian to the direction of Mecca determined.

Latitude can be obtained using equatorial and horizontal coordinates, that is, from the altitude h and declination d of the sun on the day of measurements. The declination is one of the two angles that locate a point on the celestial sphere in the equatorial coordinate system (in Figure 2, we can see equatorial and horizontal frames of reference). To determine the latitude, during the day we can use the sun at noon; by night, we can use the stars. Let us consider a circumpolar star. We can measure the two altitudes of the star when it is passing the meridian; the arithmetic mean of these two values is the latitude. Today, due to the precession of equinoxes, Polaris is almost coincident to the North Celestial Pole and therefore it is easy to find it and measure its altitude, but at Al-Biruni's time, it was a circumpolar star like others. According to [13], in the book on geographic coordinates, Al-Biruni describes the methods to determine latitude showing examples from past and contemporary literature.

As observed by Edward S. Kennedy in [13], since it is quite easy to determine latitudes, one could expect that the values we can find in medieval literature were accurate. This is not so. Let us consider, for instance, the 506 places of which Al-Kashi (about 1400 AD) is giving coordinates. 381 have values in agreement with the modern ones, that is, they differ from the modern values of just 4 minutes of arc in average. But the average of all the 506 values differs of more than a degree. This difference is explained by Kennedy in the following manner. Al-Kashi and other medieval geographers, could verify by themselves the latitude of a limited number of places, and therefore, in compiling their lists, they had to accept calculations made by other, probably not so competent, astronomers. However, let us stress that several of the data we can find in Arabic literature are providing many latitudes to within about a quarter of degree [13].

Prime meridians

Before discussing how longitude was measured, let us talk about the choice of the Prime Meridian. The geographical data coming from Arabic sources can be divided into two categories, determined by the Prime Meridian they used [13]. Ptolemy measured the longitudes eastward from the Fortunate Isles (i.e. Canaries): about a half of the Muslim sources made reference to him. This group is defined, for convenience, as the first group. The second group is following Al-Khwarizmi (ca. 820), who set the Prime Meridian of the Old World at the eastern oceanic shore, that is, 10 degrees East of the Prime Meridian of the first group. It is unknown the reason for this division. In fact, the astronomers of Al-Ma'mun decided that the longitude of Baghdad, the Abbasid capital, was 70 degrees. However, according to Ptolemy's geography, Baghdad would have 80 degrees of longitude, and more than a half of the Muslim sources gave this value. In any case, the existence of two categories is a fact. Longitudes of the same town, in the tables of these two groups, tend differing of just 10 degrees [13]. In fact, there is also a text which is reporting of a third Prime Meridian. Al-Hamdani (d. 946) states that the Orientals are measuring longitudes to the west from the east coast of China. Al-Hamdani provided the coordinates of several Indian towns and of towns located in Arabian Peninsula, including Jerusalem and Damascus too [13].

Measuring longitude

Once we have defined the Prime Meridian, finding longitude of a place becomes a problem of knowing the angle between this meridian of reference and that passing through the given place. It seems easy: because Earth is rotating of 360 degrees in 24 hours, the difference of longitude corresponds to the difference of local times. However, it is necessary a time signal, simultaneously available in these two places, and, in Middle Ages this was anything but simple [13]. A lunar eclipse can be used for this purpose, because it is possible to observe the evolution of it in the same manner from different points of the globe. Two observers, each in a different place, can establish the local times of the eclipse. The difference of these times corresponds to the difference of longitude [13]. Al-Biruni made such measurements, with the help of Abu'l-Wafa; he was in Kath and his friend in Baghdad.

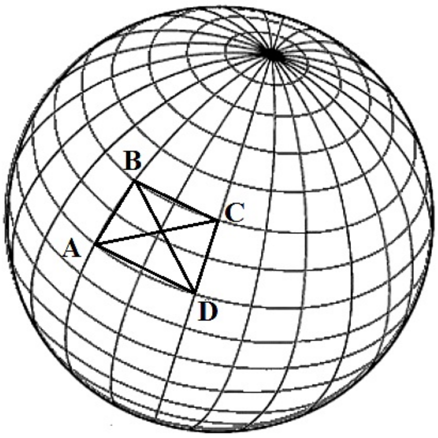


Figure 3 – An isosceles trapezoid for two places A and C on the Earth.

