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Intermittent Discounting

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Abstract

A novel theory of time discounting is proposed in which future consumption is less valuable than present consumption because of waiting costs. Waiting is intermittent as consumer's attention is periodically distracted away from future gratifications. A new axiom is introduced, called weak impatience, according to which consumers are impatient only if they pay attention to the good in the present, or when all options are in the future, a strictly positive probability exists they will pay attention to it. The model unifies and reinterprets three empirically relevant properties of intertemporal preference: decreasing impatience, present bias and sub-additive discounting. In addition, the model predicts new testable behavioral patterns, not predicted by other discounting models.

J.E.L. codes: D8, E21

Keywords : time preferences, decreasing impatience, sub-additive discounting, present bias.

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1 Introduction

Impatience is a key feature of intertemporal decisions but also a versatile property. People do not like waiting two minutes at a stoplight, but are willing to save for their retirement occurring in several decades. Those contrasting attitudes suggest that the horizon of choice interacts non-linearly with individuals' propensity to discount future outcomes. I explore this possibility and propose a novel theory of time discounting which starts from the observation that waiting for a reward requires a mental effort as people have to resist temptation and cope with some amount of frustration. The more delayed the gratification, the longer the waiting period and the less valuable future utility. In addition, introspection and casual observations indicate that people spend most of their time absorbed in daily activities during which future gratifications are not reminded. Waiting episodes can be triggered by an external event or a cue, like discussing a new model of cell-phone with a colleague, watching a tv advertisement or contemplating a piece of chocolate fudge cake at a friend's birthday.[1] Reminding may also spontaneously occur when the image of a gratification springs to mind, or when a need is felt, out of boredom, discomfort, stress, hunger, thirst or craving.

When individuals experience intermittent reminding, preferences for early vs. late gratifications depend on the waiting costs and the frequency of waiting periods. Both dimensions raise expected waiting costs and undermine consumer's willingness to delay consumption. Whereas previous models of discounting have focused on the extent to which people discount future utilities, the implications of intermittent waiting and discontinuous discounting have not been investigated so far. This second dimension of discounting is arguably as important as the size

[1] The frequency of reminders may be reinforced by biased attention toward temptation cues. For example, smokers have been found to display selective attention for smoking-related cues (Mogg, Field, and De Houwer, 2003), and heavy drinkers toward alcohol-related cues (e.g. Townshend and Duka, 2001). See Bernheim and Rangel (2004) for a theoretical analysis.

of discounting. For example, a typical question asked to people who suffer from addiction is "how many times a day do you think about ...". In less extreme situations, repeated exposure to temptation goods may lead consumers to indulge, which is routinely exploited by the advertising industry.

The goal of this paper is to investigate the implications of intermittent waiting for time preferences. To do so, I pose a general multi-period setting in which an agent derives utility from a good which may be consumed now or later. Waiting is both costly and intermittent, as reminding future consumption occurs with some probability every period. A key property of the model is that, if the decision maker expects to be distracted by activities unrelated to the coveted good over a given time interval, her discount function does not decrease over the period. It is only in waiting periods that future utility is discounted further.

The model unifies and reinterprets three important and robust features of intertemporal preferences: decreasing impatience (or hyperbolic discounting), present bias and non-additive discounting. Intermittent waiting means that the pace at which the good is discounted slows down compared to permanent waiting, which makes the consumer decreasingly impatient. Present bias, the propensity to prefer immediate gratification to future ones, appears when the present is a decision period, which is also a waiting period if consumption is delayed. Future periods differ from the present as future reminding is uncertain. As in the quasi-hyperbolic discounting model, the wait-based model is consistent with people being impatient over short delays, like a day or a week, without being implausibly impatient in long-run trade-offs. It also makes new predictions, such as the possibility of significant impatience over postponed short-delay trade-offs.

Intermittent waiting is also consistent with sub-additive discounting, according to which a sequence of trade-offs in a sub-divided interval leads to more overall discounting than a single trade-off over the whole interval. This pattern has been reported in several experiments[2] and in German representative samples (Dohmen et al., 2012; Dohmen et al. 2017). The evidence cannot be accounted for by previous delay-dependent discounting models, in which additivity holds regardless of the shape of the discount function. The intermittent discounting model proposes a theory of sub-additivity based on the premise that individuals expect more waiting in a series of short-delay trade-offs than in a long-delay trade-off. The explanation is reminiscent to the interpretation of Read (2001) according to whom sub-dividing a delay undermines people propensity to withstand waiting by making them pay more attention to every part of it.

I also show that when reminding probabilities are stationary, and waiting costs are exponentially discounted, the date $t > 0$ discount function takes a simple two-parameter functional form $D(t) = p\beta^t + (1-p)\beta$, with p the probability of reminding future rewards and β the discount factor applied to future waiting costs. The model boils down to the exponential model of Samuelson (1937) when reminding repeats every period ($p = 1$). This gives a behavioral foundation to the constant discounting model and makes clear the fact that present bias, decreasing impatience and sub-additive discounting are direct consequences of the assumption of intermittent waiting ($p < 1$).

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 lays out a general model of consumption with intermittent waiting, poses a set of axioms and studies their consequences for time preferences. Section 4 presents an analytically tractable two-parameter version of the model. Section 5 shows how the physical proximity and salience of the reward modulate impatience. Section 6 investigates the link between intermittent discounting and decreasing impatience. Section 7 explains why intermittent waiting is consistent with a bias for immediate consumption. Section 8 shows

[2] Read (2001), Read and Roelofsma (2003), Scholten and Read (2006) and Kinari et al. (2009).

that intermittent waiting costs lead to non-additive time discounting, and Section 9 concludes.

