Prejudiced Information Fusion Using Belief Functions

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Abstract. G. Shafer views belief functions as the result of the fusion of elementary partially reliable testimonies from different sources. But any belief function cannot be seen as the combination of simple support functions representing such testimonies. Indeed the result of such a combination only yields a special kind of belief functions called separable. In 1995, Ph. Smets has indicated that any belief function can be seen as the combination of so-called *generalized* simple support functions. We propose a new interpretation of this result in terms of a pair of separable belief functions, one of them modelling testimonies while the other represents the idea of prejudice. The role of the latter is to weaken the weights of the focal sets of the former separable belief function. This bipolar view accounts for a form of resistance to accept the information supplied by the sources, which differs from the discounting of sources.

1 Introduction

G. Shafer [1] has presented his theory of belief functions essentially as an approach to the fusion of unreliable elementary testimonies, each being represented by simple support functions. However many belief functions prove to be not separable, i.e., not the orthogonal sum of simple support functions. Ph. Smets [2] tries to remedy this difficulty by generalizing simple support functions, showing that any belief function is the conjunctive combination of such generalized elementary belief functions (where some masses can be negative). Using a retraction operation, he shows that any belief function can be decomposed into two separable belief functions. One represents the fusion of elementary testimonies (expressing confidence), and the other (expressing doubt) plays the role of a moderator that can annihilate, via retraction, some information supplied by the former, possibly resulting in ignorance. This pair of belief functions is called "Latent Belief Structure" by Smets.

In this paper, we present a bipolar belief function model which pushes the notion of "Latent Belief Structure" further. In a belief function, the doubt component is assumed to reflect a cognitive bias interpreted as a prejudice, pertaining to the information supplied by the confidence component. This cognitive bias leads to weaken the strength attached to the combination of some elementary

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testimonies appearing in the confidence part, thus expressing a lack of trust in the information obtained by merging these testimonies.

The organization of the rest paper is as follows. In Sect. 2, some necessary background on belief functions is introduced. In Sect. 3, we propose new results about the decomposition of belief functions, providing new insights in the weight function introduced by Smets [2], as well as conditions for separability in a simple case. Section 4 presents a generalized setting for the merging of elementary testimonies in the presence of prejudices, focusing on the process of belief attenuation by means of the retraction operation. This framework is illustrated on the Linda example [3], highlighting the difference between belief retraction and source discounting.

2 Separable Belief Functions

In Shafer evidence theory, the uncertainty concerning an agent's state of belief on a finite set of possible situations, called the frame of discernment Ω is represented by a basic belief assignment (BBA) or mass function m defined as a mapping $m : 2^{\Omega}$ to $[0, 1]$ verifying $\sum_{A \subseteq \Omega} m(A) = 1$. Each subset $A \subseteq \Omega$ such as $m(A) > 0$ is called a *focal set* of m. A BBA m is called *normal* if $\emptyset$ is not a focal set (subnormal otherwise), *vacuous* if Ω is the only focal element, *non-dogmatic* if Ω is a focal set, *categorical* if m has only one focal set different from Ω.

An elementary testimony T with strength $1-x$ in favor of a non-contradictory proposition $A \in 2^{\Omega}$ is represented by a simple BBA (SBBA) $m : 2^{\Omega} \longrightarrow [0, 1]$ such that $m(A) = 1 - x$, for $A \neq \Omega$ and $m(\Omega) = x$, with $x \in [0, 1]$ and is denoted by $m = A^x$. The value x, we call *diffidence weight*, evaluates the lack of reliability of the testimony (or the source of information). A vacuous BBA can thus be denoted by A^1 for any $A \subset \Omega$, and a categorical BBA $A \neq \Omega$ can be denoted by A^0.

