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3th August 2019

Abstract. The true difficulty of the twin paradox does not reside in the algebra that shows that the traveling twin ages less than the twin who stays at home. The truly startling part of the paradox resides in the much more difficult question why the argument cannot be reversed by symmetry, because there is no such thing as a preferred reference frame, and relative motion ought to be symmetrical. Can the traveling twin not claim with equal rights to have stayed at home while the other twin has made the journey? Most of the time text books invoke the accelerations intervening in the trip to explain the asymmetry. We will show that one can formulate and solve the paradox without making any reference to accelerations. There is actually something very simple that has been overlooked. In drafting the protocol which defines the journey, we unwittingly pick a preferred refrence frame, because we define the protocol with respect to a given frame, which thereby becomes special. It is this selection of a special reference frame which introduces the asymmetry. Hence, it is the reference frame wherein we define the protocol that will act like an absolute frame and whose unavoidable introduction breaks the symmetry between the twins. There is an infinity of protocols that can be selected to define a trip and each of these trips leads to its own corresponding twin paradox, with its own outcome as to which twin will age less. Whereas the individual trips of the two twins with respect to this protocol are asymmetrical, the set of all possible trips is symmetrical, such that the symmetry of the Lorentz group is indeed respected.

PACS. 03.30.+p Special relativity

The twin paradox [1] can be assumed to be sufficiently popularized to be even well known to the layman audience. Let us call our twins Sarah and Théo. Sarah will be the twin who travels from P to Q and then backwards from Q to P at a uniform velocity v. Her brother Théo will be the twin who stays at home in P. The true difficulty of the twin paradox is not the trivial calculation which shows that according to the theory of special relativity Sarah will turn out to have aged less than her twin brother Théo. That argument only scratches the surface of the paradox. The true paradox resides in the much more difficult question why Sarah cannot claim with equal rights that she stayed at home and that it is her brother Théo who did all the travelling and therefore should be younger than her according to the very same theory of special relativity. This is a serious, fundamental question because it addresses the self-consistency of the mathematical framework. In many text books, the asymmetry is explained on the basis of the accelerations that inevitably must intervene at the beacons which are defining the trip. Even Einstein has claimed this. This seems to make a lot of sense. It meets our physical intuition. What else is there we could get our hands on? What else is there in the form of a telltale difference that could constitute an objective cause for the asymmetry?

Most people will feel intimidated by the turn the problem is taking this way, because there are far less physicists who are fully acquainted with general relativity than there are with special relativity. To the ones who do not master general relativity this will be felt as a curse. The task - of getting fluent in general relavity first - looks so overwhelming and time-consuming that they may feel like abandoning their desire of further inquiring into the paradox all together. We want to break some good news for those people: The arguments based on accelerations are irrelevant! In fact, the accelerations are merely a self-inflicted smoke screen. We can happily ignore them as the whole argument can be developed without any reference to them.

The issue of the accelerations can be removed completely from the considerations by removing all physical objects from the description and treating the problem as a purely mathematical consequence of the Lorentz transformations of special relativity. After all, the paradox addresses a purely mathematical issue, viz. that of the self-consistency of the mathematical framework. People have presented this problem in terms of twins to render the presentation more palatable. However, this way they have also introduced the accelerations because physical objects cannot change speeds without accelerations. The resulting physical problem is then no longer equivalent to the initial, purely mathematical problem of the self-consistency of the space-time geometry. We think that by returning to the initial mathematical problem, our presentation can gain a lot in clarity.

In our approach we will severely limit the use we make of the full homogeneous Lorentz group SO(3,1), which is non-abelian and contains the group SO(3) of the rotations of $\mathbb{R}^3$. We will only use the abelian subgroup SO(1,1) of all boosts along the x-axis. This implies that we only consider linear paths and ban curved paths from the considerations all together. All this is motivated by our concern to keep things as simple as possible and to avoid introducing accelerations. In the further developments, we will need to use the extension of this group to a Poincaré-type group that allows also for translations in space and in time.

Let us consider a trip from P to Q at a velocity $\mathbf{v} = v\mathbf{e}_x$, followed by a trip from Q to P at a velocity $-\mathbf{v} = -v\mathbf{e}_x$. Of course, following standard physical thinking, there must be some short-time accelerations at P and Q. For the rest of the time, the motion could then be uniform. But we can try to think out of the box. We will consider for this purpose a Lorentz frame F_1 that does never accelerate and always travels at speed $\mathbf{v}$ with respect to Théo. Sarah's travel is considered to start when the origin of this frame F_1 passes in front of Théo who is situated at P. There is one frame F_1 in the whole set of frames that are always travelling with uniform velocity $\mathbf{v}$ with respect to Théo (in the Poincaré-type extension of SO(1,1)), that is of interest to us. The only difference between F_1 and the other frames is the moment at which it passes in P in front of Théo. We can set the clock in F_1 equal to zero at the moment it passes Théo. Now we consider also the whole set of frames that always travel with uniform velocity $-\mathbf{v}$ with respect to Théo. We select the frame F_2 that passes in Q at the very moment F_1 passes in Q. And at that moment we set the reading of the clock in F_2 equal to the reading of the clock in F_1. When F_2 will pass in P in front of Théo, we can register its reading and this will define mathematically the total traveling time for Sarah. In this scheme, which is purely mathematical and cannot really be carried out by a physical being like Sarah, there are no accelerations. Our formulation and definition are obviously exempt of any reference to accelerations. Admittedly, in this abstraction we no longer discuss real physical trips with real physical twins, but we can still discuss the basic issues. It is for the abstract problem formulated this way that we will solve the paradox.

