Seismic inverse problem using multi-components data with Full Reciprocity-gap Waveform Inversion
Florian Faucher

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Seismic inverse problem using multi-components data with Full Reciprocity-gap Waveform Inversion

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ICIAM 2019, Valencia, Spain
July 19\textsuperscript{th}, 2019

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Overview

1. Introduction

2. Time-Harmonic Inverse Problem, FWI
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   - Iterative reconstruction algorithm

3. Reconstruction procedure using dual-sensors data

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Florian Faucher – Reciprocity-gap Waveform Inversion – July 19th, 2019
Reconstruction of subsurface Earth properties from seismic campaign: collection of wave propagation data at the surface.

- Reflection (back-scattered) partial data,
- \textit{nonlinear, ill-posed inverse problem}. 

\begin{itemize}
  \item Source \quad \text{Surface } \Gamma \\
  \text{Receivers set } \Sigma \\
  \text{Subsurface area of interest } \Omega
\end{itemize}
Seismic exploration inverse problem

Reconstruction of subsurface Earth properties from seismic campaign: collection of wave propagation data at the surface.

- Reflection (back-scattered) partial data,
- nonlinear, ill-posed inverse problem.
Plan

2 Time-Harmonic Inverse Problem, FWI
- Dual-sensors data
- Iterative reconstruction algorithm
Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler’s equations, heterogeneous medium parameters $\kappa$ and $\rho$:

\[
\begin{align*}
-\imath \omega \rho(x) \mathbf{v}(x) &= -\nabla p(x), \\
-\imath \omega p(x) &= -\kappa(x) \nabla \cdot \mathbf{v}(x) + f(x).
\end{align*}
\]

$p$: scalar pressure field, $\kappa$: bulk modulus, $\rho$: density, $\mathbf{v}$: vectorial velocity field, $\omega$: angular frequency, $f$: source term,
We consider propagation in acoustic media, given by the Euler’s equations, heterogeneous medium parameters \( \kappa \) and \( \rho \):

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\end{align*}
\]

\( p \): scalar pressure field, \( \kappa \): bulk modulus, \\
\( \mathbf{v} \): vectorial velocity field, \( \rho \): density, \\
\( f \): source term, \( \omega \): angular frequency.

The system reduces to the Helmholtz equation when \( \rho \) is constant,

\[
(-\omega^2 c(x)^{-2} - \Delta) p(x) = 0,
\]

with \( c(x) = \sqrt{\kappa(x)\rho(x)^{-1}} \).
The quantitative inverse problem aims the recovery of the physical parameters from **surface field measurements**.

**Dual-sensors** record the pressure and vertical velocity:

\[ \mathcal{F}(m = (\kappa, \rho)) = \{ p(x_1), p(x_2), \ldots, p(x_{n_{rcv}}) \}; \]
\[ \{ v_n(x_1), v_n(x_2), \ldots, v_n(x_{n_{rcv}}) \}. \]

---

**D. Carlson, N. D. Whitmore et al.**  
Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer  
SEG, 2007 – 2010

**CGG & Lundun Norway (2017 – 2018)**  
TopSeis acquisition (www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis)
Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters by solving a minimization problem,

\[
\min_{m \in \mathcal{M}} J(m) = \frac{1}{2} \| \mathcal{F}(m) - d \|^2.
\]

- \(d\) are the observed data,
- \(\mathcal{F}(m)\) represents the simulation using an initial model \(m\):

- P. Lailly
  The seismic inverse problem as a sequence of before stack migrations
  Conference on Inverse Scattering: Theory and Application, SIAM, 1983

- A. Tarantola
  Inversion of seismic reflection data in the acoustic approximation
  Geophysics, 1984

- A. Tarantola
  Inversion of travel times and seismic waveforms
  Seismic tomography, 1987
FWI, iterative minimization

Observations → Initial model $m_0$  $k = 0$ → Forward problem $F_\omega(m_k)$ → Misfit functional $\mathcal{J}$
FWI, iterative minimization

