Emerging gravitation in an accelerated expansion of the Universe – Probable natures of inertia, dark matter and dark energy

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***Abstract*— Since the 2000s, many astrophysical observations have led to establishment of a standard model of cosmology, based on the existence of dark matter and dark energy to explain formation and the future of the Universe. Others theories like MOND (Modified Newtonian Dynamics) theory [1] or entropic gravity theory [2] give different explanations on universal gravitation theory either in order to explain galaxy rotation curve regardless existence of dark matter or to explain the origin of gravitational field and curvature of space-time by the mass. In the theory of relativity, the curvature of the space–time is imputed to the presence of mass or energy but no explanation is given to link presence of mass and curvature of the space-time. In other words, how, fundamentally, mass distorts space-time? How to explain, fundamentally, the equality between gravitational and inertial masses? This paper proposes to establish some theories to explain origin of inertia and by consequent, explain how mass distorts space-time and creates gravitational field. For that, this article proposes first to establish a relation linking the gravitational constant G to the acceleration of the expansion of the Universe called Ψ. Moreover, this article proposes to model the evolution of the global Universe's "scale factor" without taking account of general relativity. That permits to explain nature of dark energy and unifies Hubble constant H to gravitational constant G as well as retrieving the literal value of the cosmological constant. Furthermore, explanation of origin of inertia needs to introduce a new form of gravitation field similar to magnetic field in the Maxwell electromagnetic theory and inspired by gravitoelectromagnetism theory. The new gravitational field, whose origin is linked to the movement of mass, permits to retrieve some general relativity's results including polarization of gravitational waves predicted by Einstein as well as positions of photon sphere or innermost stable circular orbit in the case of non-rotary and electrically neutral central mass like a Schwarzschild black hole. It even permits to retrieve the general relativity's calculation of apsidal precession of an astronomical body's orbit in case of weak field approximation. Finally, this article proposes a model able to explain galaxy rotation curve as well as the evolution of their characteristic size related to the evolution of scale factor of the Universe and regardless existence of dark matter as unknown matter or regardless MOND theory.**

***Index Terms*— Gravitational constant, Acceleration of the expansion of the Universe, Energy density of the quantum vacuum, Cosmological constant, Dark mater, Dark energy, Hubble constant, Extraordinary gravitation field, Galaxy rotation curve**

I. General introduction

Two major discoveries have permitted to advance the cosmology science during the last 70 years without counting the advent of discoveries of Big Bang and the cosmic microwave background. The first one is the discovery of the non-ordinary mass distribution in galaxy M31 by Van de Hulst **[3]** and in galaxy M33 by Louise Volders **[4]**. The second one is the discovery of the acceleration of the expansion of the Universe by two independent projects in 1998 (the Supernova Cosmology Project and the High-Z Supernova Search Team) in measuring type Ia supernovae redshift and their apparent magnitude **[5]**. These two discoveries have led to consider that Universe is probably composed of dark matter and dark energy permitting respectively to explain formation of large-scale structure of the Universe and the current positive measurement of acceleration of the expansion of the Universe despite the fact that Universe is composed of around 10^{80}massive particles **[6]**, which should normally decelerate its expansion. However, nowadays, natures of dark matter and dark energy remain unknown and no dark matter particles as unknown particles have been yet detected even with advanced sensor technology **[7]**. Initial performance of the modern COSINE-100 experiment reproducing the DAMA/LIBRA experiment questions its conclusions about detection of annual modulation signal due to presence of dark matter particles **[8]**. Moreover, we can ask ourselves why Milky Way's dark matter halo which is supposed to be around 6.7 to 33 times more massive than radiant matter of the Milky Way **[9][10]** do not collapse to form dark matter black holes (even in form of a cloud) instead of having a spherical distribution? Even if that is not possible to form a compact object of dark matter, it could however be possible theoretically to collect enough dark matter particles in a volume included into a Schwarzschild radius to curb enough space-time to form massive black holes in the Universe.

This affirmation is all the more relevant that dark matter does not interact electromagnetically and its compaction should be much easier than ordinary matter (baryon) for which, electromagnetic interactions and electron degeneracy pressure impeach gravitational collapse.

Indeed, for a given density of matter ρ, Schwarzschild black hole radius is given by:

$$R_{SBH}^{*} = \sqrt{\frac{3c^2}{8\pi G\rho}} \quad (0)$$

With G, the gravitational constant and c, the celerity of light in the vacuum. Even if we do not exactly know global density of dark matter in the Universe, an estimated density of dark matter could be $\rho_{DM} \sim 3\times 10^{-29}\mathrm{g}.cm^{-3}$ **[11]**, which is potentially higher than critical density. That means dark matter cloud could form a black hole in our observable Universe of nearly radius of 8 billion light year, which is less than the radius of the observable Universe. Moreover, existence of this density of dark matter involves a curved shape of the Universe. Therefore, imposing existence of dark matter, as a cloud unable to collapse like ordinary matter, would lead to contradiction considering results from WMAP (Wilkinson Microwave Anisotropy Probe) revealing that Universe is flat with 0.4% margin of error **[11]**.

Our article does not try to deny existence of dark matter but questions about its nature. It proposes a debate about potential nature of what dark matter could really be. This article proposes also to establish the probable nature of dark energy and highlights potential existence of a new gravitational field with different physical properties, compare to the classical known gravitational field's ones.

II. The mass as inertia of the expansion of space-time

A. *Physical concept using weak field approximation*

Let be an electrically neutral and non-rotary spherical mass m with a radius R_0. An infinitesimal volume in spherical coordinate, if we supposed to be in a Euclidian space, has for expression $dV = 4\pi r^2 dr$. In a Schwarzschild metric, Because of the central mass m this infinitesimal volume is expanded of value $\Delta(dV)$ as:

$$\Delta(dV) = \Delta(4\pi r^2) \times dr + 4\pi r^2 \times \Delta(dr) \quad (1)$$

Considering m as a weak mass ($\frac{2Gm}{c^2} \ll R_0 \leq r$), expression of $\Delta(dV)$ becomes:

$$\Delta(dV) \cong \frac{8\pi Gmr}{c^2}\left(2 \times \ln\left(\frac{r}{R_0}\right) + 1\right) \times dr \quad (2)$$

Generating expansion of the volume dV has consequence to accumulate energy into expanded volume $\Delta(dV)$ due to the work of a supposed constraint applied on slice thickness dr of the space. In Newton gravitation theory of weak field, gravitational force is in r^{-2}, so by a reaction mechanism, we can consider that for $r = R_0$ constraint applied on the volume dV surrounding mass m has for expression:

$$F(r = R_0) = \frac{\eta}{R_0^2} \quad (3)$$

With η, a homogenous parameter that we are not trying to express. Metric constraint for $r = R_0$ applied to the constraint F, noted F_m, is given by the following expression:

$$F_m(R_0) = -\left.\frac{dF}{dr}\right|_{r=R_0} = \frac{2F(r = R_0)}{R_0} \quad (4)$$

Let be a cubic parallelepiped at $r = R_0$ with an area per face $s \ll A = 4\pi R_0^2$ with a total volume of $s^{\frac{3}{2}}$. Considering the holographic principle **[20]**, total physical information of any ratio of the expanded volume $\Delta(dV)$ is encoded on the surface surrounding it. Thus, because of the presence of the mass m, the expansion of this cubic parallelepiped volume would be a ratio of $\Delta(dV)$ linked to value of its area s such as at $r = R_0$ the expansion of its volume is worth $\frac{s}{A}\Delta(dV)_{r=R_0}$. Applied to the cubic parallelepiped at $r = R_0$, the metric constraint then becomes $\frac{s}{A}F_m(R_0)$. We can define an equivalent work of the constraint $F(r = R_0)$ on slice thickness dr of the space-time as:

$$dW = \frac{1}{2}F_m(R_0)R_0 dr \quad (5)$$

We can note that in a non-relativist gravitation, this energy dW (proportional to $1/R_0^2$) is equivalent in mathematical expression to gravitational energy contained in a volume dV with energy density u as:

$$dW \equiv u \times dV \propto \frac{dr}{r^2} \quad (6)$$

With:

$$u = \frac{g^2}{8\pi G} \quad (7)$$

With g, the local gravity. Of course, we can note that dW and udV are not of the same physical nature.

Considering now the work, done by metric constraint $\frac{s}{A}F_m$, applied on the cubic parallelepiped (cp). As metric constraint is distributed on each of the six faces of the cubic parallelepiped, the total work, done by metric constraint $\frac{s}{A}F_m$, becomes:

$$dW_{cp} = \frac{6s}{A}dW \quad (5\ bis)$$

If we consider that, energy density of quantum vacuum is scale invariant **[12]** thereby; we can suppose a proportional mathematical relation between dW_{cp} and $\Delta(dV)$ as:

$$dW_{cp} = \sigma \times \frac{s}{A}\Delta(dV)_{r=R_0} \quad (8)$$

With σ a homogenous parameter as energy density in $J.m^{-3}$ that supposed to be the average energy density of quantum vacuum. Equation (8) stipulates that expansion of the slice of space with a thickness dr due to presence of mass must accumulate energy because of the non-variant scale factor of vacuum energy density.

Thus (2), (5 bis) and (8) involve:

$$F_m(R_0) = \frac{8\pi\sigma Gm}{3c^2} \quad (9)$$

We can note that $F_m(R_0)$ is independent of R_0.

B. *The theory*

Our theory is based on the idea that the matter as inertia, interacts with space-time in its accelerated expansion and caused by the global acceleration of the expansion of the Universe. Without presence of mass, space-time normally accelerate its expansion but, in presence of any mass, entanglement between matter and space-time impeaches acceleration of expansion of space-time inside the matter. The space-time remains "trapped" into matter and is curved outside of it because of its accelerated expansion. Mathematically, our theory stipulates that it exists a direct relation between $F_m(R_0)$ and Ψ representing global acceleration of the expansion of the Universe (in s^{-2}) as:

$$F_m(R_0) = m \times \Psi \quad (10)$$

Equations (9) and (10) permit to write:

$$G = \frac{3c^2}{8\pi\sigma}\Psi \quad (11)$$

In addition, we can note that acceleration of the Universe is given by:

$$\Psi = \frac{H^2\sigma}{\rho_c c^2} \quad (11\ bis)$$

With H, the Hubble constant and ρ_c, the critical density of the matter. According to our theory, evolution of the value of Ψ from inflation epoch to present involves that the gravitational constant G has undergone evolution during time **[13]**. Linking G to Ψ can explain, notably during inflation epoch to current Universe that gravitational force has changed intensity from much higher values in the past to its current value. Our theory could thereby explain how large-scale structure in our current Universe were formed regardless dark matter existence. Our theory can also explain anisotropic measurement of temperature from cosmic microwave background due to quantum vacuum fluctuation of σ value and, thanks to (11), its implication to the possible spatial anisotropic value of G in the primordial Universe. With higher value of G in the past Universe, past stellar evolution could be very different with a stellar lifetime much shorter and with a number and emissivity of massive stars higher than current astrophysical observations. It involves that probable candidate for dark matter could be massive compact halo objects like black holes and primordial black holes. Our theory explains also how first galaxies could be formed so early in the Universe. We can also remark that our theory involves that, in case of deceleration of the expansion of the Universe, gravitational force could be repulsive. If our theory permits to know origin of gravitation in the Universe, it does not explain origin of Ψ itself whether it is positive or negative.