Al-Biruni proposed also a mathematical method for calculating the difference of longitude. Let us assume we know the latitudes of two places (A and C in the Figure 3) and their distance AC. In each of these two points are passing a meridian and a parallel, and therefore we have four circles, which intersect determining an isosceles trapezoid (ABCD). An isosceles trapezoid can be inscribed in a circle, and therefore we can use a theorem of Ptolemy on cyclic quadrilaterals. This theorems links sides and diagonals of trapezoids (it is given in the Figure 4).

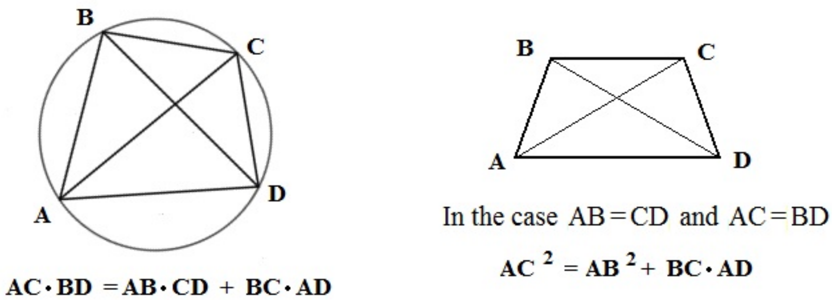


Figure 4 – Ptolemy's theorem applied to a cyclic quadrilateral (left) and to an isosceles trapezoid (right).

In applying the Ptolemy's theorem, Al-Biruni used the chord function (Figure 5). Chords were the preferred objects of early trigonometry. We find them in the first trigonometric table, the Hipparchus' table, where values of chord function are given for every 7.5 degrees. Ptolemy compiled a more extensive table of chords in his book on astronomy, giving the values by increments of half a degree. In the Figure 5 we see how angle, chord and sine functions are connected. Let us note that the name of trigonometric function 'sine' is coming from Latin 'sinus', which means ‘bend, curve, bosom’. Gherardo of Cremona used ‘sinus’ in his Latin translation of Arabic geometrical text to render Arabic ‘jiba’, ‘chord of an arc’, which he confused with ‘jaib’ ‘bundle, bosom’ [14].

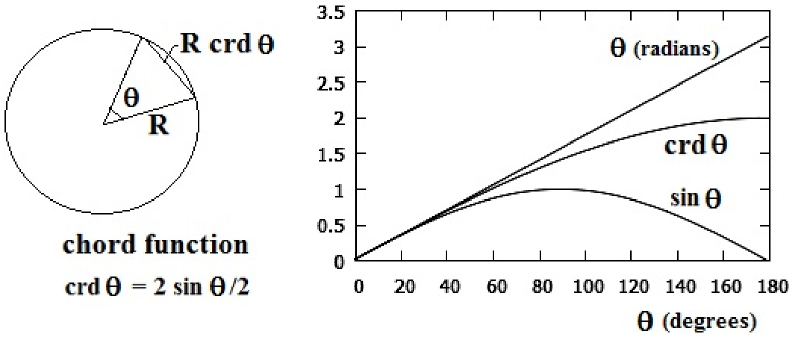
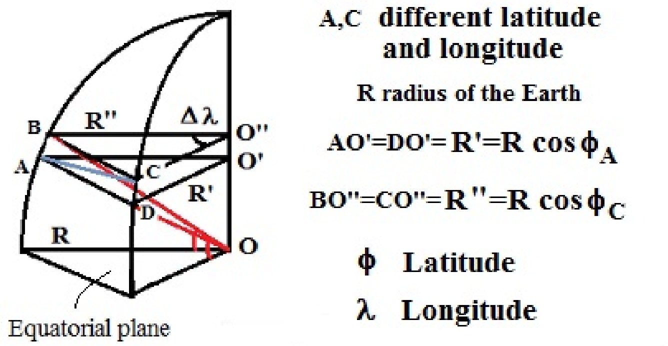


Figure 5 – The Chord Function and the Sine Function.