A belief function $Bel(A)$ is a non-additive set function which represents the total quantity of belief in the subset A of Ω and is defined by $Bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B)$. A BBA m can be equivalently represented by its associated plausibility and commonality functions respectively defined for all $A \subseteq \Omega$ by $Pl(A) = \sum_{A \cap B \neq \emptyset} m(B) = 1 - Bel(\overline{A})$ and $Q(A) = \sum_{B \supseteq A} m(B)$.

The conjunctive combination of BBA's m_j derived from k distinct sources, denoted by $m_{\bigcirc\!\!\!\!\cap}$ is expressed by $m_{\bigcirc\!\!\!\!\cap}(A) = \sum_{A_1 \cap \ldots \cap A_k = A} \left(\prod_{j=1}^{k} m_j(A_j) \right)$. Note that $m_{\bigcirc\!\!\!\!\cap}$ is not always normal. Dempster's rule, denoted by $\oplus$, is a normalized version of the conjunctive combination rule and is defined such that: $m_{\oplus}(\emptyset) = 0$ and $m_{\oplus}(A) = K \cdot m_{\bigcirc\!\!\!\!\cap}$ for $A \neq \emptyset$. The normalization factor K is of the form $(1 - c(m_1, \ldots m_k))^{-1}$ where $c(m_1, \ldots m_k) = \sum\limits_{A_1 \cap \ldots \cap A_k = \emptyset} \left(\prod_{j=1}^{k} m_j(A_j) \right) < 1$ represents the amount of conflict between the sources. These two combination rules are commutative, associative, and generally used to combine BBAs from distinct sources. The Dempster rule is simply expressed using the commonality functions as: $Q_1 \oplus \ldots \oplus Q_k = K.Q_1 \cdot Q_2 \cdots Q_k$.

In Shafer's view [1], a separable BBA is the result of Dempster's rule of combination of simple BBAs: $m = \bigoplus_{\emptyset \neq A \subset \Omega} A^{w(A)}, w(A) \in [0,1],\ \forall A \subset \Omega, A \neq \emptyset$. We call the mapping $w : 2^{\Omega} \setminus \{\Omega\} \to (0,1]$ a *diffidence function*. If the BBA is non-dogmatic ($m(\Omega) > 0$), this representation is unique, and $w(A) > 0, \forall A \subset \Omega$. Denœux [4] has extended this concept to the conjunctive combination of subnormal BBA's $\bigcirc\!\!\!\!\cap_{\emptyset \neq A \subset \Omega} A^{w(A)}, w(A) \in [0,1]\ \forall A \subset \Omega$.

Shafer [1] [Th. 7.2 p.143] shows that if Bel is a separable belief function, and A and B are two of its focal sets such as $A \cap B \neq \emptyset$, then $A \cap B$ is a focal set of Bel. The condition $A \cap B \neq \emptyset$ can be dropped if we allow for sub-normalized belief functions. But the converse is not true. This necessary condition clearly indicates that not all belief functions are separable. To overcome this difficulty, Smets [2] generalized the concept of simple support function, considering A^x such that $x \in (0, +\infty)$. Smets has shown that any non dogmatic BBA can be decomposed into the conjunctive combination of generalized BBA's: $m = \bigcirc\!\!\!\!\cap_{\emptyset \neq A \subset \Omega} A^{w(A)}$, extending the range of diffidence functions w to $(0, +\infty)$. For every $A \subset \Omega$, the weights $w(A)$ are obtained from the commonality function of m as: $w(A) = \prod_{B \supseteq A} Q(B)^{(-1)^{|B|-|A|+1}} = \frac{\prod_{C \cap A = \emptyset, |C| \text{odd}} Q(A \cup C)}{\prod_{C \cap A = \emptyset, |C| \text{even}} Q(A \cup C)}$.