III. Global expansion of the Universe

A. *Physical concept and hypothesis*

In order to study and quantify theoretically expansion of the Universe, we are going to adopt five major hypothesis almost all accepted by astrophysics science. The first one is the cosmology principle, which considers that at any given epoch, Universe is homogeneous and isotropic (at large scale), electrically neutral, composed of energy and mass of formed or not still formed compact objects (galaxies, clusters, quasars…). In this case, compact objects are supposed to have no proper velocity due to movement of mass compared to an observer (and so they have a null kinetic energy) except recessional velocity due to expansion of the Universe. The second hypothesis is that an observer on Earth could thought that he is at the center of his own spherical Universe composed of massive matter with global density called ρ with a scale factor noted R representing the theoretical global radius of the whole Universe (and not only the observable Universe). The third hypothesis is that we consider Universe as a closed system that is to say no transfer of mass or energy is possible inward or outward our Universe. The fourth hypothesis is that Universe is flat **[11]** and if we consider the perfect cosmological principle, the flatness of the global Universe has occurred in all the ages of the Universe. It means that we can use Euclidian geometry to describe classical geometry evolution of the Universe through past and future time. Finally, we will consider that any evolution of state of one part of the Universe included in a sphere of volume V, with the observer at its center, can influence simultaneously states of all the other parts of this volume. This assertion permits to respect the first law of thermodynamics. This assertion puts forward the fact that information could spread across Universe to compensate what we call horizon problem **[14]** at any epoch of the Universe and not only before inflationary epoch. The holographic principle could explain this **[20]**, considering that a surface S surrounding any compact volume of space V could contain all the necessary information of state of matter inside of the volume V. This assertion permits to characterize state of a given matter (density of matter, potential gravitational energy, gravitational energy density…) inside a volume $V=\frac{4}{3}\pi r^3$ by decoding the necessary information on its surface $S=4\pi r^2$, with r, the distance from the observer without taking into account propagation time of information. Quantum entanglement phenomenon has showed that states of particles can be correlated and state's affectation of one affects simultaneously the state of the other one independently of their distance.

We have also these following known equations implying the Hubble constant H:

$$H=\frac{\dot{R}}{R}\quad(12)$$

$$\Psi=H^2+\dot{H}\quad(13)$$

From (12) and (13), we have:

$$\Psi=\frac{\ddot{R}}{R}\quad(14)$$

If we call by $\mathcal{E}(t)$, the total quantity of energy from thermic and non-thermic photons in the Universe and $P(t)$, the total quantity of energy per unit of time by radiation (Radiative power of emission) from stellar or no stellar objects, or more commonly called luminosity, in the Universe, so we can write:

$$\frac{d\mathcal{E}(t)}{dt}=P(t)-H\mathcal{E}(t)\quad(15)$$

The gravitational potential energy of the Universe is worth:

$$E_p=\frac{4}{15}\pi^2G\rho^2R^5\quad(16)$$

The gravitational field inside a global volume $V=\frac{4}{3}\pi R^3$ generated by mass $\frac{4}{3}\pi R^3\rho$ create a gravitational energy density given by (7). Integrating this all over the Universe give global energy as:

$$E_s=\frac{8}{45}\pi^2G\rho^2R^5\quad(17)$$

The displacement of mass because of expansion of the Universe create a current density of matter $\vec{j}$ at radius r from the observer as:

$$\vec{j}(r)=\rho H\vec{r}\quad(18)$$

At radius r from the observer, the gravity field applied on matter is worth:

$$\vec{g}(r)=-\frac{4}{3}\pi G\rho\vec{r}\quad(19)$$

This global displacement of mass generate a work done by gravitational interaction between matters and given by its power as:

$$P_W(t)=\int 4\pi r^2\vec{j}(r)\times\vec{g}(r)\,dr\quad(20)$$

Hence, according to (19) and (20):

$$P_W(t)=-\frac{16}{15}\pi^2G\rho^2\dot{R}R^4\quad(21)$$

Finally, it is necessary to introduce existence of energy from quantum vacuum and its global energy in the universe is worth:

$$E_v=\frac{4}{3}\pi\sigma R^3\quad(22)$$

Expansion of the Universe must obey to conservation of the energy stated by the first law of thermodynamics. Breaking this law should obey to another unknown law. We suppose that, contrary to global thought, energy is a constant of time physical quantity of Universe from Big Bang to present and it cannot be otherwise.

According to the first law of thermodynamics applied to the Universe, we have for a dt flow of time between two instants t and $t+dt$:

$$dE_p+dE_s+dE_v+d\mathcal{E}(t)+dMc^2=P_W(t)dt\quad(23)$$

With M, the mass of the Universe.

B. *Global equations and their involvements*

According to (11), the paradigm that gravitation is linked to the accelerated expansion of the Universe involves that G is variant as function of time. Moreover, we can express temporal evolution of the mass of the Universe as:

$$dM = \frac{4}{3}\pi R^3 d\rho + 4\rho\pi R^2 dR \quad (24)$$

Thus, (12) and (15) to (24) permit to write global equation of evolution of the scale factor of the Universe as:

$$\left((\sigma + \rho c^2) \times 4\pi R^2 + \frac{37\pi c^2 \rho^2 R^4}{30\sigma}\Psi\right)\frac{dR}{dt} + \left(\frac{4}{3}\pi R^3 c^2 + \frac{\pi c^2 \rho R^5}{3\sigma}\Psi\right)\frac{d\rho}{dt} + \frac{\pi \rho^2 R^5 c^2}{6\sigma} \times \frac{\mathrm{d}\Psi}{\mathrm{d}t} + P(t) - H \times \mathcal{E}(t) = 0 \quad (25)$$

If we consider in our Universe that main mass loss is due to stellar or no stellar activities thanks to their radiative emission, thus, we can write:

$$P(t) + \frac{dM}{dt}c^2 \approx 0 \quad (26)$$

From(24) and (26), we can conclude that:

$$\frac{d\rho}{dt} \approx \frac{-3P(t)}{4\pi R^3 c^2} - 3\rho H \quad (27)$$

Thus, (27) represents Friedmann equation of evolution in time of density ρ, where quantity $\frac{P(t)}{4\pi H R^3}$, called pressure p in Friedmann-Lemaitre equations, represents the ratio between total stellar power emission in the Universe (Luminosity of the Universe) and $4\pi R^2 \dot{R}$ representing the volumetric flow rate of Universe expansion. So, we can write:

$$p = \frac{P(t)}{4\pi H R^3} \quad (27\ bis)$$

According to (26) and (27), we can rewrite (25) as:

$$\left(4\pi\sigma R^2 + \frac{7\pi c^2 \rho^2 R^4}{30\sigma}\Psi\right)\frac{dR}{dt} - \frac{\rho R^2 P(t)}{4\sigma}\Psi + \frac{\pi\rho^2 R^5 c^2}{6\sigma} \times \frac{\mathrm{d}\Psi}{\mathrm{d}t} - H \times \mathcal{E}(t) = 0 \quad (28)$$

From (28), the Hubble constant is worth:

$$H = \frac{\frac{\rho R^2 P(t)}{4\sigma}\Psi - \frac{\pi \rho^2 R^5 c^2}{6\sigma} \times \frac{\mathrm{d}\Psi}{\mathrm{d}t}}{4\pi\sigma R^3 + \frac{7\pi c^2 \rho^2 R^5}{30\sigma}\Psi - \mathcal{E}(t)} \quad (29)$$

1) *Evolution of the current Universe*

If we consider that H is in current time, independent of ρ and R, and also independent of global stellar power emission, so we have to consider the following approximation:

$$\left|\frac{\rho R^2 P(t)}{4\sigma}\Psi\right| \ll \left|\frac{\pi\rho^2 R^5 c^2}{6\sigma} \times \frac{\mathrm{d}\Psi}{\mathrm{d}t}\right| \quad (30)$$

In addition, we need also to suppose that:

$$|4\pi\sigma R^3 - \mathcal{E}(t)| \ll \left|\frac{7\pi c^2 \rho^2 R^5}{30\sigma}\Psi\right| \quad (31)$$

According to (29) and approximations (30) & (31):

$$H \approx -\frac{1}{1.4\Psi} \times \frac{d\Psi}{dt} \quad (32)$$

According to (32), approximation (30) becomes:

$$P(t) \ll HMc^2 \quad (30\ bis)$$

With M, as a reminder, the mass of the Universe. Considering the Universe composed of 10^{80} equivalent proton particles [**6**] and, composed of around 2000 billion of galaxies [**16**] themselves composed of around 100 billion stars emitting a power radiation of the order of magnitude of $10^{27}W$, so we can consider that $P \sim 10^{50}W$. In this case, (30 bis) is true.

According to (11) and (32), we can also write H as:

$$H \approx -\frac{1}{1.4G} \times \frac{dG}{dt} \quad (33)$$

According to current value of $H \approx 2.2 \times 10^{-18} s^{-1}$ (67,8Km.s^{-1}Mpc^{-1}), and regarding (33), we have current value of $\dot{G}$ as:

$$\frac{dG}{dt} \approx -2.06 \times 10^{-28} m^3.s^{-3}.\mathrm{Kg}^{-1} \quad (34)$$

From (32), we have a direct relation between Ψ and R as:

$$\Psi = \frac{A}{R^{1.4}} \quad (35)$$

With A a constant as:

$$A = \frac{8\pi\sigma G R_0^{1.4}}{3c^2} \approx 8.7 \times 10^{-13} m^{1.4}.s^{-2} \quad (36)$$

Numerical value of A in (36) is given for a value of $\sigma \approx 10^{-29} g.cm^{-3}$ and in considering that value of $R_0 \approx$ 13.8 billion light-years, used to define the current observed radius of the observable Universe **[15]**.

Thus, according to (14) and (35), we can write:

$$\dot{R} = \sqrt{\frac{10}{3}A(R^{0.6} - R_0^{0.6}) + \dot{R}_0^2} \quad (37)$$

If we consider that $R \gg R_0$ and $AR^{0.6} \gg \dot{R}_0^2$ so, we can write R as function of time t:

$$R(t) \approx \left(0.7\sqrt{\frac{10}{3}A} \times (t - t_0) + R_0^{0.7}\right)^{\frac{10}{7}} \quad (38)$$

Thus, future of the Universe is the Big Rip scenario.

Hence, according to (12), H can be written as a function of time:

$$H(t) \approx \frac{1}{0.7(t - t_0) + \frac{1}{H_0}} \quad (39)$$

With H_0, the value of Hubble constant at $t = t_0$ as:

$$H_0 = \frac{1}{R_0^{0.7}}\sqrt{\frac{10}{3}A} \quad (39\ bis)$$

According to (35) and (38), Ψ can be written as function of time:

$$\Psi(t) = \frac{0.3}{\left(0.7(t - t_0) + \frac{1}{H_0}\right)^2} \quad (40)$$

Thus, the current acceleration of the expansion of the Universe is linked to H as:

$$\Psi = 0.3H^2 \quad (40\ bis)$$

Thus, according to (11 bis) and (40 bis) we can estimate value of σ from physical parameters:

$$\sigma \approx 0.3\rho_c c^2 \quad (40\ ter)$$

With, as a reminder, ρ_c the critical density that is a parameter defining flatness of the spatial geometry of the Universe.