Observations

Initial model $m_0$

$k = 0$

Forward problem $F_\omega(m_k)$

Misfit functional $\mathcal{J}$

Optimization procedure

1. Gradient
2. Search direction $s_k$
3. Line search $\alpha_k$

$k = k + 1$

update model

$m_{k+1} = m_k + \alpha_k s_k$

Update $\omega$
FWI, iterative minimization

Observers \rightarrow \text{Initial model } m_0 \rightarrow \text{Forward problem } F_\omega(m_k) \rightarrow \text{Misfit functional } J \rightarrow \text{Optimization procedure}

1. Gradient
2. Search direction \( s_k \)
3. Line search \( \alpha_k \)

\[ m_{k+1} = m_k + \alpha_k s_k \]

\[ k = k + 1 \]

Update \( \omega \)

Numerical methods

- Adjoint-method for the gradient computation, L-BFGS method,
- **Hybridizable Discontinuous Galerkin** discretization method,
- elasticity, anisotropy, viscosity.
Hybridizable Discontinuous Galerkin discretization:

- **global matrix** with the **faces d.o.f.** only,
- **local problem** to have the volume solution on DG d.o.f.

- Global matrix needs **less memory** than FE and DG (order),
- the local problems are small and embarrassingly parallel,
- 1st order: same accuracy for \( p \) and \( v \),
- topography, sub-surface shapes.
Plan

3 Reconstruction procedure using dual-sensors data
Iterative reconstruction with dual-sensors data

- Compare the pressure and velocity fields separately ($L2$):

$$J_{L2} = \sum_{source} \frac{1}{2} ||F_p^{(s)} - d_p^{(s)}||^2 + \frac{1}{2} ||F_v^{(s)} - d_v^{(s)}||^2.$$
Iterative reconstruction with dual-sensors data

- Compare the pressure and velocity fields separately ($L2$):

$$\mathcal{J}_{L2} = \sum_{\text{source}} \frac{1}{2} \| \mathcal{F}^{(s)}_p - d_p^{(s)} \|^2 + \frac{1}{2} \| \mathcal{F}^{(s)}_v - d_v^{(s)} \|^2.$$ 

- Compare the reciprocity-gap:

$$\mathcal{J}_G = \frac{1}{2} \sum \sum \| d_v^{(s_1)}^T \mathcal{F}^{(s_2)}_p - d_p^{(s_1)}^T \mathcal{F}^{(s_2)}_v \|^2.$$ 

G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich
Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization
ESAIM: M2AN, 2019.
Reciprocity-gap waveform inversion

\[ \mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d^{(s_1)}_v F^{(s_2)}_p - d^{(s_1)}_p F^{(s_2)}_v \|^2. \]

- Motivated by Green’s identity (using variational formulation).
Reciprocity-gap waveform inversion

\[ \mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_v^{(s_1)^T} F_{p}^{(s_2)} - d_p^{(s_1)^T} F_{v}^{(s_2)} \|^2. \]

- Motivated by Green’s identity (using variational formulation).
- Reciprocity-gap functional from inverse scattering with Cauchy data.

R. Kohn and M. Vogelius
Determining conductivity by boundary measurements II. Interior results

D. Colton and H. Haddar
An application of the reciprocity gap functional to inverse scattering theory
Inverse Problems 21 (1), 2005, 383398.

G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich
Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization
ESAIM: M2AN, 2019.

T. van Leeuwen and W. A. Mulder
A correlation-based misfit criterion for wave-equation traveltime tomography
Lipschitz-type stability for the Helmholtz equation with partial data,
\[ \| m_1 - m_2 \| \leq C (J_G(m_1, m_2))^{1/2}, \]
Stability results

Lipschitz-type stability for the Helmholtz equation with partial data,

\[ \| m_1 - m_2 \| \leq C(\mathcal{J}_G(m_1, m_2))^{1/2}, \]

- for piecewise linear parameters.
- Using back-scattered data from one side in a domain with free surface and absorbing conditions,

G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich
Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data

G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich
Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization
ESAIM: M2AN, 2019.
Main feature

It allows the separation of numerical and observational sources:

\[ J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_v^{(s_1)^T} F_p^{(s_2)} - d_p^{(s_1)^T} F_v^{(s_2)} \|^2. \]
Main feature