The time dilatation is a purely mathematical effect of special relativity. What we have done in searching for our approach is finding out under which conditions we can formulate the problem under its pure mathematical form, such that it reveals the intrinsic truth within the mathematical theory beyond discussion. The discussion without the accelerations is so to say an idealized mathematical theory. It takes the form of a Gedanken experiment from which we have banned any pollution of the purely mathematical argument by what we consider as physical limitations of the experimental set-up for the measurement of the ideal result. We relegate the accelerations to the status of imperfections in the design of the experimental set-up. Classical theories are always idealized in the sense that the imperfections of the measuring device do not enter the theory. This way of presenting things is of course a purely mental view. But it permits to exclude the accelerations from the idealized theory by considering them as imperfections of our experimental set-up. The set-up must be designed such as to make the relative influence of the imperfections as small as possible.

The following argument shows how we can render the relative contribution of the accelerations to the age difference negligible in a physical experiment with real twins. We can make the accelerations take place over short distances PP_1, Q_1Q, QQ_1 and P_1P. The motions over P_1Q_1 and Q_1P_1 are then uniform. Whatever the effect of the accelerations may be, we can choose the distance P_1Q_1 and make it so long that the effect of the accelerations becomes negligible with respect to the age difference that builds up during the uniform motions over P_1Q_1 and Q_1P_1. This is is because the age difference builds up linearly along the distance P_1Q_1. Due to the homogeneity of space, increasing the distance P_1Q_1 will not change the effect of the accelerations over PP_1, Q_1Q, QQ_1 and P_1P. We can thus make the age difference as large as we like by increasing the distance P_1Q_1, a fact that must condemn any quantitative attempt to account for the assymetry between the two twins on the basis of the accelerations. This shows conclusively that attributing the asymmetry to the accelerations is logically flawed and raises the question of the true origin of the asymmetry.

We consider frames F_1 and F_1' that are passing in P and in Q when the time t on Théo's clock is zero. In the whole set of frames F_1^j with velocity $\mathbf{v}$ the distance PQ is Lorentz contracted, which explains why Sarah ages less. This is due to the fact that the whole trip is defined by points P and Q in Théo's frame. As a matter of fact, *the end points P and Q of Sarah's trip are at rest in his frame.* The whole trip is defined in Théo's frame. It is the definition of the trip which introduces the asymmetry. To reverse the argument to make it symmetrically valid for Sarah, we must define the trip differently. We must define the trip by beacons in space-time that are not defined in Théo's frame but in Sarah's frame. They should be at rest in Sarah's frame. There are no physical objects available to us that could play the rôle of the beacons required. All the physical objects we have at our disposal and that could play the rôle of beacons belong to Théo's frame. We must therefore introduce purely mathematical beacons rather than true physical objects in order to define the "symmetrical trip" of Théo with respect to Sarah. The beacons must also be traveling frames. We would have to launch rockets from remote positions in space at well chosen times to obtain objects that could serve as physical objects embodying the beacons which Sarah needs to define Théo's symmetrical trip. As this is not very practical, we consider the beacons as mathematical objects. With the appropiate mathematically defined beacons, it will be Théo who turns out to be the younger twin at the end of the trip.

We can even define the end points of the trip in a frame that travels at such a speed that Théo seems to travel at a speed $-\mathbf{w}$ and Sarah at a speed $\mathbf{w}$ with respect to it. Following Galilean logic this frame would have velocity $w = v/2$, but relativistically the value of w is different, due the addition rule for velocities in the Lorentz group, such that w is rather defined by the second-degree equation $v = 2w/(1 + w^2/c^2)$. This equation applies both to the to and fro parts of the relative trip. The equation has two algebraic solutions $w = \frac{c^2}{v}\frac{\gamma\pm 1}{\gamma}$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ as usual. The solution with the plus sign must be discarded because it would lead to superluminal velocities $w \geq c$. One can easily convince oneself that the other solution always implies $w \leq c$. If we defined the trip appropriately with beacons at rest in a frame with this velocity w, then both Théo and Sarah would have to travel and they would end up with the same age.