2) *Example of a specific past evolution of the Universe: The inflationary epoch*

Considering a primitive age evolution of the Universe with a time constant mass M due to successive creation and disintegration of matter and without any presence of baryogenesis physical process. The value of the primitive Universe's mass M is much lower than the current one. The energy of the primitive Universe was mainly in the form of thermic photons. Globally, we can consider that (26) is true which means that average value of emitting power P is null (there is as much radiative emission from massive matter's disintegration as photon absorption to create mass). Thus, we are considering that $P \approx 0$ as mass is conserving during this Universe's specific primitive age. We consider that even in this epoch of the Universe, space is flat considering that curvature of the Universe is undetectable at any epoch (known as flatness problem) that means that for any epoch of the Universe, we can consider that $\frac{c}{HR} \ll 1$. According to (15), with $P = 0$, energy of the free photons is worth as function of the scale factor R:

$$\mathcal{E} = \frac{\mathcal{E}_0 R_0}{R} \quad (41)$$

With $\mathcal{E}_0$ and R_0 the "initial" values of photon energy and scale factor of the Universe. We can admit that the value of $\mathcal{E}_0$ is almost equal to the value of the total energy of the past and present Universe if we consider that R_0 has almost Planck length value and Mc^2, energy of the primitive Universe's mass, negligible compare to $\mathcal{E}_0$.

As mass remains constant, so density of matter ρ evolves as function of R:

$$\rho = \frac{3M}{4\pi R^3} \quad (42)$$

In addition:

$$\frac{d\rho}{dt} = -\frac{9M}{4\pi R^4}\dot{R} \quad (43)$$

According to (25), (41) to (43), we have :

$$\left(4\pi\sigma R^2 + \frac{21M^2c^2}{160\pi\sigma R^2}\Psi\right)\dot{R} + \frac{3M^2c^2}{32\pi\sigma R}\dot{\Psi} - \frac{\dot{R}}{R^2}\mathcal{E}_0 R_0 = 0 \quad (44)$$

From (44), Hubble constant is worth for this Universe's epoch:

$$H = \frac{-\frac{3M^2c^2}{32\pi\sigma R} \times \frac{d\Psi}{dt}}{\sigma \times 4\pi R^3 + \frac{21M^2c^2}{160\pi\sigma R}\Psi - \frac{\mathcal{E}_0 R_0}{R}} \quad (45)$$

Density of quantum vacuum is a constant of time, so for a primitive Universe, we can consider that:

$$|\sigma \times 4\pi R^3| \ll \left|\frac{21M^2c^2}{160\pi\sigma R}\Psi - \frac{\mathcal{E}_0 R_0}{R}\right| \quad (46)$$

Thus, from (45) and (46) we can write H as:

$$H \approx \frac{-\frac{3M^2c^2}{32\pi\sigma} \times \frac{d\Psi}{dt}}{\frac{21M^2c^2}{160\pi\sigma}\Psi - \mathcal{E}_0 R_0} \quad (47)$$

From (47), we can see that expansion of the Universe is impossible without presence of mass even if its value is small compare to the current mass of the Universe.

From (14) and (47), we deduce that :

$$\ddot{R} = KR + \frac{R_0^{1.4}}{R^{0.4}}(\Psi_0 - K) \quad (48)$$

With:

$$K = \frac{160\pi\sigma\mathcal{E}_0 R_0}{21M^2c^2} \quad (49)$$

Hence, we obtain from (48):

$$\dot{R} = \sqrt{K(R^2 - R_0^2) + \frac{10}{3}R_0^{1.4}(\Psi_0 - K)(R^{0.6} - R_0^{0.6}) + \dot{R}_0^2} \quad (50)$$

Thus, if we consider that KR^2 is much higher than the rest of what is under the root square expression, evolution of R as function of time is ascending exponential type:

$$R(t) \approx R_0^* \exp\left[\sqrt{K}(t - t_0^*)\right] \quad (51)$$

With R_0^* a specific scale factor respecting:

$$KR_0^{*2} \gg \frac{10}{3}R_0^{1.4}(\Psi_0 - K)\left(R_0^{*0.6} - R_0^{0.6}\right) + \dot{R}_0^2 - KR_0^2 \quad (52)$$

With t_0^* the time of Universe after Big Bang to reach R_0^* scale factor.

We can observe that $\frac{1}{\sqrt{K}}$ is the characteristic duration of the inflationary epoch, proportional to the mass of the Universe at inflationary epoch.

3) *Potential nature of dark energy*

According to (29), for the current Universe, Ψ is worth:

$$\Psi = \frac{-\frac{2}{3}\pi\rho^2 R^5 c^2 \dot{\Psi} - 4\sigma H\left(4\pi\sigma R^3 - \mathcal{E}(t)\right)}{\frac{14}{15}\pi c^2 \rho^2 R^5 H - \rho R^2 P(t)} \quad (53)$$

As a reminder, if we consider that, our observable Universe is composed of around 2×10^{12} galaxies for $z < 8$ **[16]** composed of around 10^{11} stellar objects emitting around $10^{27} W$, so numerically, we can suppose that $P \approx 10^{50} W$. Moreover, if we consider that Universe is composed barely of tree hydrogen atoms per cubic meter of Universe (without counting dark matter), for $R >$ 13.8 billion light year, we have:

$$\frac{\frac{14}{15}\pi c^2 \rho^2 R^5 H}{\rho R^2 P(t)} > 21 \quad (54)$$

Therefore, in neglecting $\rho R^2 P(t)$ compare to $\frac{14}{15}\pi c^2 \rho^2 R^5 H$, we can express acceleration of the expansion of the Universe from (53) as:

$$\Psi \approx -\frac{5}{7} \times \frac{\dot{\Psi}}{H} - \frac{30\sigma}{7} \times \frac{4\pi\sigma R^3 - \mathcal{E}(t)}{\pi c^2 \rho^2 R^5} \quad (55)$$

The equality between Ψ and the first term of (55) give equation (32). So, (32) is true if value of R permits to neglect the second term of (55).

Moreover, the NASA's Fermi Gamma-ray Space telescope has estimated that number of total photons in Universe is around 4×10^{84} **[17]** and if we consider that all of them are in average wavelength of $0.5nm$ (even it is much higher in average), and for $R >$ 13.8 billion light year, we have:

$$\frac{\mathcal{E}}{4\pi\sigma R^3} < 5.7 \times 10^{-2} \quad (56)$$

According to (11), and (56), equation (55) becomes:

$$\Psi \approx -\frac{40\pi\sigma}{21Hc^2} \times \frac{dG}{dt} - \frac{120}{7} \times \frac{\sigma^2}{c^2\rho^2 R^2} \quad (57)$$

Thus, equation (57) shows that acceleration of the expansion of the Universe is mainly linked to two terms including one that represents potential dark energy and linked to the evolution of value of gravitational constant G as function of time. The other term contributes to deceleration of the expansion of the Universe. According to (57), considering a baryon density of nearly tree hydrogen atom per cubic meter and $R > 13.8$ billion light year and if we consider that σ is worth exactly 1nJ per cubic meter, therefore we have the following inequalities:

$$2.1 \times 10^{-28} < \left|\frac{dG}{dt}\right| < 1.49 \times 10^{-26} (m^3.s^{-3}.Kg^{-1}) \quad (58)$$

Considering Friedmann equation:

$$\Psi = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \quad (59)$$

With p the pressure and Λ, the cosmological constant, we have then, by comparison of (57) and (59) we can write:

$$p = -\frac{\rho c^2}{3} + \frac{30\sigma^2}{7\pi G\rho^2 R^2} \quad (60)$$

Considering our current Universe with a density of tree hydrogen atoms per cubic meter and for $R = 13.8$ billions light year, value of p is:

$$p \approx 4.76 \times 10^{-8} Pa \quad (60\ bis)$$

In addition, by comparison of (57) and (59) we can write:

$$\Lambda = -\frac{40\pi\sigma}{7Hc^4} \times \frac{dG}{dt} \quad (61)$$

Therefore, according to (61) and for the current Universe, according to (33), the cosmological constant can be written as:

$$\Lambda = \frac{8\pi G\sigma}{c^4} \quad (62)$$

Thus, Λ is consistent with the literature **[17]**.
Its current value is:

$$\Lambda \approx 2.07 \times 10^{-52} m^{-2} \quad (62\ bis)$$

Thus, the cosmological constant is linked to the density of the vacuum energy and its value is a function of time.
According to $(27\ bis)$ and (60), the total electromagnetic power emission of Universe is a determinist function as:

$$P(t) = -\frac{4}{3}\pi\rho R^3 Hc^2 + \frac{120}{7} \times \frac{H\sigma^2 R}{G\rho^2} \quad (63)$$

Let be η the following ratio:

$$\eta = \frac{\frac{14}{15}\pi c^2 \rho^2 R^5 H}{\rho R^2 P(t)} \quad (64)$$

From (64), we can note that η is worth most commonly:

$$\eta = 0.7\frac{Mc^2}{P(t)} \times H \quad (65)$$

With M, the current mass of the Universe. Therefore, we can see that η is a measurement of the ratio between the total current energy in the form of mass and the total current energy in the form of photons in the Universe. If we consider that, our Universe is composed of average photon with a length wave less than $500nm$ and around 4×10^{84} in number **[18]** and if we consider that matter density is around 3 hydrogen atoms per cubic meter, so η must verify:

$$\eta > 1.84 \times 10^3 \quad (66)$$

With the presence of dark matter, minimum value of η would be greater than the one given in (66).
We can note that (54) and (66) are not in accordance given that the digital data for its quantification are not the same.
According to (63) and (64), we can write R as function of η:

$$R = \sqrt{\frac{\frac{120}{7}\sigma^2\eta}{\left(\frac{4}{3}\eta + \frac{14}{15}\right)\pi G c^2 \rho^3}} \quad (67)$$

Thus, according to (66) (or even (54)) and (67), it is necessary to have a current Universe with an approximated scale factor of:

$$R \approx \sqrt{\frac{90\sigma^2}{7\pi G c^2 \rho^3}} \quad (68)$$

If we still consider a density of matter of tree hydrogen atom per cubic meter, we must have a current Universe with a scale factor of around 246 billion light years. In this case, according to (63), the total radiative power emission must verify:

$$P \leq 3.4 \times 10^{51} W \quad (69)$$

Now, if we consider density of matter equivalent to around 17 hydrogen atoms per cubic meter (presence of dark matter), the current scale of the Universe must be around 18 billion light-years. In this case, according to (63), the total radiative power emission must verify:

$$P \geq 2.67 \times 10^{49} W \quad (70)$$

In these cases ((69) and (70)) relations (32) and (33) are true. Whether it is for (69) or (70) conditions, we can note that theirs results are coherent considering that we estimated $P \sim 10^{50} W$ for our observable Universe with its scale factor of 13.8 billion light years.
Moreover, by comparison of (53) and (59) if we suppose that condition (56) remains true, we have the quasi-exact expression of pressure p:

$$p = -\frac{\rho c^2}{3} + \frac{4c^2\sigma^2 HR}{G \times \left[\frac{14}{15}\pi c^2\rho^2 R^3 H - \rho P(t)\right]} \quad (71)$$

Utilizing $(27\ bis)$ for our current Universe, total radiative power emission $P(t)$ of the Universe is solution of equation:

$$P(t) = -\frac{4}{3}\pi\rho R^3 Hc^2 + \frac{16\pi c^2\sigma^2 H^2 R^4}{\left[\frac{14}{15}\pi c^2\rho^2 R^3 H - \rho P(t)\right] G} \quad (72)$$