It allows the separation of numerical and observational sources:

\[ J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_v^{(s_1)} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)} \mathcal{F}_v^{(s_2)} \|^2. \]

- \( s_1 \) is fixed by the observational setup,
- \( s_2 \) is chosen for the numerical comparisons,
- arbitrary positions of computational source,
- no need for a priori information on the observational source: position and wavelet are not required,
- not possible with the traditional misfit.
Numerical experiments

- Experiments for acoustic media
- Comparison of misfit functions
- Changing the numerical acquisition with $\mathcal{J}_G$
- Extension toward elasticity
Experiment setup

3D velocity model $2.5 \times 1.5 \times 1.2$km using dual-sensors data.
Experiment setup

We work with time-domain data acquisition.
Experiment setup

We work with time-domain data (pressure and velocity).

For the reconstruction, we apply a Fourier transform of the time data.
Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $J_{L2}$ or $J_G$, frequency from 3 to 15Hz.

$$J_{L2} = \sum_{\text{source}} \frac{1}{2} \| \mathcal{F}_p^{(s)} - d_p^{(s)} \|^2 + \frac{1}{2} \| \mathcal{F}_v^{(s)} - d_v^{(s)} \|^2.$$

$$J_G = \frac{1}{2} \sum_{\text{source}} \sum_{\text{source}} \| d_v^{(s_1)^T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)^T} \mathcal{F}_v^{(s_2)} \|^2.$$
Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $J_{L2}$ or $J_g$, frequency from 3 to 15Hz.

(a) True velocity

(b) Starting velocity
Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $I_{L2}$ or $I_G$, frequency from 3 to 15Hz.

(a) Using $I_{L2}$

(b) Using $I_G$
Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $J_{L2}$ or $J_G$, frequency from 3 to 15Hz.

But the major advantage of $J_G$ is the possibility to consider alternative acquisition setup.
Experiment with different obs. and sim. acquisition

\[
\min J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \left\| d_v^{(s_1)} T F_p^{(s_2)} - d_p^{(s_1)} T F_v^{(s_2)} \right\|^2.
\]

Acquisition for the measures \( s_1 \)

- 160 sources,
- 100 m depth,
- point source,
Experiment with different obs. and sim. acquisition

$$\min \mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_{s_1}^T F_{p}^{(s_2)} - d_{p}^{(s_1)} F_{v}^{(s_2)} \|^{2}.$$ 

Acquisition for the measures $s_1$

- 160 sources,
- 100 m depth,
- point source,

Arbitrary numerical acquisition $s_2$

- 5 sources,
- 80 m depth,
- multi-point sources,

- No need to known observational source wavelet.
- Differentiation impossible with least squares types misfit.
Experiment with different obs. and sim. acquisition

Data from frequency between 3 to 15 Hz, domain size $2.5 \times 1.5 \times 1.2$ km, **Simulation using 5 sources only**.
Experiment with different obs. and sim. acquisition

Frequency from 3 to 15 Hz, $2.5 \times 1.5 \times 1.2$ km, **Simulation using 5 sources only.** -33% computational time.

(a) True velocity

(b) 15 Hz reconstruction
Reciprocity-gap for elasticity

$$-
\n\n\frac{\nabla \cdot \sigma(x)}{\nabla \cdot \sigma(x)} - \frac{\omega^2 \rho(x)}{\nabla \cdot \sigma(x)} u(x) = g(x),
$$

$\sigma$ is the stress tensor; elastic isotropy, Lamé parameters $\lambda$ and $\mu$:

$$
\sigma = \lambda \text{Tr} (\epsilon) I + 2 \mu \epsilon.
$$

Three (heterogeneous) parameters to characterize the medium:

- $\lambda$ and $\mu$, or $c_p = \sqrt{\frac{\lambda + 2 \mu}{\rho}}$, $c_s = \sqrt{\frac{\mu}{\rho}}$
- Density $\rho$.