3 The Bipolar Decomposition of a Belief Function

We can write the decomposition of a non-dogmatic belief function as $m = (\bigcirc\!\!\!\!\cap_{A \in \mathcal{C}} A^{w^+(A)}) \oslash (\bigcirc\!\!\!\!\cap_{B \in \mathcal{D}} B^{w^-(B)})$, where

- w^+ and w^- are standard diffidence functions in $(0,1)$ defined from the original one w associated to m, such that: $w^+(A) = \min(1, w(A))$, and $w^-(A) = \min(1, 1/w(A)), \forall A \subset \Omega$.
- $\mathcal{C}$ and $\mathcal{D} \subseteq 2^{\Omega}, w(A) < 1$ if $A \in \mathcal{C}$ and $w(B) > 1$ if $B \in \mathcal{D}$.
- $\oslash$ defined by $m_1 \oslash m_2 = (\bigcirc\!\!\!\!\cap_{\emptyset \neq A \subset \Omega} A^{w_1(A)}) \bigcirc\!\!\!\!\cap (\bigcirc\!\!\!\!\cap_{\emptyset \neq B \subset \Omega} B^{\frac{1}{w_2(B)}})$ is the *retraction operation*, also obtained by the division of commonality functions: $Q_1 \oslash Q_2(X) = \frac{Q_1(X)}{Q_2(X)}, \forall X \subseteq \Omega$, called *decombination* [2] or removal [5]. Ginsberg [6] and Kramosil [7] have exploited this division rule.
- Factors of the form $A^{w(A)}$ represent testimonies in favor of A if $w(A) < 1$, and will be called *prejudices* against believing A if $w(A) > 1$.

A belief function is separable if and only if $w(A) \leq 1, \forall A \subset \Omega$ in the above decomposition. In that case, the set of focal sets of m contains Ω and is closed under conjunction [1]. So a separable belief function will be of the unique form: $m = \bigcirc\!\!\!\!\cap_{A \in \mathcal{C}} A^{w^+(A)}$.

A mass function m can thus be decomposed in a unique irredundant way as a pair (m^+, m^-), of separable belief functions induced by BBAs m^+ and m^-, such that $m = m^+ \oslash m^-$. The confidence component denoted by m^+ is a BBA obtained from the merging of SBBAs, with focal sets in $\mathcal{C}$, and the diffidence component denoted by m^- is a BBA obtained likewise, with focal sets in $\mathcal{D}$. By construction, $\mathcal{C} \cap \mathcal{D} = \emptyset$. The pair (m^+, m^-) of separable BBAs is called a latent belief structure [2] more recently studied in [4,8,9]. The existence of positive and negative information is generally coined under the term *bipolarity* [10], an idea applied to latent belief structures in [11]. A general study of the canonical conjunctive decomposition of a belief function was realised by Ke et al. [12] and Pichon [13], albeit without focusing on its possible meaning.

In the following we are interested in retrieving the mass function m from its diffidence function w via the commonality function rather by the conjunctive combination. First, note that the expression $\prod_{B \supseteq A} Q(B)^{(-1)^{|B|-|A|+1}}$ makes sense for $A = \Omega$, and we get $w(\Omega) = 1/Q(\Omega)$. So function w can be extended to the whole of 2^S, even if only sets $A \subset \Omega$ appear in the decomposition formula. In previous studies, $w(\Omega)$ remained undefined. Of course, $w(\Omega) > 1$ but this will be also the case for the diffidence weights of other subsets for non-separable belief functions.

Noticing that $m(A) = \sum_{A \subseteq B}(-1)^{|B|-|A|}Q(B)$, and moreover $\log w(A) = \sum_{A \subseteq B}(-1)^{|B|-|A|+1} \log Q(B)$, it is clear that m is to Q what $-\log w$ is to $\log Q$. Since $Q(A) = \sum_{A \subseteq B} m(B)$, we have $\log Q(A) = \sum_{A \subseteq B} \log(1/w(B)) = \log \prod_{A \subseteq B} \frac{1}{w(B)}$. Hence,

$$Q(A) = \frac{1}{\prod_{A \subseteq B} w(B)} \tag{1}$$

Note that in (1), the weight $w(\Omega)$ appears explicitly in all the expressions of $Q(A)$ for all subsets A. Hence we can retrieve the BBA m, from the diffidence function w computed from it, directly as $m(E) = \sum_{E \subseteq A}(-1)^{|B|-|A|}(\frac{1}{\prod_{A \subseteq B} w(B)})$. In particular we can have the following result:

Proposition 1. *A diffidence function computed from m via (1) is such that $\prod_{A \subseteq \Omega} w(A) = 1$.*

Proof. We know that commonalities satisfy $Q(\emptyset) = 1$. Using (1) yields $Q(\emptyset) = \frac{1}{\prod_{\emptyset \subseteq B} w(B)} = 1$. So, $\prod_{A \subseteq \Omega} w(A) = 1$.

It gives a general definition of a diffidence function as a mapping $w : 2^\Omega \to (0, +\infty)$, such that $\prod_{A \subseteq \Omega} w(A) = 1$ and $w(\Omega) \geq 1$. Note that the mass function m_w derived from any function w defined in this way is not always positive. Indeed, suppose that $w(A) = \lambda < 1, w(B) = \mu > 1, w(C) = 1, C \neq \Omega$ otherwise (so $w(\Omega) = 1/\lambda\mu > 1$). By means of the conjunctive rule, one gets the BBA: $m(A \cap B) = (1-\lambda)(1-\mu), m(A) = \lambda(1-\mu), m(B) = (1-\lambda)\mu, m(\Omega) = \lambda\mu$. It is clear that $m(A \cap B), m(A)$ are negative, in general. So the mapping $m \mapsto w$ is injective, but it is not surjective. Namely, given a diffidence function w such that $\prod_{A \subseteq \Omega} w(A) = 1$ and $w(\Omega) \geq 1$, Q_w obtained by (1) is a decreasing set-function that ranges on $[0, 1]$, but decreasingness is not sufficient to ensure that masses obtained from function Q_w are all positive, i.e., Q_w is not always a commonality function. On the other hand, diffidence functions such that $w(A) \leq 1, \forall A \subset \Omega$ are in one to one correspondence with BBAs of separable belief functions.

Example: Two Overlapping Focal Sets on a 4-Element Frame. Let $\Omega = \{a, b, c, d\}$. We denote $\{a\}$ by a, $\{a, b\}$ by ab, etc. Consider m with $m(ab) = \beta; m(ac) = \gamma; m(a) = \alpha$ with $\alpha + \beta + \gamma < 1$, (hence $m(\Omega) = 1 - (\alpha + \beta + \gamma)$). Note that $Q(a) = 1$, $Q(ab) = 1 - \alpha - \gamma$, $Q(ac) = 1 - \alpha - \beta$, $Q(B) = 1 - \alpha - \beta - \gamma$ for other non-empty sets B.

We can decompose m as a combination $m = \{ab\}^{w(ab)} \bigcirc\!\!\!\!\!\cap \{ac\}^{w(ac)} \bigcirc\!\!\!\!\!\cap \{a\}^{w(a)}$. Its diffidence function is given in Table 1.

Table 1. Decomposition with focal sets: ab, ac, a and Ω

A	m	w	Inverse solution
a	α	$w(a) = \frac{(1-\alpha-\gamma)(1-\alpha-\beta)}{1-\alpha-\beta-\gamma}$	$1 - (w(ab) + w(ac) - w(ab)w(ac))w(a)$
ab	β	$w(ab) = \frac{1-\alpha-\beta-\gamma}{1-\alpha-\gamma}$	$(1 - w(ab))w(ac)w(a)$
ac	γ	$w(ac) = \frac{1-\alpha-\beta-\gamma}{1-\alpha-\beta}$	$(1 - w(ac))w(ab)w(a)$
$abcd$	$1 - \alpha - \beta - \gamma$	$w(\Omega) = \frac{1}{1-\alpha-\beta-\gamma}$	$w(ac)w(ac)w(a)$
other subsets	0	1	0