Still by comparison of (53) and (59), we can show that the cosmological constant is worth:

$$\Lambda = \frac{-16\pi^2\rho R^3\sigma}{\frac{14}{5}\pi c^2\rho R^3 H - 3P(t)} \times \frac{1}{c^2}\frac{dG}{dt} \quad (73)$$

However, approximate expressions of p (pressure), $P(t)$ (radiative power emission) and Λ given respectively in equations (60), (62) and (63) can give value close to reality accounting approximations done to reach their literal value.
Thus, according to (63) and (67), we can write:

$$P(t) = \frac{H\sigma^3}{\sqrt{\pi} c G^{\frac{3}{2}} \rho^{\frac{7}{2}}} K(\eta) \quad (74)$$

With:

$$K(\eta) = \left[-\frac{4}{3}\left(\frac{\frac{120}{7}\eta}{\frac{4}{3}\eta + \frac{14}{15}}\right)^{\frac{3}{2}} + \frac{120}{7}\sqrt{\frac{\frac{120}{7}\eta}{\frac{4}{3}\eta + \frac{14}{15}}}\right] \quad (75)$$

Let be the parameter e defined as:

$$e = \frac{\frac{120}{7}\eta}{\frac{4}{3}\eta + \frac{14}{15}} \quad (76)$$

According to (65), (74), (75) and (76), e is solution of equation:

$$\frac{\frac{14}{15}e}{\frac{120}{7} - \frac{4}{3}e} \times \left[-\frac{4}{3}e^{\frac{3}{2}} + \frac{120}{7}\sqrt{e}\right] = \frac{0.7Mc^3\sqrt{\pi}G^{\frac{3}{2}}\rho^{\frac{7}{2}}}{\sigma^3} \quad (77)$$

The approximate solution for e is then:

$$e \approx \frac{\pi R^2\rho^3c^2G}{\sigma^2} \quad (78)$$

Hence,

$$\eta = \frac{\frac{14}{15}\pi R^2\rho^3c^2G}{\frac{120}{7}\sigma^2 - \frac{4}{3}\pi R^2\rho^3c^2G} \quad (79)$$

We deduce that η goes to zero as ρ goes to zero with the expansion of Universe. In this case, for future Universe, it is necessary to solve equation (72) to find the exact literal value for $P(t)$. Therefore, we can deduce a new expression of $P(t)$ whatever the value of η:

$$P(t) = \sqrt{\frac{289}{225}\pi^2\rho^2R^6H^2c^4 - \frac{16\pi c^2\sigma^2H^2R^4}{\rho G}} - \frac{1}{5}\pi\rho R^3Hc^2 \quad (80)$$

To ensure $P \geq 0$, it is necessary that density of matter verify the following inequality:

$$\rho \geq \left(\frac{90\sigma^2}{7\pi c^2GR^2}\right)^{\frac{1}{3}} = \rho_{min} \quad (81)$$

We can note that (81) ensures that η remains positive.

Hence, for our current Universe, the scale factor R must be greater than 108 billion light year if we consider that $\rho \approx \rho_c$ representing around 5.2 hydrogen atom per cubic meter.

Inequality (81) states that the mass of the Universe must be greater than a minimum value dependent of the scale factor R in order to generate radiation from massive matter as what happen in the current core of the stars:

$$M \geq \left(\frac{640\sigma^2\pi^2R^7}{21c^2G}\right)^{\frac{1}{3}} \quad (82)$$

Hence, the current mass of the Universe must be greater than 3.8×10^{55}Kg if we suppose that scale factor of the Universe is greater than 108 billion light-year.

From (73) and (80) we can write the quasi-exact expression of the cosmological constant without neglecting the quantity $P(t)$:

$$\Lambda = \frac{-\frac{80\pi\sigma}{17c^4H}\frac{dG}{dt}}{1 - \sqrt{1 - \frac{280}{289}\frac{\rho_{min}^3}{\rho^3}}} \quad (73\ bis)$$

We can note that we obtain the Λ literal value given in (62) if we consider $\rho = \rho_{min}$ (equivalent to say that $P = 0$) and considering equation (33). Thus, Λ is a physical parameter whose value is variable with time.

4) Discussion around potential nature of dark matter and scenario of formation of galaxies

According to our theory, the value of G could be higher in the past Universe. Contrary to what is admitted, our theory supposes that fluctuation of energy density of quantum vacuum in the primordial Universe; whose σ is, as reminder, a measurement of its average; had a direct impact on the spatial anisotropy of the value of G and consequently on the spatial anisotropy of the density of matter. It is because size of the primordial Universe was at the same order of magnitude as the spatial coherence of virtual particles composing quantum vacuum that the anisotropy of the value of G had large-scale effect for the primordial Universe and thereby, any proportion conserved, for the current Universe. It is resulting the current large-scale measurement of the spatial anisotropy of the density of matter in the cosmic microwave background map. A greater value of G permitted to form first galaxies without taking into account existence of dark matter as extraordinary matter that currently, we supposed it is. It implies also that first stars were much more massive than the current ones **[19]** with a shorter life cycle. Thus, with a greater value of G, a larger number of generation of stars occurred. A significant quantity of ordinary matter remained in the form of red and brown dwarf but especially in the form of black holes and primordial black holes. With the decline in the value of G as function of time and because of their primary rapid rotation curves, galaxies begun to grow larger and residual ordinary matter from past stellar evolution has remained gravitationally trapped into them. We suppose that the current density of total matter is larger than the supposed value of the current radiant baryon density such as $0.6\rho_c$ but its nature remains the same than the one of ordinary matter. If we imagine far in the future, the value of P, given in (80), should decrease because of the expansion of the Universe. The current ordinary matter should therefore no longer irradiate. What future of Universe is for us today should be the same than today for the past of the Universe. That means a portion of radiant matter in the first ages of the Universe is no longer radiating today and compose what we call dark matter. The current value of the critical density is around 5.2 hydrogen atom per cubic meter. To explain formation of large-scale structure in the Universe with the current value of $G = 6.674 \times 10^{-11}\text{Kg}^{-1}m^3s^{-2}$, it is necessary to introduce presence of dark matter with a density of around 5.2 time more than the density of ordinary matter **[11]**. That involves Universe could not be flat. However, the universe is flat according to the latest observations of satellite WMAP **[11]**. How to explain that? If Universe is flat, that means that matter density is less or equal to critical density according to general relativity. If density of the matter is less or equal to critical density, how could we explain formation of large-scale structure of the Universe? Establishing a proportional relationship between G and Ψ, assuming that a higher past value, than the current value, of Ψ occurred and it has decreased with time, permit to answer to the previous questions.

IV. CONSEQUENCES OF THE ACCELERATION OF THE EXPANSION OF THE UNIVERSE

*A. **The origins of inertia and equality between gravitational and inertial masses***

The equivalence principle of Einstein states that an acceleration is equivalent to gravitation and any experiment, even that based on gravitation itself, cannot permit to distinguish an accelerated referential to a gravitational field. Therefore, an accelerated person could not distinguish a gravitational field from an accelerated referential. Measurement of Ψ respects the equivalence principle of Einstein given the fact that, according to our theory, its measurement needs to evaluate the perfect value of G. Our theory states that any mass M as inertia, plays a role of impeaching the space-time to expand in accelerating and, the main consequence of it is space-time's curvature around mass M. Thereby, as an inertia, if a mass is able to curb space-time by creating gravitation field so, an accelerated mass can also distort space-time and generate a "felt" gravitational field proportional to its acceleration. Based on the idea of Dennis Sciama **[21]** and trying to demonstrate Mach's principle, Woodward had developed an analogy between gravitation and electromagnetism **[22]**. Thus, if we call by ϕ the gravitational potential energy per unit of mass, we can define the gravitational field $\vec{g}$ as:

$$\vec{g} = -\overrightarrow{\mathrm{grad}}\phi - \frac{1}{c}\frac{\partial \vec{A}_g}{\partial t} \quad (83)$$

With $\vec{A}_g$ the gravitation potential equivalent to the magnetic potential and $\overrightarrow{\mathrm{grad}}\phi$ is the mathematical gradient of potential ϕ. By analogy with electromagnetism calculation of magnetic potential, in case of non-relativistic masses, $\vec{A}_g$ can be calculated at a distance r from mass source as:

$$\vec{A}_g(t,r) = -\frac{2G}{c}\iiint \frac{\mu\vec{v}\left(t-\frac{r}{c}\right)}{r}d\tau \quad (84)$$

With μ as the local density of matter into volume $d\tau$ and $t-\frac{r}{c}$ notation to symbolizing a retarded potential due to the fact that propagation of gravitational potential is, as we will see in IV-C, celerity of light c. Moreover $\vec{v}$ is the vector velocity of the local mass $\mu d\tau$.

According to Reissner-Nordström metric **[23]**, in case of non-relativistic masses, $\vec{A}_g$ becomes with electrical charged mass:

$$\vec{A}_g(t,r) = -\frac{2G}{c}\iiint \frac{\mu\vec{v}\left(t-\frac{r}{c}\right)}{r}d\tau + \frac{G}{4\pi\varepsilon_0 c^3}\iiint \frac{\kappa\vec{v}\left(t-\frac{r}{c}\right)}{r^2}d\tau \quad (84\ bis)$$

With $\kappa = \frac{dQ^2}{d\tau}$, the squared charge density of matter into volume $d\tau$ and ε_0, the vacuum permittivity. A generalized expression of $\vec{A}_g$ is given thereafter in equation (O) according to Kerr-Newman metric.

Gravitation is attractive and thereby is a centripetal field like electric field generate by negative electric charge. As $\mu > 0$, thus it is necessary to include the sign "minus" in the expression of $\vec{A}_g$. At a distance r from a punctual electrically neutral mass M, according to (84), $\vec{A}_g(t,r)$ is worth:

$$\vec{A}_g(t,r) = \frac{-2GM\vec{v}\left(t-\frac{r}{c}\right)}{rc} \quad (85)$$

From (83) and (85), we can deduce that:

$$\vec{g} = -\overrightarrow{\mathrm{grad}}\phi + \frac{2G}{rc^2}\times M\vec{a}\left(t-\frac{r}{c}\right) - \frac{2GM}{c^2r^2}v_r\left(t-\frac{r}{c}\right)\times\ \vec{v}\left(t-\frac{r}{c}\right) \quad (86)$$

With $\vec{a}$, the acceleration of mass M and $v_r = \frac{dr}{dt}$ the value of its radial velocity from the point where the gravitational field $\vec{g}$ is measured. Thus, the measured gravitational field noted $\vec{g}_m = -\overrightarrow{\mathrm{grad}}\phi$ can be expressed from (86) as:

$$\vec{g}_m = \vec{g}\left(t-\frac{r}{c}\right) - \frac{2G}{rc^2}\times M\vec{a}\left(t-\frac{r}{c}\right) + \frac{2GM}{c^2r^2}v_r\left(t-\frac{r}{c}\right)\times\ \vec{v}\left(t-\frac{r}{c}\right) \quad (87)$$