In Figure 6, we can see how Al-Biruni used Ptolemy’s Theorem. Let us consider the isosceles trapezoid of Figure 3. Knowing the distance AC and the difference of latitude between these two places, he found the chord of the difference of longitude, crd(Dλ), and using Ptolemy’s tables, the corresponding angle.



$$AC^2 = AB^2 + BC \cdot AD$$

$$AC^2 = R^2 crd^2 \Delta\phi + R \cos\phi_A \cdot R \cos\phi_C \, crd^2 \Delta\lambda$$

$$crd^2 \Delta\lambda = \frac{AC^2 - R^2 crd^2 \Delta\phi}{R \cos\phi_A \cdot R \cos\phi_C}$$

Figure 6 – The method of Al-Biruni for determining longitude, applied to isosceles trapezoid ABCD, made by chords corresponding to arcs of meridians and parallels.

To have the distance AC, Al-Biruni used the lengths of caravan routes, renormalized by means of a certain coefficient depending on whether the route was more or less direct or difficult [13]. Let us note that, in applying the Ptolemy’s theorem, AC is a chord, not an arc. Al-Biruni, using the lengths of caravan routes was estimating the arc, not the chord. However, assuming arc and chord differing of a negligible quantity, we can evaluate crd(Dλ) as in Figure 6 and convert the result in degrees.
To find the difference of longitude between Baghdad and Ghazni (in modern Afghanistan), Al-Biruni applied his own method using three different paths. The result he proposed was the arithmetic mean of these three results. In [13], it is told that the final value of longitude had an accuracy of about 1.5%. Between Baghdad and Ghazni, there is a difference of longitude of about 24 degrees. Given an Earth’s radius of 6371 km, for an angle of 24 degrees at a latitude of 33 degrees, the corresponding arc is 2238.1 km long and the chord 2221.8 km long. A difference of less than 20 km

is negligible, when compared to the uncertainty of a distance obtained from the lengths of caravan routes. However, it is my opinion that, in some manner, Al-Biruni considered the difference between arc and chord in renormalizing the distance.

Lists of places and cartography

A collection of lists of places with their latitudes and longitudes reveals the large geographical knowledge of the medieval Muslim world. We can find these lists in books on astronomy, geography and compilations used to prepare maps [13] or to determine the Qibla [15]. In classical antiquity, maps were drawn by Anaximander, Hecataeus of Miletus, Herodotus, Eratosthenes and Ptolemy, and were prepared after the observations by explorers in a mathematical framework. It is almost certain that the world map of Ptolemy was available to geographers of Abbasid empire. During Al-Ma'mun reign, the geographers of his House of Wisdom prepared a new large map of the world. This was one of the fruits of the collaboration between scientists stimulated by this Caliph.

Around 1005, Al-Biruni wrote a little work on the mapping of the globe, entitled 'The Flat Projection of Figures and Balls' [13]. In this treatise, Al-Biruni discussed eight types of map projections. One is the azimuthal equidistant projection, today used for polar projections, which shows all meridians as straight lines (the flag of United Nations is an example of such a polar azimuthal projection). Al-Biruni describes this projection in mechanical terms as rolling a ball over the paper. More than five centuries later, a primitive example of such a method was provided by the map of the world drawn by Ahmad Al-Sharafi Sarafi of Sfax in 1571 [13]. He did not know the work of Al-Biruni, and the same was for Guillaume Postel, the first to apply this method in Europe in 1581 [13].

Another method proposed by Al-Biruni was that which is known as Nicolosi globular projection. This method had no significant application in the East. It reappeared in Europe in 1660, when Giambattista Nicolosi published examples of it in two representations of east and west hemispheres. Another example appeared in Paris in 1676, and then others followed [13]. In 1701, the French scientist Philippe de la Hire described a mapping method very similar to the globular projection. The English cartographer Aaron Arrowsmith, in the notes accompanying a world map of 1794, tells that he is using the method proposed by de la Hire because it is the best. Moreover, he describes the construction of the grid of coordinates in exactly the same manner as Al-Biruni did [13]. Let us conclude with Kennedy's words [13]; it is not appropriate to say that Al-Biruni would directly influenced Arrowsmith, but it is astonishing that two scientists, one of the eleventh and the other of the eighteenth century, found the same good solution to represent the globe.

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