It is a separable belief function if the diffidence weights are ≤ 1. Note that $w(ab) = \frac{1-\alpha-\beta-\gamma}{1-\alpha-\gamma} < 1$ and $w(bc) = \frac{1-\alpha-\beta-\gamma}{1-\alpha-\beta} < 1$ but it is not always the case for $w(a) = \frac{(1-\alpha-\gamma)(1-\alpha-\beta)}{1-\alpha-\beta-\gamma}$. The condition of separability of the belief function m is $\alpha^2 + \alpha(-1 + \beta + \gamma) + \beta\gamma \leqslant 0$. Fixing β, γ, this condition is of the form $\alpha_1 \leq \alpha \leq \alpha_2$, with $\alpha_1 = \frac{(1-\beta-\gamma)-\sqrt{(-1+\beta+\gamma)^2-4\beta\gamma}}{2}$ and $\alpha_2 = \frac{(1-\beta-\gamma)+\sqrt{(-1+\beta+\gamma)^2-4\beta\gamma}}{2}$, provided that $(1 - \beta - \gamma)^2 \geq 4\beta\gamma$. The latter condition is valid only if β and γ are small enough, that is if $\sqrt{\beta} + \sqrt{\gamma} \leq 1$. Besides note that $0 \leq \alpha_1 \leq \alpha_2 \leq 1 - \beta - \gamma$.

It is of interest to consider the special case when $\beta = \gamma$. It is easy to verify that $\alpha_1 = \frac{1-2\beta-\sqrt{1-4\beta}}{2}$ and $\alpha_2 = \frac{1-2\beta+\sqrt{1-4\beta}}{2}$. We must have $\beta \leq 0.25$ otherwise the belief function cannot be decomposable (α_1 and α_2 are not defined). For $\beta = 0.25$ we have that $\alpha_1 = \alpha_2 = 0.25$. See the graph of the functions giving α_1 and α_2 in terms of β on Fig. 1. It indicates the zone of non-separability under the line $1 - 2\beta$ and on the right-hand side of the curve for α_1 and α_2.

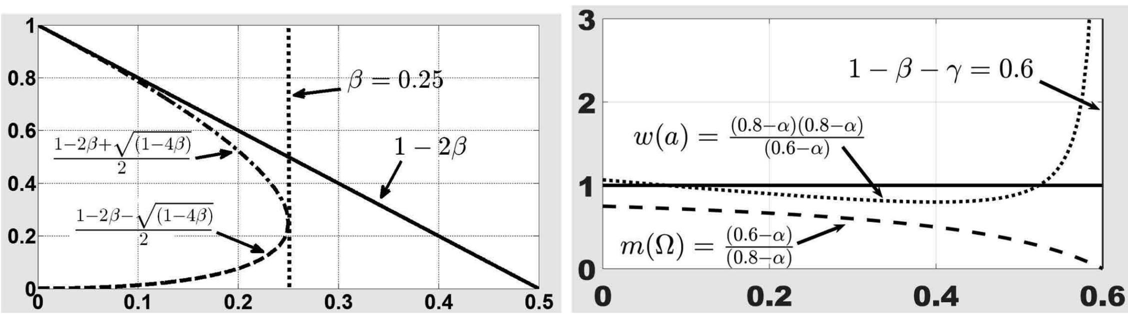

Fig. 1. Left: $m(a)$ in terms of $\beta = \gamma$ if $w(a) = 1$. Right: diffidence weights in terms of α

It may sound strange that there are two separability thresholds α_1 and α_2. Actually, it means that, fixing β and γ, there are still two possibilities for choosing $m(a)$ such that m is the conjunctive combination of two SBBAs m_b and m_c respectively focused on ab and ac. Let $\lambda = m_b(ab)$ and $\mu = m_c(ac)$. By definition, we have $\beta = \lambda(1-\mu)$ and $\gamma = (1-\lambda)\mu$. Suppose without loss of generality that $\lambda\mu \leq (1-\lambda)(1-\mu)$. There are two possible choices for m_b and m_c:

- $m_b^1(ab) = \lambda$ and $m_c^1(ac) = \mu$. Then $\alpha_1 = \lambda\mu$, where a is weakly supported.
- $m_b^2(ab) = 1-\mu$ and $m_c^2(ac) = 1-\lambda$. Then $\alpha_2 = (1-\lambda)(1-\mu)$, where a is strongly supported ($\lambda\mu$ is small). Note that $m_b^1(ab) = 1 - m_c^2(ac)$.

When m defined by parameters α, β, γ is separable, we get $w(a) = 1$, which leads to the condition $(\alpha+\beta)(\alpha+\gamma) = \alpha$. Hence $w(ab) = 1-\alpha-\beta$ and $w(ac) = 1-\alpha-\gamma$. So we can define $m_b(ab) = \alpha+\beta$, $m_b(\Omega) = 1-\alpha-\beta$; and $m_c(ac) = \alpha+\gamma$, $m_c(\Omega) = 1-\alpha-\gamma$. Choosing $\alpha = \alpha_1$ or α_2 leads to respective pairs of SBBAs (m_b^1, m_c^1) and (m_b^2, m_c^2). We can check that indeed these pairs are related by the condition $m_b^1(ab) = 1 - m_c^2(ac)$, that is, $\alpha_1 + \beta + \alpha_2 + \gamma = 1$.

Finally, we can study the variation of the diffidence weights when α ranges from 0 to its maximum $1-\beta-\gamma$. Note that $w(a)$ is the mass of Ω for the SBBA m_a focusing on a, when considering the decomposition of the BBA m. The less $w(a)$ the stronger is the testimony pointing to a, the testimony is not present if $w(a) = 1$, and it becomes a prejudice against a when $w(a) > 1$. It can be checked (see Fig. 1 right) that:

- For $\alpha = 0$, we get $w(a) = \frac{(1-\beta)(1-\gamma)}{1-\beta-\gamma} > 1$.
- $w(a)$ decreases with α until a value $\underline{\alpha} = 1-\beta-\gamma-\sqrt{\beta\gamma}$ where the derivative vanishes. The minimal value of $w(a)$ is $\frac{(\beta+\sqrt{\beta\gamma})(\gamma+\sqrt{\beta\gamma})}{\sqrt{\beta\gamma}}$ and it is less than 1 only if $\sqrt{\beta}+\sqrt{\gamma} \leq 1$, as seen earlier. When α_1 and α_2 exist, $\alpha_1 \leq \underline{\alpha} \leq \alpha_2$, and they coincide if and only if $\sqrt{\beta}+\sqrt{\gamma} = 1$.
- $w(a)$ increases with $\alpha \geq \underline{\alpha}$ and $\lim_{\alpha\to 1-\beta-\gamma} w(a) = +\infty$.

Looking at the right part of Fig. 1, we note that when $\alpha = 0$, $w(a) > 1$ and testimonies in favor of ab and ac are weak; so the prejudice against a is strong enough to erase the focal set a from m. When $w(a)$ reaches its minimal value, the prejudice in favor of a is maximal. When α is close to its maximum value $1-\beta-\gamma$, testimonies in favor of ab and ac are less and less challenged since their diffidence weights get close to 0, while the prejudice against a rapidly increases to infinity. At the limit, we get a dogmatic belief function with $m(a) = 1-\beta-\gamma$ and the prejudice no longer compensates the elementary testimonies in favor of ab and ac.