The first term $\vec{g}\left(t-\frac{r}{c}\right)$ is from classical gravitation attraction due to, according to our theory, acceleration of the expansion of the Universe and, in case of electrically neutral mass M, and is worth:

$$\vec{g} = -\frac{GM}{r^3}\vec{r} \quad (88)$$

The second term is linked to the fact that the punctual mass has a specific acceleration compared to the referential of measurement. As massive matter interacts with space-time, its acceleration must also distort space-time, and generate an attraction or a repulsion depending on its vector's orientation. Finally, the third term is linked to the fact that classical Newtonian gravitation $\vec{g}$ changes in value because of radial velocity of the mass m and has mathematical consequence to be always an attractive field. As said, equation (87) implies that a punctual mass M, undergoing a movement compare to an inertial frame of reference, must deform space-time of the same nature that a classical gravitation field does it. Thus, we can postulate that, in its movement, inertia is given to matter because of its interaction with space-time. According to (25) a Universe devoid of matter and energy has a static scale factor. According to our theory, this kind of Universe is therefore devoid of gravitation with $G = 0$ and, according to (87), preventing matter to distort space-time. Thus, in this kind of Universe devoid of gravitation, interaction between matter and space-time is inexistent which means that interaction between matter and space-time exists only in an accelerated expansion of Universe. According to (53), acceleration expansion of the Universe occurs because of decreasing of value of Ψ (or G) as function of time and presence of density of matter. Thereby, with the absence of matter in a specific Universe, any isolated mass m does not distort space-time in its movement compare a frame referential. It is also important to note, thanks to Reissner-Nordström metric, that a particle with a given mass and with an electrical charge generates less inertia than the same particle devoid of electrical charge **[24]** such as for $Q \neq 0$:

$$m_c = \frac{m_{nc}}{2} - \frac{1}{2}\sqrt{m_{nc}^2 - \frac{Q^2}{4\pi\varepsilon_0 G}} \quad (A)$$

In equation (A), m_{nc} is the mass of an electrically neutral matter and m_c is the apparent mass (its inertial mass) of the same matter carrying electrical charge Q). From (A), we can deduce that:

$$m_c \leq m_{nc} \quad (B)$$

Inversely, we have:

$$m_{nc} = \frac{Q^2}{16\pi\varepsilon_0 G m_c} + m_c \quad (C)$$

From (C), we can note that for $Q = 0, m_{nc} = m_c$. and in the case of the electron, if it becomes electrically neutral, its mass would be around 9.5×10^{11}Kg. For a proton, its electrically neutral mass would be around 5.2×10^{8}Kg. In (A), the following condition must be verified:

$$Q^2 \leq 4\pi\varepsilon_0 G m_{nc}^2 \quad (D)$$

With G tending to zero, Q is constraint to tend to zero which implies that value of electrical charge must be dependent of G. Thus, in a static Universe, no electrically charged particle can exist. If we admit that m_{nc} is given to matter thanks to combined Higgs mechanism and gluon mechanism and m_c a constant of time mass for a given particle (electrical charged elementary particles are supposed to have constant apparent masses independent of the Universe's epochs), so according to (C), the elementary electrical charge is linked to G as:

$$Q^2 = 4\pi\varepsilon_0 G\Gamma^2 \quad (E)$$

With $\Gamma^2 = 4(m_{nc}m_c - m_c^2)$ a constant of time depending of the nature of the elementary particle. For electron and proton, value of Γ is around 1.857×10^{-9}Kg. Then, we will call Γ, the charged equivalent mass.
We can suppose that for any known elementary particle:

$$\Gamma \leq m_p \quad (F)$$

With m_p the Planck mass. Thus, maximum elementary charge of any elementary particle cannot exceed the value Q_{max} such as:

$$Q_{max} = \sqrt{4\pi\varepsilon_0 G m_p^2} = \sqrt{2\varepsilon_0 hc} \quad (G)$$

With h, the Planck constant. Thus, value of Q_{max} is worth for any epoch of the Universe (if we suppose ε_0, h and c as constants of time) $Q_{max} \approx 1.875 \times 10^{-18} C$ representing the Planck charge.
Existence of Q_{max} involves that G have a maximum value G_{max} such as:

$$G_{max} = \frac{Q_{max}^2}{4\pi\varepsilon_0\Gamma^2} \quad (H)$$

With Γ, the charged equivalent mass of proton or electron accounting the fact that there are no other elementary particle in the standard model, with higher charged equivalent mass than proton or electron's one.
According to (E), (G) and (H), the ratio between the current value of G and G_{max} is worth:

$$\frac{G}{G_{max}} = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \alpha \quad (I)$$

With e, the elementary charge; with $\hbar$, the reduced Planck constant and α, the fine-structure constant.
According to our theory, the physical quantity G varies as function of time. Therefore, according to (I), α must also be a time variant physical quantity. Current still non-detection of value's variation of α as function of time has the same origin than non-detection of value's variation of Hubble constant as function of time whereas it is accepted that H must be a time variant physical quantity and demonstrated in this article. Indeed, measurement of H based on the study of different kind of celestial objects lead to, without counting uncertainties of measurements, a quasi-constant value of H even when measurement is done with the cosmic microwave background itself by WMAP **[11]**. This, lead us to ask the following question: "Do any measurements by any Universe's observations are able to detect variability of any physical quantity, including variation in the values of physical constants, as function of time?" The answer of this question seems to be "no". Thus, no measurements made through Universe's observations may permit us to probe the past physical characteristics of our Universe.

Thanks to Kerr metric, we obtain the same result of reduction of inertia and interaction with space –time for a matter with a rotating axially symmetric. As for a charged particle, a matter with a mass M given by combined Higgs and gluon mechanisms could not have, if spinning, an angular momentum J exceeding a maximum value of angular momentum J_{max} such as:

$$J_{max} = \frac{G}{c}M^2 \quad (J)$$

Thus, for example, measured 1Kg (equivalent to m_c) of pure proton has a combined Higgs mass mechanism and gluon mechanism of 3.1×10^{35}Kg (equivalent of m_{nc}). Hence, an object composed of pure proton with a measured mass of 1Kg could not have currently an angular momentum higher than $2.16 \times 10^{53}\text{Kg}.m^2.s^{-1}$. We can note that in a static Universe with $G = 0$, $J_{max} = 0$ which means that angular momentum of any matter is null even if a moment of force is applied on it. It implies that inertial forces are potentially inexistent and Mach's principle is true. Besides electroweak and strong interactions, other forces have electrostatic and gravitational origins. In a static Universe, if gravitational interaction is inexistent, electrostatic interaction is, according to(E), also nonexistent. Thus, in a static Universe, moment of force is also inexistent.
We can note also that for an elementary particle, angular momentum of spin is at the same order of magnitude as:

$$\hat{J} = \frac{G}{c}m_p^2 \quad (K)$$

Thus, a matter, having an electrical charge or/and spinning around an axis, has lower interaction between it and space-time and therefore, provide it with less inertia than if this same matter is electrically neutral without rotary movement.
Thus, by postulating that interaction between matter and space-time generates less inertia, we postulate that origin of inertia may be directly related to the fact that matter interacts with space-time through physical quantity Gm_c (and not only quantity m_c) with m_c, for reminder, the apparent (or inertial) mass of a given matter. Reciprocally, interaction between matter and space-time must generate curvature of space-time whose equivalent Newtonian attraction between two apparent (inertial) masses m_1 and m_2 should be proportionate to $(Gm_1) \times (Gm_2)$ accounting the fact that it is physical quantity "Gm_c", which is at the origin of space-time curvature. Thus, for gravitational interaction between apparent masses m_1 and m_2 separated of distance r, the second law of Newton applied to apparent mass m_1 should fundamentally be written, in a specific inertial frame of reference called $\mathcal{R}$, as:

$$Gm_1\vec{a}_{1/\mathcal{R}} = \frac{Gm_1 \times Gm_2}{r^3}\vec{r}_{1\to 2} \quad (L)$$

In the case of electrostatic interaction between charges q_1 and q_2 separated of distance r, the second law of Newton applied to charge q_1 with apparent mass m_1 should fundamentally be written, in a specific inertial frame of reference called $\mathcal{R}$, as:

$$Gm_1\vec{a}_{1/\mathcal{R}} = -\frac{Gq_1q_2}{4\pi\varepsilon_0 r^3}\vec{r}_{1\to 2} \quad (M)$$

Introducing G in Coulomb's law in (M) is due to Reissner-Nordström metric **[23]**, which combines the vacuum permittivity with the gravitational "constant".

We can note that in replacing q_1 and q_2 by their charged equivalent masses Γ_1 and Γ_2, previous equation (M) becomes equivalent to gravitational interaction such as:

$$Gm_1\vec{a}_{1/\mathcal{R}} = \pm\frac{G\Gamma_1 \times G\Gamma_2}{r^3}\vec{r}_{1\to 2} \quad (N)$$

In the practical case, simplifying equations (L) and (M) lead to admit that m_1 plays directly as well role of inertia as role of attraction (as well as m_2) in the case of gravitational interaction. However, virtues of the primary writings given thanks to (L) and (M) permit to highlight nature of inertia and the type of physical evolution of the two kind of interactions in case of variation of the value of G. Therefore, we can note that (84) or $(84\,bis)$ can be written more generally, in case of non-relativistic masses, such as:

$$\vec{A}_g(t,r) = -\frac{2}{c}\iiint \frac{\vec{v}\left(t-\frac{r}{c}\right)}{r} dI \quad (O)$$

With dI, a term of repercussion of the interaction between matter and space-time at distance r from the source, such as:

$$dI = G\left[\mu - \frac{\varrho c^2}{2Gr} - \frac{\kappa}{8\pi\varepsilon_0 rc^2}\right]d\tau \quad (P)$$

With ϱ the squared Kerr parameter (or length scale) density such as:

$$\varrho = \frac{d\vec{\gamma}^2}{d\tau} \quad (Q)$$

With:

$$\vec{\gamma} = \frac{1}{\mu c} \times \frac{d\vec{\mathrm{J}}_m}{d\tau} \quad (R)$$

With $\vec{\mathrm{J}}_m$ the angular momentum of the matter.

As explained above and thanks to (P), there is less interaction between charged & rotary matter with space-time than between electrically neutral and non-rotary matter with space-time.

On the one hand, interaction between matter and space-time permits to a given mass, to undergo constraint of distorted space-time as gravitational attraction. In the other hand, interaction between matter and space-time generates inertia for the same given mass. It is because, gravitational attraction as well as inertia are consequences of the same physical process (interaction of matter with space-time) that, gravitational and inertial masses are both equals.

*B. **Examples of calculated theoretical results confirmed by the general relativity theory***

Considering a punctual central non-rotary electrically neutral mass M $(\varrho = 0, \kappa = 0)$ and, a punctual non-rotary electrically neutral mass m $(m \ll M)$ orbiting with any trajectory around mass M. According to (87), in neglecting retarded effect, the gravitational field "felt" by the mass m is:

$$\vec{g}_m = \vec{g} - \frac{2GM}{rc^2}\vec{a}_{M|m} + \frac{2GM}{r^3c^2}\left(\vec{v}_{M|m}.\vec{r}\right)\vec{v}_{M|m} \quad (89)$$

With $\vec{a}_{M|m}$ and $\vec{v}_{M|m}$ the acceleration and the velocity of the mass M measured in the referential of the mass m. With $\vec{r}$ which represents the position of mass M compare to m (i.e. $\vec{r} =$ "$\overrightarrow{mM}$"). Finally, quantity $\left(\vec{v}_{M|m}.\vec{r}\right)$ represents the dot product between $\vec{v}_{M|m}$ and $\vec{r}$. If we consider that, the central mass M is an inertial frame of reference (Galilean reference frame), according to the second law of Newton, we can write:

$$m\vec{a}_{m|M} = m\vec{g}_m \quad (90)$$

With $\vec{a}_{m|M}$ the acceleration of mass m measured in the referential of the mass M supposed, as a reminder, to be an inertial frame of reference. Indeed, even if $\vec{g}_m$ is the gravitational field measured in the frame of reference of mass m, it is curvature of the local space-time which generates $\vec{g}_m$. The same curvature is measured from inertial frame of reference of the mass M. Thus, it is as if, in the frame of reference of mass M, mass m undergoes gravitational field $\vec{g}_m$. Here why equation (90) is true.