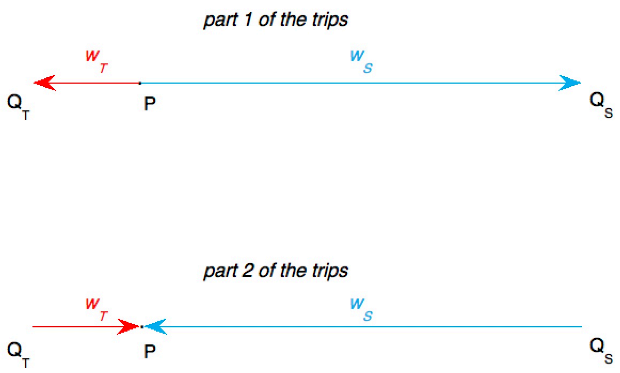

Fig. 1. The generalized scheme for the trips of Théo and Sarah in the reference frame wherein we define the protocol. In this protocol frame the beacons at Q_T and Q_S are fixed. In the first part, the twins move out. In the second part they join one other again. Both trips have the same duration within the protocol frame. The classical twin paradox is obtained in the limit $w_T \to 0$. The point Q_T coincides then with P. The symmetrical journey, where Théo ages less, is obtained in the limit $w_S \to 0$. Q_S coincides then with P. But this symmetrical journey does not correspond to Théo's relative motion with respect to Sarah in the journey that corresponds to the limit $w_T \to 0$. We can consider a whole continuum whereby the relative motion occurs with velocity $v = (w_s + w_T)/(1 + w_S w_T/c^2)$, and the ratio $w_T/w_S \in [0, \infty[$. One can even extend this to $w_T/w_S \in]-\infty, \infty[$, if we allow the twins to move out in the same direction.

To make an in-depth study of the asymmetry (see Fig. 1), we can thus choose first a reference frame wherein the two beacons one will use to define the trip are at rest. Towards this frame Sarah can have a speed w_S and Theo a speed w_T, while the relative speed of Sarah to Theo is $v = (w_S + w_T)/(1 + w_S w_T/c^2)$. We must then adjust the lengths of the trips of Sarah and Theo in such a way that we prevent that also the relative velocity $v = (w_S - w_T)/(1 - w_S w_T/c^2)$ enters the scene. Having two relative velocities within the protocol is actually not forbidden, but we want to keep things as simple as possible. This way the two twins will travel the same amount of time in the protocol frame. In the more specific cases $w_T = 0$ and $w_T = w_S$ treated above these constraints on the lengths are implicit, because one length is zero, or both lengths are equal. In the traditional twin paradox we are thus identifying the protocol frame with Théo's frame. We can also define the problem in the reverse way, by imposing v and a ratio w_S/w_T. After having fixed this problem we can then have $w_S > w_T$ such that Sarah ages less, $w_S = w_T$ such that both twins age at the same rate, or $w_S < w_T$ such Theo ages less. Each of these cases entails a different twin paradox. This will make it much more obvious that we are bound to pick a biased set of beacons to define a trip, and that this surreptitiously introduces a priviliged frame that seems to define some absolute rest. The bias we have fallen prey to in the standard presentation of the twin paradox is due to the contents of the set of physical objects that are available to us. With the mathematical beacons we can define trips at will. (One could even define a trip by two beacons that are not at rest in a same frame, but we will not consider this complication). For each paradox one can eliminate the accelerations by replacing Sarah and Theo by a number of frames whose origins pass the beacons at the right time. We can define an infinity of trip protocols, leading to an infinity of twin paradoxes. There is thus not just one twin paradox, but an infinity of them. Allowing for accelerations in the calculations is certainly possible but only adds algebraic complexity.

Some confusion can be created by our classical intuition based on Newtonian mechanics, where the relative motions conceptually just take the mathematical expressions $\mathbf{r}(t)$ (for Sarah's motion with respect to Théo) and $-\mathbf{r}(t)$ (for Théo's motion with respect to Sarah). We may embark that intuition unwittingly with us at the back of our mind in our quest for understanding. This is a red flag! Because in relativity this is plain wrong, as the two twins have different schemes of simultaneity. For the problem at hand, we must rule out any such inconscious appeal to our classical intuition right from the start by a conscious act of rejection. The symmetry between the journeys the two twins are carrying out is not of the Newtonian or Galilean type. The symmetry that prevails in relativity is of a different type. In relativity, the set S of all possible journeys is symmetrical just like in Galilean logic such that it contains both $\mathbf{r}(t)$ and $-\mathbf{r}(t)$. But in relativity the symmetry does not follow the Galilean logic all the way, because for a given trip, $-\mathbf{r}(t)$ is no longer the journey of Théo with respect to Sarah if the journey of Sarah with respect to Théo is $\mathbf{r}(t)$. It is rather of a type $\mathbf{r}'(t')$, where $\mathbf{r}'$ and t' must be carefully detailed, and certainly is not equal to $-\mathbf{r}(t)$. In other words, the expected relativistic symmetry duly exists but it applies to the complete set S of possible trips, while the detailed Galilean symmetry $\mathbf{r}(t) \leftrightarrow -\mathbf{r}(t)$ between two travelers in a single trip does no longer prevail. Very obviously, we should never have doubted about all this because SO(1,1) and SO(3,1) are groups and therefore automatically symmetrical. After introducing a prefered frame, the group will still remain symmetrical with respect to this frame. We have just been victims of a bias that exists within the set of the physical objects available to us in the physical world that surrounds us and which could serve as possible beacons for trips. But the correct full set of all possible beacons must be unbiased and very obviously reflect the full symmetry of the Lorentz group.

References

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