4 Prejudiced Information Fusion

A generalized SBBA focused on a subset E with diffidence weight x represents the idea that "one has some reason to believe that the actual world is in E (and nothing more)" when x is small ($x < 1$), whereas, when $x > 1$, it expresses the idea that "one has some reason not to believe that the actual world is in E" [2], what we called *prejudice*. Note that the latter does not mean that we have a reason to believe the complement $\overline{E}$ of E (which would mean assigning a weight $x < 1$ to $\overline{E}$). In this section, we try to provide an interpretation of non-separable belief functions in terms of merging elementary testimonies with prejudices that weaken the result of the former merging. The idea is that the agent possessing a prejudice of strength $y > 1$ against believing E is ready to doubt about the truth of E whenever receiving a testimony claiming that E is true. More generally, the combination $A^x \bigcirc\!\!\!\!\cap\, B^y$ of a simple BBA $A^x, x < 1$ with a simple prejudice $B^y, y > 1$ yields the diffidence function $w(\cdot)$ such that:

$$\text{if } B \neq A, w(E) = \begin{cases} x & \text{if } E = A \\ y & \text{if } E = B \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

and $w(A) = xy$ if $A = B$. So, it is a belief function if and only if $A = B$ and $xy < 1$. It is equivalent to *erode* the testimony A^x with another testimony $A^{1/y}$ using retraction. In particular, $A^x \bigcirc\!\!\!\!\cap\, A^{1/x}$ yields total ignorance. However, erosion cannot alter A^x by retracting $B \neq A$.

We can compare the erosion with discounting an SSB A^x: the discounting procedure reduces the mass $1-x$ bearing on A with a factor $\delta \in [0,1]$ and yields $m_\delta(A) = A^{(1-\delta)+\delta.x}$, which is equal to A^{xy} provided that $0 < \delta = \frac{1-xy}{1-x} \leq 1$ since $y > 1$, that is $1 < y < 1/x$.

More generally we can retract a focal set B from a separable mass function m. Consider $m = \bigcirc\!\!\!\!\cap_{i=1}^{k} A_i^{w_i}$ and its combination with a prejudice $B^x, x > 1$. Focal sets of m are of the form $E_I = \cap_{i\in I} A_i, I \subseteq \{1, \dots k\}$ with masses $m(E_I) = \prod_{i\in I}(1-w_i)\prod_{i\notin I} w_i$ (where we allow that some E_I's may be identical). Combining this mass function with E_J^x yields a mass function m' such that $m'(E_J) = xm(E_J) + (1-x)(\sum_{I\subseteq J} m(E_I))= xm(E_J) + (1-x)Bel(E_J)$ (where $E_\emptyset = \Omega$). So E_I is erased from the focal sets of m by E_J^x if and only if $x = \frac{Bel(E_J)}{Bel(E_J)-m(E_J)} = \frac{\sum_{I\subseteq J} m(E_I)}{\sum_{I\subset J} m(E_I)} = 1/(1-\prod_{i\in J}(1-w_i))$, which is clearly more than 1. Note that we can erode a single focal set via retraction, while discounting affects all focal sets to the same extent. Similarly, it can be checked that if $J \subset I$, $m'(E_J) = (\prod_{i\notin I} w_i)\prod_{i\in I\setminus J}(1-w_i)(1-x+x\prod_{i\in J}(1-w_i)) = 0$ if and only if $x = 1/(1-\prod_{i\in J}(1-w_i))$ again, while if $J \not\subset I$, $m'(E_I) = xm(E_I)$, which is provably less than 1. In other words, retracting the focal set E_J erases all focal sets $E_I \subset E_J$ as well, namely all combinations between the merging of information from sources indexed in J, with information from other sources.