Acceleration of the central mass M in the referential of orbiting m around M is worth:

$$\vec{a}_{M|m} = \frac{\partial\vec{v}_{M|m}}{\partial t} \quad (91)$$

As we have:

$$\vec{v}_{M|m} = -\vec{v}_{m|M} \quad (92)$$

We deduce from (91) and (92) that:

$$\vec{a}_{M|m} = -\vec{a}_{m|M} \quad (93)$$

According to (90) and (93), we can conclude that:

$$\vec{a}_{m|M} = \frac{\vec{g} + \frac{2GM}{r^3c^2}\left(\vec{v}_{M|m}.\vec{r}\right)\vec{v}_{M|m}}{1-\frac{2GM}{rc^2}} \quad (94)$$

In case of relativistic mass m and a punctual central non-rotary electrically neutral mass M, previous equation (94) can be written as:

$$\frac{d\left(\frac{\vec{v}_{m|M}}{\sqrt{1-\frac{v_{m|M}^2}{c^2}}}\right)}{dt} = \frac{\vec{g} + \frac{2GM}{r^3c^2}\left(\vec{v}_{M|m}.\vec{r}\right) \times \frac{\vec{v}_{M|m}}{\sqrt{1-\frac{v_{m|M}^2}{c^2}}}}{1-\frac{2GM}{rc^2}} \quad (94.0)$$

1) The photon sphere and the innermost stable circular orbit for a Schwarzschild black hole

The equation (94) (or even equation(94.0)) permits to retrieve results of photon sphere and the innermost stable circular orbit for a massive particle around a Schwarzschild black hole. Indeed, in case of circular orbit $\vec{v}_{M|m}.\vec{r} = 0$. Adopting Binet coordinates $u = \frac{1}{r}$, $u' = \frac{du}{d\theta}$ with θ, the polar angle and $\vec{a}_{m|M} = -L^2u^2(u" + u)\vec{e}_r$, with $\vec{e}_r$ the unitary vector of radial trajectory of even relativistic mass m compare to M, we can write from (94) (or from (94.0)) :

$$L^2u^2(u" + u) = \frac{GMu^2}{1-\frac{2GM}{c^2}u} \quad (94.1)$$

With L, the angular momentum per unit of mass. The set of circular trajectories is given, considering $u" = 0$, by:

$$L^2u = \frac{GM}{1-\frac{2GM}{c^2}u} \quad (94.2)$$

In case of photon sphere, we can write $L = \frac{c}{u}$. Thus, from (94.2), position of photon sphere is solution of equation:

$$\frac{2}{u} = \frac{r_s}{1-r_su} \quad (94.3)$$

With r_s, the Schwarzschild radius. Thus, position of photon sphere is at radius $r = \frac{3}{2}r_s$.

In a general context, solving (94.2) gives general solutions as:

$$u = \frac{1}{2r_s} \pm \sqrt{\frac{1}{4r_s^2} - \frac{c^2}{2L^2}} \quad (94.4)$$

In the case of the innermost stable circular orbit, we admit that its angular momentum per unit of mass is equal to photon sphere's one: $L = \frac{3}{2} r_s c$. Thus, in taking this value for L, (94.4) has 2 solutions: $r = \frac{3}{2} r_s$ and $r = 3r_s$. We can deduce that position of the innermost stable circular orbit is at radius $r = 3r_s$. The literal values of these solutions are consistent with the literature **[25]**.

2) The apsidal precession expression with a weak field approximation

From (94), we can deduce that for a non-necessary circular orbit, $\vec{a}_{m|M}$ cannot be radial. Thus, according to (92) and (94), the tangential component of $\vec{a}_{m|M}$ is worth:

$$\left(\vec{a}_{m|M}.\vec{e}_\theta\right)\vec{e}_\theta = -\frac{\frac{2GM}{r^3c^2}\left(\vec{v}_{M|m}.\vec{r}\right) \times \left(\vec{v}_{m|M}.\vec{e}_\theta\right)}{1 - \frac{2GM}{rc^2}}\vec{e}_\theta \quad (94.a)$$

With $\vec{e}_\theta$, the unitary vector of tangential trajectory of mass m compare to M.

As $\vec{r}$ represents the position of mass M compare to m so by noting by $\vec{e}_r$, as a reminder, the unitary vector of radial trajectory of non-relativistic mass m compare to M, we can write $\frac{\vec{r}}{r} = -\vec{e}_r$. Thereby, $(94.a)$ becomes according to (92):

$$\left(\vec{a}_{m|M}.\vec{e}_\theta\right)\vec{e}_\theta = -\frac{\frac{2GM}{r^2c^2}\left(\vec{v}_{m|M}.\vec{e}_r\right) \times \left(\vec{v}_{m|M}.\vec{e}_\theta\right)}{1 - \frac{2GM}{rc^2}}\vec{e}_\theta \quad (94.b)$$

From classical derivative, we can write:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(\vec{a}_{m|M}.\vec{e}_\theta\right) \quad (94.c)$$

From $(94.b)$ and $(94.c)$ we can write:

$$\frac{d\left(\dot{\theta}^2\right)}{dt} + \frac{4\dot{r}\dot{\theta}^2}{r} = -\frac{\frac{4GM}{r^2c^2}\dot{r}\dot{\theta}^2}{1 - \frac{2GM}{rc^2}} \quad (94.d)$$

In removing dependence on time in $(94.d)$, we can write:

$$\frac{d\left(\dot{\theta}^2\right)}{dr} + \frac{4\dot{\theta}^2}{r} = -\frac{\frac{4GM}{r^2c^2}\dot{\theta}^2}{1 - \frac{2GM}{rc^2}} \quad (94.e)$$

By solving $(94.e)$, we can note that angular momentum per unit of mass of mass m is slightly different from its classical Newtonian value L and is worth L' such as:

$$L' = \dot{\theta}r^2 = \frac{L}{1 - \frac{2GM}{rc^2}} \quad (94.f)$$

Thus, the real angular momentum per unit of mass is variant as function of r.

We can note that:

$$L' > L \quad (94.g)$$

Thus, the fact that angular momentum per unit of mass is slightly higher than its classical Newtonian value implies existence of precession of the orbit (advance of the perihelion) of any celestial object.

By adopting Binet coordinates, the radial acceleration of $\vec{a}_{m|M}$ is worth in case of weak field approximation ($L' \approx L$):

$$\left(\vec{a}_{m|M}.\vec{e}_r\right) = -L^2u^2(u" + u) + o(L^2u^2) \quad (94.h)$$

From (92), (94), and $(94.h)$, we can write:

$$L^2(u" + u) = \frac{GM + \frac{2GM}{c^2}\left(\vec{v}_{m|M}.\vec{e}_r\right)^2}{1 - \frac{2GM}{c^2}u} \quad (94.i)$$

The mechanical energy of mass m noted E_m, is a constant of time. In weak field approximation, we can write $\phi = -GmMu$, thus:

$$\left(\vec{v}_{m|M}.\vec{e}_r\right)^2 = \frac{2E_m}{m} - L^2u^2 + 2GMu \quad (94.j)$$

In weak field approximation, with a Taylor series to order 1 in $\frac{GM}{c^2}u$, we can write from $(94.i)$ and $(94.j)$:

$$L^2(u" + u) \approx \left[GM + \frac{2GM}{c^2}\left(\frac{2E_m}{m} - L^2u^2 + 2GMu\right)\right]\left(1 + \frac{2GM}{c^2}u\right) \quad (94.k)$$

In taking into account only terms in u (Taylor series in order 1 in u), we can write from $(94.k)$:

$$u" + u \approx \frac{GM}{L^2}\left(1 + \frac{4E_m}{mc^2}\right) \times \left(1 + \frac{2GM}{c^2}u\right) + \frac{4G^2M^2}{c^2L^2}u \quad (94.l)$$

In neglecting $\frac{4E_m}{mc^2}$ in front of 1, $(94.l)$ can be written as:

$$u" + u \approx \frac{GM}{L^2} + \frac{6G^2M^2}{c^2L^2}u \quad (94.m)$$

Thus general solutions in $r(\theta)$ of $(94.m)$ are:

$$r(\theta) = \frac{1}{\frac{GM}{L^2\omega^2} + A\cos(\omega\theta + \phi)} \quad (94.n)$$

With $\omega = \sqrt{1 - \frac{6G^2M^2}{L^2c^2}}$ and $A = \frac{GMe}{L^2\omega^2}$, with e, the eccentricity of the orbit.

Considering Taylor series to order 1, in $\frac{6G^2M^2}{L^2c^2}$, in case of weak mass M in front of $\frac{Lc}{G}$, the apsidal precession is given as:

$$\varepsilon = \frac{2\pi}{\omega} - 2\pi \approx \frac{6\pi G^2M^2}{L^2c^2} \quad (94.o)$$

In weak field approximation, we can show that:

$$L^2 = GMa(1 - e^2) \quad (94.p)$$

With a, the elliptical semi major axis. We can note that neglecting M in front of $\frac{Lc}{G}$ means, for $e = 0$, neglecting quantity $\sqrt{\frac{GM}{a}}$ in front of c. As velocity of mass m is considered as non-relativistic, the approximation $(94.o)$ is good.

Moreover, the third law of Kepler links the orbital period T with a as:

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM} \quad (94.q)$$

From $(94.o)$, $(94.p)$ and $(94.q)$ we can deduce:

$$\varepsilon = \frac{24\pi^3a^2}{T^2c^2(1 - e^2)} \quad (94.r)$$

The literal value of ε is consistent with the literature **[26]**.

We can note that even if calculations are done with the hypothesis of weak field approximation, equation (87) is valid in strong gravitational field due to the presence of Schwarzschild singularity in equation (94). With (87) (or (94)), other results of general relativity can be obtained if considering equation (O), as the Lense-Thirring precession or even existence of gravitational waves (see IV-B-3 Conclusion). Typically, a punctual non-rotary electrically neutral mass m orbiting circularly around an observer at a distance r from him and with velocity v would distort a length ℓ of a quantity $\Delta\ell$, in accordance with (87) in taking value of radial velocity $v_r = 0$, such as:

$$\Delta\ell = \frac{2Gmv^2}{c^4 r}\ell \quad (94.s)$$

For example, even if moon is not a punctual-non-rotary mass, calculation from $(94.s)$ shows that it distorts, at its apsis, a length $\ell = 1m$ situated at the center of Earth of value $\Delta\ell \approx 10^{-24}m$.