Increased computational requirement and the ill-posedness of the inverse problem.
Reciprocity-gap for elasticity

Wave propagation in elastic media

\[-\nabla \cdot \sigma(x) - \omega^2 \rho(x)u(x) = g(x),\]

\(\sigma\) is the stress tensor; elastic isotropy, Lamé parameters \(\lambda\) and \(\mu\):

\[\sigma = \lambda \text{Tr}(\epsilon)I_d + 2\mu\epsilon.\]
Reciprocity-gap for elasticity

Wave propagation in elastic media

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\(\sigma\) is the stress tensor; elastic isotropy, Lamé parameters \(\lambda\) and \(\mu\):

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Three (heterogeneous) parameters to characterize the medium:

- \(\lambda\) and \(\mu\), or \(c_p = \sqrt{(\lambda + 2\mu)/\rho}\), \(c_s = \sqrt{\mu/\rho}\)
- Density \(\rho\).

Increased computational requirement and the ill-posedness of the inverse problem.
Elastic reciprocity formula

In elasticity, reciprocity needs measurements of $\sigma$ and $u$

$$\mathcal{F}(m := (\lambda, \mu, \rho)) = \{ u(x) \mid_{\Sigma}, (\sigma(x) \cdot n) \mid_{\Sigma} \}. $$
Elastic reciprocity formula

In elasticity, reciprocity needs measurements of $\sigma$ and $u$

$$\mathcal{F}(m := (\lambda, \mu, \rho)) = \left\{ u(x) \mid \Sigma, \ (\sigma(x) \cdot n) \mid \Sigma \right\}.$$

$$\mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \left\| d_u^{(s_1)} T \mathcal{F}_{\sigma \cdot n}^{(s_2)} - d_{\sigma \cdot n}^{(s_1)} T \mathcal{F}_{u}^{(s_2)} \right\|^2.$$
Pluto model

2D elastic models of size $31 \times 7$ km.

(a) P-wave speed  
(b) S-wave speed  
(c) Density
Pluto model

2D elastic models of size $31 \times 7\text{km}$.

(a) P-wave speed

(b) S-wave speed

(c) Density
Reconstruction setup

- The density remains fixed; frequency from 0.50 to 7 Hz,
- Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.

### Observational acquisition:
- 150 sources,
- 20 m depth.

### Computational acquisition:
- 30 sources,
- 10 m depth.
Reciprocity waveform inversion

- The density remains fixed; frequency from 0.50 to 7 Hz,
- Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.
Reciprocity waveform inversion

- The density remains fixed; frequency from 0.50 to 7 Hz,
- Low frequency could be replaced by complex frequency (Laplace domain) or a priori information.
Plan

5. Conclusion
**Conclusion**

**Quantitative time-harmonic inverse wave problem:**
- Hybridizable Discontinuous Galerkin discretization, HPC,
- large scale optimization scheme using back-scattered data,
- acoustic, elastic, anisotropy, 2D, 3D, attenuation,
- stability and convergence analysis.

**Reciprocity-gap waveform inversion:**
- minimal information on the acquisition setup,
- reduced computational cost,
- other applications (elasticity, seismology, helioseismology),
- **perspective:** design efficient setup; data (rotational seismology).
Conclusion

Quantitative time-harmonic inverse wave problem:
- Hybridizable Discontinuous Galerkin discretization, HPC,
- large scale optimization scheme using back-scattered data,
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Thank you
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H. Barucq, H. Calandra, G. Chavent, F. Faucher
A priori estimates of attraction basins for velocity model reconstruction by time-harmonic Full Waveform Inversion and Data Space Reflectivity formulation
G. Alessandrini, S. Vessella
Lipschitz stability for the inverse conductivity problem
Adv. in Appl. Math., 2005
E. Beretta, M. V. de Hoop, F. Faucher, O. Scherzer
Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates
SIAM Journal on Mathematical Analysis, 2016
G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich
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Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data

M. Grote, M. Kray, U. Nahum
Adaptive eigenspace method for inverse scattering problems in the frequency domain

M. Grote, U. Nahum
Adaptive eigenspace for multi-parameter inverse scattering problems

H. Barucq, F. Faucher, O. Scherzer
Eigenvector Model Descriptors for Solving an Inverse Problem of Helmholtz Equation