So we can consider that any belief function comes from merging unreliable elementary testimonies, with prejudices that weaken the weights pertaining to the conjunctions of information items coming from sources. It is indeed natural to consider that information we receive from the outside is challenged by our prior information taking the form of stereotypes, or prejudices that one is often unaware of. The receiver is reluctant to consider the result of such conjunction valid. For instance, consider a variant of the Linda problem [3]. In this case, the bank teller Linda, depicted as a philanthropist, is found by participants to a psychological experiment, more likely to be a philanthropist bank teller than a bank teller, because the former looks more "representative" or typical of persons who might fit the description of Linda. Here we consider the case when we receive two testimonies, namely one (B^v) claiming that Linda is a banker and another one A^w that she is a philanthropist. The fusion process leads us to allocate a belief degree $(1-v)(1-w)$ to the fact that she is a philanthropist bank teller. However, a prejudiced individual would hardly believe that a bank teller can be philanthropist, and would like to erode, possibly erase, this belief by combining the result of the fusion with the generalized SSB $(A \cap B)^u$ with $1 < u \leq 1/(v+w-vw)$, which leads to a belief degree equal to $1-(u+v-vw)u$, that is all the lesser as the prejudice is strong.

5 Conclusion

This paper revisits the decomposition of a belief function into a combination of generalized simple support functions proposed by Smets [2] showing that it can be viewed as the merging of uncertain testimonies and of prejudices against the results of their partial conjunctions. We have laid bare new formal properties of the diffidence function w and shown how to reconstruct the BBA m from it via Moebius-like transforms. Our results strengthen the approach to belief function based on the merging of pieces of evidence, as opposed to the approach based on upper and lower probability. Future research can be a study of the information ordering based on diffidence functions, introduced by Denœux [4], on which our results can shed more light.

References

1. Shafer, G.: A Mathematical Theory of Evidence. Princeton Univiersity Press, Princeton (1976)
2. Smets, P.: The canonical decomposition of a weighted belief. In: Proceedings 14th International Joint Conference on Artificial Intelligence (IJCAI), Montreal, vol. 2, pp. 1896–1901, 20–25 August 1995
3. Tversky, A., Kahneman, D.: Extensional versus intuitive reasoning: the conjunction fallacy in probability judgment. Psychol. Rev. **90**, 293–315 (1983)
4. Denœux, T.: Conjunctive and disjunctive combination of belief functions induced by nondistinct bodies of evidence. Artif. Intell. **172**(2), 234–264 (2008)
5. Shenoy, P.P.: Conditional independence in valuation-based systems. Int. J. Approx. Reason. **10**(3), 203–234 (1994)
6. Ginsberg, M.L.: Non-monotonic reasoning using Dempster's rule. In: Proceedings of National Conference on Artificial Intelligence, Austin, TX, pp. 126–129, 6–10 August 1984
7. Kramosil, I.: Probabilistic Analysis of Belief Functions. Kluwer, New York (2001)
8. Pichon, F., Denoeux, T.: On Latent belief structures. In: Mellouli, K. (ed.) ECSQARU 2007. LNCS (LNAI), vol. 4724, pp. 368–380. Springer, Heidelberg (2007). https://doi.org/10.1007/978-3-540-75256-1_34

9. Schubert, J.: Clustering decomposed belief functions using generalized weights of conflict. Int. J. Approx. Reason. **48**(2), 466–480 (2008)
10. Dubois, D., Prade, H.: An introduction to bipolar representations of information and preference. Int. J. Intell. Syst. **23**(8), 866–877 (2008)
11. Dubois, D., Prade, H., Smets, P.: "Not impossible" vs. "guaranteed possible" in fusion and revision. In: Benferhat, S., Besnard, P. (eds.) ECSQARU 2001. LNCS (LNAI), vol. 2143, pp. 522–531. Springer, Heidelberg (2001). https://doi.org/10.1007/3-540-44652-4_46
12. Ke, X., Ma, L., Wang, Y.: Some notes on canonical decomposition and separability of a belief function. In: Cuzzolin, F. (ed.) BELIEF 2014. LNCS (LNAI), vol. 8764, pp. 153–160. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-11191-9_17
13. Pichon, F.: Canonical decomposition of belief functions based on Teugels representation of the multivariate Bernoulli distribution. Inf. Sci. **428**, 76–104 (2018)