3) Conclusion

By analogy to the electromagnetic field, it must exist another field different from classical gravitational field $\vec{g}$ and equal to $\overrightarrow{\text{rot}}\vec{A}_g$, with $\overrightarrow{\text{rot}}$ the vector operator curl. Contrary to gravitoelectromagnetism theory, we postulate the principle that in terms of physics, this field has the same nature as $\vec{g}$ as a gravitational field thanks to its dimensional analysis. However, this new gravitational field is different from $\vec{g}$ in its physical behavior more analogous to a magnetic field. Thus, instead of calling this new field "gravitomagnetic field", we will prefer to call this new gravitational field, "extraordinary gravitational field" (opposite to ordinary gravitational field $\vec{g}$), with the notation $\vec{\zeta}$. As space-time can only be distorted and, consequence of any distortion of space-time is emergence of gravitation field, so nature of this new field cannot be different from a gravitational field. Existence of this new gravitational field explains also the existence of gravitational waves as well as their polarizations.

C. The local equations of gravitational fields

By analogy to the Maxwell electromagnetic local equations, inspired by the gravitoelectromagnetism equations **[27]**, and according to previous equations, we admit equivalent gravitational local equations in case of electrically neutral and non-rotary matter density, as:

$$\text{div}\vec{g} = -4\pi G\mu \quad (95)$$

$$\text{div}\vec{\zeta} = 0 \quad (96)$$

$$\overrightarrow{\text{rot}}\vec{g} = -\frac{1}{c}\frac{\partial\vec{\zeta}}{\partial t} \quad (97)$$

$$\overrightarrow{\text{rot}}\vec{\zeta} = -\frac{8\pi G}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{g}}{\partial t} \quad (98)$$

With div, the divergence vector operator, μ and $\vec{j}$ are respectively the density of matter and the matter current density vector such as $\vec{j} = \mu\vec{v}$, with $\vec{v}$, the velocity of the matter's current.

From Kerr-Newman metric, for charged and rotary matter current density (μ, ϱ, κ), only equations (95) and (98) are modified as respectively (95.1) and (98.1) and we admit their following expressions:

$$\text{div}\vec{g} = -4\pi G\mu + \frac{2\pi\varrho c^2}{r} + \frac{G\kappa}{2\varepsilon_0 r c^2} \quad (95.1)$$

$$\overrightarrow{\text{rot}}\vec{\zeta} = -\frac{8\pi G}{c}\left(1 - \frac{\varrho c^2}{2\mu G r} - \frac{\kappa}{8\pi\varepsilon_0 \mu r c^2}\right)\vec{j} + \frac{1}{c}\frac{\partial\vec{g}}{\partial t} \quad (98.1)$$

With r distance from the source.

For non-rotary electrically neutral matter, equations of propagation of gravitational fields are:

$$\Delta\vec{g} - \frac{1}{c^2}\frac{\partial^2\vec{g}}{\partial t^2} = -4\pi G\overrightarrow{\text{grad}}\mu - \frac{8\pi G}{c^2}\frac{\partial\vec{j}}{\partial t} \quad (99)$$

$$\Delta\vec{\zeta} - \frac{1}{c^2}\frac{\partial^2\vec{\zeta}}{\partial t^2} = \frac{8\pi G}{c}\overrightarrow{\text{rot}}\vec{j} \quad (100)$$

In introducing a gauge relation like:

$$\text{div}\vec{A}_g = -\frac{1}{c}\times\frac{\partial\phi}{\partial t} \quad (101)$$

We can introduce the propagation equation of the gravitational potential vector as:

$$\Delta\vec{A}_g - \frac{1}{c^2}\frac{\partial^2\vec{A}_g}{\partial t^2} = \frac{8\pi G}{c}\vec{j} \quad (102)$$

From equations (97) and (98) we can demonstrate, thanks to Poynting's theorem that the local conservation of the energy of the gravitational fields is:

$$\frac{\partial u}{\partial t} = \vec{j}.\vec{g} - \text{div}\left(\frac{c}{8\pi G}\vec{g}\times\vec{\zeta}\right) \quad (103)$$

With $\vec{j}.\vec{g}$, the dot product between $\vec{j}$ and $\vec{g}$ and with $\vec{g}\times\vec{\zeta}$ the cross product between $\vec{g}$ and $\vec{\zeta}$.

In (103), u represents the density of gravitational fields as:

$$u = \frac{g^2 + \zeta^2}{16\pi G} \quad (104)$$

We can see that equation (104) is not consistent with (7), but if we consider that energy density of field $\vec{\zeta}$ in the Universe contributes equally to that of $\vec{g}$ (as well as magnetic field has the same energy density as the electric field's one in an electromagnetic field) we can then consider that equation (17) is correct.

From (103), we can deduce the gravitational Poynting vector as:

$$\vec{\Pi}_g = \frac{c}{8\pi G}\vec{g}\times\vec{\zeta} \quad (105)$$

Like an accelerated charged particle, an accelerated mass lose energy in the form of gravitational waves. By analogy with electromagnetic field, the gravitational Larmor formula **[28]** of an accelerated mass losing energy, in the form of gravitational wave, with a power emission of P_L is:

$$P_L = \frac{2Gm^2a^2}{3c^3} \quad (106)$$

With a, the proper acceleration of the mass m in a given inertial frame of reference. This loss of energy contributes to a gain of inertia for matter.

V. SPIRAL GALAXY ROTATION CURVE

A. Introduction and hypothesis

According to Hubble sequence, barred or regular spiral galaxies are final formation of galaxies from elliptical galaxies through lenticulars **[29][30][31][32][33]**. In this chapter, we want to model the curve of rotation of spiral galaxies given that spiral galaxies are the most quasi-static state in the evolution of galaxy formation. Any celestial object belonging to a spiral galaxy undergoes gravitation field given thanks to equation (87). We define the galaxy rotation curve as the set of tangential velocities of celestial objects composing the spiral galaxy as function of distance at its center r. If we consider any object with a mass m orbiting with a circular trajectory around a central mass $M \gg m$, supposed to be an inertial frame of reference, and with a tangential velocity v at distance r, we can then write:

$$\vec{a}_{m|M} = -\frac{v^2}{r^2}\vec{r} \quad (107)$$

Thus, according to (89), (93) and (107), we can express the gravity field $\vec{g}_m$ applied to mass m as:

$$\vec{g}_m = \vec{g} + \frac{2GM}{r^3} \times \frac{v^2}{c^2}\vec{r} \quad (108)$$

As reminder, $\vec{g}$ is given by equation (88).
Thus, (108) can be written as:

$$\vec{g}_m = \vec{g} \times \left(1 - \frac{2v^2}{c^2}\right) \quad (108\ bis)$$

As we suppose that celerity of celestial objects are not relativistic, we will assume for the rest that:

$$\vec{g}_m \approx \vec{g} \quad (109)$$

We can observe that (109) stays true if celestial objects have elliptical trajectories with a non-zero eccentricity (it is also the case for value of the third term of (87) which can be considered as negligible compare to the value of g).
Moreover, as said, we can calculate extraordinary gravitation field as:

$$\vec{\zeta} = \overrightarrow{\mathrm{rot}}\vec{A}_g \quad (110)$$

As we consider a quasi-static evolution of spiral galaxy, we can consider that $\frac{\partial \vec{g}}{\partial t} \equiv \vec{0}$. Then, we can write in quasi-static state for gravities $\vec{g}$ and $\vec{\zeta}$ (at a distance r from the center of the galaxy, values of $\vec{g}$ and $\vec{\zeta}$ are considered as time-invariant) the equivalent Biot and Savart law **[34]** for the extraordinary gravitational field in case of $\kappa = 0$ and $\varrho = 0$:

$$\vec{\zeta} = -\frac{2G}{c}\iiint \frac{\vec{J} \times \vec{r}}{r^3} d\tau \quad (111)$$

Thus, we can write in order of magnitude that:

$$\|\vec{\zeta}\| \equiv \frac{v}{c}\|\vec{g}\| \quad (112)$$

As celestial objects in spiral galaxy are not relativistic, (112) permits us to neglect the effects of extraordinary gravitation field, compare to those of ordinary gravitation field.

B. *Spiral galaxies rotation curve modelling*

If we call by ϕ the gravitational potential energy per unit of mass introduced in (83), we suppose that density of matter μ in any spiral galaxy is in accordance with the equivalent Maxwell-Boltzmann statistics applied to matter; in the supposed inertial frame of reference, which is the center of the galaxy; and has for expression:

$$\mu = \mu_0 \exp(-\beta\phi) \quad (113)$$

With β a homogeneous physical quantity in $s^2.m^{-2}$ and μ_0 the density of matter for $\phi = 0$.
According to (95) and (113), we can write:

$$\mathrm{div}\vec{g} = -4\pi G\mu_0 \exp(-\beta\phi) \quad (114)$$

According to (109), we can write:

$$\vec{g} = -\overrightarrow{\mathrm{grad}}\phi \quad (115)$$

Thus, we can simply write from (114) and (115):

$$\overrightarrow{\mathrm{grad}}\ln(-\mathrm{div}\vec{g}) = \beta\vec{g} \quad (116)$$

With ln( ), the natural logarithm function to the base of the mathematical constant $e \approx 2.71828$ ...

As we suppose that quantity $\vec{g}$ is only dependent of radial distance r from the center of the spiral galaxy, we can rewrite (116) in cylindrical coordinate system as:

$$\frac{\partial \ln\left(\frac{1}{r}\frac{\partial(rg)}{\partial r}\right)}{\partial r} \times \frac{\vec{r}}{r} = \beta\vec{g} \quad (117)$$

With $g = \|\vec{g}\|$. Thus, from (117), we can write:

$$\frac{\left(\frac{\partial^2(rg)}{\partial r^2}\right)}{\left(\frac{\partial(rg)}{\partial r}\right)} = -\beta g + \frac{1}{r} \quad (118)$$

At distance r from the center of the spiral galaxy, the tangential velocity of any object as function of local gravity g can be written as:

$$v^2(r) = r \times (g + \ddot{r}) \quad (119)$$

At a given distance r, the average value of $v^2(r)$ on all the celestial objects of the galaxy at distance r from its center, noted $\langle v^2\rangle(r)$ is:

$$\langle v^2\rangle(r) = r \times (g + \langle\ddot{r}\rangle) \quad (120)$$

Then, considering the great number of potential celestial objects situated at position r from the center of the spiral galaxy, we can suppose that:

$$\langle\ddot{r}\rangle \approx 0 \quad (121)$$

From (118), (120) and (121) we can write:

$$r\frac{\partial^2\langle v^2\rangle}{\partial r^2} = (1 - \beta\langle v^2\rangle) \times \frac{\partial\langle v^2\rangle}{\partial r} \quad (122)$$

Considering the approximation $\lim_{r\to 0}\langle v^2\rangle = 0$ **[35],** we deduce from (122) that:

$$\langle v^2\rangle(r) = \frac{4}{\beta} \times \frac{r^2}{r_0^2 + r^2} \quad (123)$$

With r_0 a characteristic value of radial distance.
Thus, we can conclude that:

$$\lim_{r\to+\infty}\langle v^2\rangle = \frac{4}{\beta} \quad (124)$$

From (120), (121) and (123) we can deduce:

$$\vec{g}(r) = -\frac{4}{\beta} \times \frac{\vec{r}}{r_0^2 + r^2} \quad (125)$$

From (115) and (125), the gravitational potential energy ϕ can be expressed as:

$$\phi(r) = \frac{2}{\beta}\ln\left(\frac{r_0^2 + r^2}{A}\right) \quad (126)$$

With A, a constant of homogenization.
The gravitational potential energy ϕ is null for $r = 0$. Indeed, according to (113) and given the fact that maximum value for density μ is reached for $r = 0$ **[36] [37]**, and because $\phi(r)$ is a monotonically increasing function dependent of r, so we can conclude that $\phi(0) = 0$. Therefore, we have:

$$A = r_0^2 \quad (127)$$

From (113), (126) and (127) we can deduce that:

$$\mu = \frac{\mu_0}{\left(1 + \frac{r^2}{r_0^2}\right)^2} \quad (128)$$

From (95), (125) and (128) we can deduce that:

$$2 = \pi G\mu_0 r_0^2\beta \quad (129)$$

If we consider that a spiral galaxy has a constant thickness E independent of r, its mass called M_g and is worth:

$$M_g = \int_0^{+\infty} \mu(r) 2\pi r E dr \quad (130)$$

According to (128) and (130), we can deduce that:

$$M_g = \pi\mu_0 E r_0^2 \quad (131)$$

From (129) and (131) we can show that:

$$\beta = \frac{2E}{GM_g} \quad (132)$$

Moreover, the angular momentum J of any spiral galaxy is quasi invariant in time if we consider no exchange of matter and no major gravitation interactions between galaxies.

Considering that, all the celestial objects, in a spiral galaxy, have quasi-circular orbit, the angular momentum of a spiral galaxy is given as:

$$\mathrm{J} = \int_0^{+\infty} \mu(r) 2\pi r^2 E v(r) dr \quad (133)$$

According to (123), (128) and (133), we can write:

$$\mathrm{J} = \frac{4}{3}\pi\mu_0 r_0^3 E \sqrt{\frac{4}{\beta}} \quad (134)$$

According to (129) and (132), we can write that:

$$\mu_0 = \frac{M_g}{\pi E r_0^2} \quad (135)$$

From (132), (134) and (135), we can express the spiral galaxy's angular momentum as:

$$\mathrm{J} = \frac{4M_g r_0}{3}\sqrt{\frac{2GM_g}{E}} \quad (136)$$

From (136), if we consider that values of M_g and E are quasi-invariant in time, even if it is not real the case, we can deduce that value of r_0 increases in time as value of G decreases. Thus, galaxies have become bigger and bigger over time.

Moreover, from (135) and (136), we can write:

$$\mu_0 = \frac{32GM_g^4}{9\pi E^2 \mathrm{J}^2} \quad (137)$$

We can consider that evolution of the mass of a galaxy per unit of time $\frac{dM_g}{dt}$ is due to algebraic agglomeration of mass per unit of time $\frac{\partial M_g}{\partial t}$ and its stellar radiative emission P_g such as:

$$\frac{dM_g}{dt} = \frac{\partial M_g}{\partial t} - \frac{P_g}{c^2} \quad (138)$$

Thus, in considering that J is a time-invariant quantity and if we still suppose that E is also a quasi-invariant of time quantity therefore, from (33), (136) and (138) we can deduce that evolution in time of r_0 is given by the following equation:

$$\frac{dr_0}{dt} = \left(-\frac{3}{2M_g} \times \frac{\partial M_g}{\partial t} + \frac{3P_g}{2M_g c^2} + \frac{7}{10}H\right) r_0 \quad (139)$$

We can consider that evolution of mass due to stellar radiation is globally negligible compare to H. Indeed, orders of magnitude are such as:

$$\frac{P_g}{M_g c^2} \sim \frac{L_0}{M_0 c^2} \approx 10^{-21} s^{-1} \ll H \quad (140)$$

With L_0 and M_0, respectively the solar luminosity and mass.

In addition, according to (39), from (139) and hypothesis (140) we can write:

$$r_0(t) \approx r_0(t_0) \times [1 + 0.7H_0(t - t_0)] \times \left(\frac{M_g(t_0)}{M_g(t)}\right)^{\frac{3}{2}} \quad (141)$$

Thus, considering (140) as true, therefore (141) permits to say that characteristic size r_0 of spiral galaxies have an evolution depending on how they accreted mass from surrounding celestial objects during these last billion years. Their size's evolution could be, over time, nearly linear and quasi-proportional to the expansion of the observable Universe's scale factor. Their size could also have a time non-linear evolution in growing or shrinking due to mass gain or loss following galactic fusions and contribution of hydrogen matter due to cosmic filaments between galaxies clusters in different regions of the Universe. However, if galaxies 'size has grown, residual primordial stars as well as their gas has migrated and are situated currently, at the edge of galaxies in the galactic halo. This is maybe why, for spiral galaxies, some physical properties of galactic halo's stars, as star's metallicity and average age, are so different from the properties of less peripheral stars **[38]**. According to (141), galaxy's size can shrink or grow as function of time depending respectively on contribution of matter **[39]** in one side or expansion of the Universe in the other side. According to (13) and (89) we can show that, in addition to the conventional gravitation force, it exists a force density applied on any volume of any cosmic filament with density ρ whose origin is linked to the presence of galaxy cluster of mass M located at a filament node and worth f_v such as:

$$f_v = \frac{2G\rho M}{c^2}\dot{H} \quad (142)$$

Thus, f_v is independent of distance and is either attractive or repulsive according to the sign of $\dot{H}$. The current gravitational density force is higher than current value of f_v in our observable Universe. However, past value of $\dot{H}$ implies that f_v was higher than gravitation density force and allowed matter to be torn out from one galaxy cluster to another one. Nowadays, because of weak value of $\dot{H}$, flow of matter in cosmic filaments must be faded or exist only due to attractive gravitational forces.

VI. SYNTHESIS AND CONCLUSION

This article was intended to give, modestly, solutions to some enigma of modern cosmology. This article proposes a new theory to explain origin of inertia and why matter, with a mass, curves space-time. According to our theory, acceleration of the expansion of the Universe gives inertia to matter in permitting it to interact with space –time. Hence, it is because any massive matter interacts with space-time that it tends to keep its velocity. Consequently, because space-time is in constant accelerated expansion and interacts with massive matter as inertia of expansion, that space-time is curved. Moreover, it is because matter interacts with space-time that it undergoes its curvature effects in the form of gravitational interaction. The more matter interacts with space-time, the more its gravitational interaction with other masses is strong. However, the more matter interacts with space-time, the more its inertia is strong. Thus, it is because gravitation interaction and inertia of a massive matter are both from the same physical phenomenon that gravitational mass equals to inertia mass. Concretely, in order to conceptualize interaction between mass and space-time, this article proposes a theory, which unifies gravitational constant G with the acceleration of the expansion of the Universe Ψ and density of vacuum energy σ in a single equation. This theory permits to consider that the gravitational constant must be a time-variant physical quantity. Moreover, this article stipulates that the total energy of the Universe must be a constant of time. Thanks to this consideration, our article shows that the constant of Hubble is linked to gravitational constant as well as to its first derivative as a function of time. It shows also that gravitation constant is a decreasing function of time and that dark energy is just a consequence of the decline over time in the value of the gravitational constant. Moreover, thanks again to this consideration and thanks to Friedmann-Lemaitre equations, this article shows that radiative emission power (or luminosity) of the entire Universe is a determinist quantity depending on the knowledge of only few physical parameters like density of matter, Hubble constant or scale factor of the Universe. Furthermore, as gravitational constant is supposed to have greater value in the past of the Universe, this article explains how current observed large-scale structure of the Universe was formed thanks to higher value of gravitational interaction without taking into account necessarily of the presence of dark matter as a different nature matter from ordinary baryonic one. The link between gravitational constant and acceleration of the expansion of the Universe can also explain anisotropy in the (black body temperature) mapping of the cosmic microwave background at a time when fluctuation of quantum vacuum energy density generated spatial fluctuation in the value of gravitational constant G resulting in occurrence of primordial matter density's anisotropy. Moreover, a past higher value of gravitational constant has many consequences on past stellar characteristics and evolutions. It is include the fact that past stars are more massive, much bigger and with a shorter life cycle than current ones. Consequently, this article proposes to reconsider nature of what we call dark matter and it stipulates that the hidden mass of the Universe is in what remain of previous generations of stars disappeared since the last 13 billion years, mostly in the form of black holes or primordial black holes and are probably located in the halos of galaxies. Consequence of this property is that visible stars in the galactic halo, including those of our milky way, ceased their formation long ago, due to impoverishment of hydrogen gas, and tend to be old and metal poor **[38]**. In an explanatory approach to the nature of inertia, this article postulates the fact that inertial mass of any elementary particle remains constant in time but it is necessary to consider that their elementary charge is linked to gravitational constant and thus, evolves as function of time. This leads to consider that G has an upper bound and the ratio between current value of G and its maximum value is worth the fine-structure constant. This article also proposes to take account of presence of two different kinds of gravitational field in the Universe. One is the classical ordinary gravitation field created by presence of mass but, the second one, called in this article, extraordinary gravitation field, is, in its physical behavior, more analogous to the magnetic field and, is generated by current mass density. This extraordinary gravitation field is different in nature from gravitomagnetism field in the gravitoelectromagnetism theory accounting the fact that it is a gravitational field generated from space-time distortion and its existence permits to retrieve some relativistic effects. Finally, this article models mass distribution in spiral galaxies. It permits to first show that tangential velocity of celestial peripheral objects converges to a non-zero value in galactic halo. Then, it highlights the fact that size of the spiral galaxies has probably increased over time due to the decrease of the value of the gravitational constant and with the increase of the scale factor of the Universe. Thus, as said, evolution of galaxies 'size may explain why stars in current galactic halo are so poor-metal, old **[38]** and reflect the past of current spiral galaxies. Beyond theses knowledge, we can note that equation (11) may have a different physical signification if written as:

$$c^2 = \frac{8\pi}{3} \times \frac{\sigma G}{\Psi} \quad (11)$$

Indeed, this equation links four fundamental physical quantities (without taking account of the number π as a physical quantity). According to our theory, the quantity $\sigma \times G$ is like a coefficient of interaction between matter and space-time that symbolizes the "rigidity" of space-time from expansion due to presence of mass and caused by physical quantity Ψ, proper to Universe kind of expansion. Thus, interpretation of the given equation (11) above is the link between value of the celerity of any propagation wave based on the space-time structure, including the electromagnetic wave as well as gravitational wave, and the physical characteristic of space-time, which are its "rigidity" and the acceleration of its expansion. Thus, equivalent to propagation of "classical" waves like acoustic wave or a vibrating wire in a material medium, electromagnetic wave as well as gravitational wave propagate through space-time as immaterial medium of propagation with celerity c given in the equation (11). It is because origin of inertia is interaction between space-time and matter that any mass cannot reach celerity of gravitational waves. As photons are massless and so have no inertia that they can reach the same celerity of propagation of the gravitational waves. We consider in our theory that variation of Ψ as function of time influences only the value of G and not the value of celerity c. Indeed, if acceleration of the expansion of the Universe permits matter to interact with space-time and give it inertia via the physical quantity G, therefore only value of G evolves with time via evolution of Ψ. Thus, in our theory, we consider that quantity c is a real constant of time independent of G and Ψ evolutions.

Of course, like all any others theories, those postulated in this article have to be verified, either by simulations or by observations, before being confirmed or invalidated. Observations can be possible in the near future thanks to the ESA and the NASA James Webb Space telescope replacement of the Hubble telescope which will be able to study formation of first stars and galaxies **[40]** as well as to measure density of dark matter from gravitational lensing **[41]**. In parallel, the future Euclid Spacecraft by ESA has objective to understand nature of the dark energy by measuring acceleration of the expansion of the Universe as well as measuring distribution of the dark matter and galaxies **[42]** in the Universe.

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VIII. References

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