2 Related Literature

The exponential discounting model is by far the most used framework in intertemporal models. It is parsimonious and normatively appealing, yet has a limited descriptive validity.[3] Several discounting models deviate from the exponential functional by introducing decreasing impatience or a strong preference for immediate consumption. The quasi-hyperbolic models of Phelps and Pollak (1968) and Laibson (1997) assume a modified exponential discount factor $\alpha\beta^{-t}$ with $0 < \alpha, \beta < 1$, and $D(0) = 1$, in which the parameter $1/\alpha > 1$ can be interpreted as an extra weight applied to present utility. Benhabib and Bisin (2004) introduce a separate cost of delaying consumption interpreted as "the psychological restraint from the impulse of choosing the immediate reward." Contrary to the model with waiting, the cost of delay is a fixed cost independent of the size of the reward and of the length of the delay. Laibson (2001) and Bernheim and Rangel (2004) propose models of addiction in which temptation effects endogenously depend on past associations between cues (e.g. the sight of a lighter) and rewards (smoking a cigarette). The wait-based model does not make explicit what lead people to remind consumption and focuses on consequences of intermittent reminding for time preferences.

The paper is also related to the vast literature in psychology on waiting, distractions, and time perception. A body of consistent evidence shows that the perception of duration is affected by attention. The father of American psychology William James already noted in 1890: "The tracts of time (...) shorten

[3] See Frederick, Loewenstein and O'Donoghue (2002) and Cohen et al. (2016) for surveys.

in passing whenever we are so fully occupied with their content as not to note the actual time itself. (...) On the contrary, a day full of waiting, of unsatisfied desire for change, will seem a small eternity". Closer to us, experimental evidence shows that the ratio of judged to real duration increases when attention is stimulated.[4] People who are paying attention to time itself, e.g. when they are waiting in a queue, or when they have been told in advance to estimate a period of time, feel the time passing more slowly. On the contrary, the ratio of judged to real time decreases when subjects are kept busy by a cognitively demanding task (Zakay and Block, 1997). If attention is distracted by non-temporal information, less capacity is available for processing temporal information (Kahneman, 1973). Katz, Larson and Larson (1991) find that distractions like watching a news board or television while waiting make the wait more acceptable for customers. The evidence is consistent with the model's assumption that consumers pay attention to time in waiting states. The process of waiting causes a lengthening of the perceived temporal distance, which deepens the discount on delayed utility.

Existing models of discounting have difficulties in explaining why waiting is more aversive when the reward is physically close, visible, or can be examined. In the famous "marshmallow experiment" by Mischel and Ebbesen (1970) and Mischel, Ebbesen and Raskoff Zeiss (1972), pre-school children were given the choice between one treat immediately or two if they waited for a short period. They found that children waited much longer for a preferred reward when they were distracted from it than when they attended to them directly. When the rewards were out of sight, 75% of children were able to wait the full time (15 minutes). When it was exposed, the mean delay time was only about 1 minute. Successful children developed strategies of diversion like singing songs or thinking

[4]See Fraisse, (1963) and Thomas and Brown (1974) for evidence. Hicks, Miller and Kinsbourne (1976) and Thomas and Weaver (1975) provide an attention-based theory of this phenomenon. Another interpretation is that people use a subjective internal timer which is slowed down when they are kept busy (Taatgen, Hedderik and Anderson, 2007).

aloud. Mischel, Ebbesen and Raskoff Zeiss (1972) conclude that "attentional and cognitive mechanisms which enhanced the salience of the rewards shortened the length of voluntary delay, while distractions from the rewards, overtly or cognitively, facilitated delay." Multiple follow-up studies have confirmed that keeping in mind the reward hinders, not facilitates, the ability to control one-self (Metcalfe and Mischel, 1999).

More recently, Hofmann et al. (2012) investigate with an experience sampling method how often desires in everyday life, like eating, sleeping or drinking, are felt and how often they are enacted or inhibited. They find that people who were the best at self-control reported fewer episodes of temptation rather than better ability to resist temptations. Ent et al. (2015) also show that self-control is linked to avoiding, rather than merely resisting temptation. Traditional theories of intertemporal choice have difficulties in accounting for those observations as pure time preferences are not distinguished from the frequency of temptations.

Relatedly, some researchers argue that decreasing impatience reflects non-linear perception of time. Ebert and Prelec (2007) report that making people pay more attention to the time dimension of the choice (e.g. by letting people focus on the arrival date of an item) has the effect of increasing discounting of the far future. Zauberman et al. (2009) find that making duration more salient to participants lead them to be more sensitive to time horizon, resulting in less similar preference between short and long time horizons.[5]

[5] See also Radu et al. (2011).

3 Wait based preferences

I begin by posing some general axioms before investigating a wait-based expected utility model of intertemporal consumption.

3.1 Axioms

A consumer decides at which date $t \in \{0, ..., T\}$ a good is consumed. Periods are reminding episodes if the DM pays attention to the consumption good. They are waiting episodes if reminded consumption is postponed. Reminding is exogenous. It occurs in period $s = 0, 1, ..., T$ with probability $p_s \in [0, 1]$. Waiting happens before, but not during or after consumption: $p_s = 0$, $s = t, ..., T$. Hence, assuming that the good is always consumed at one point, the terminal date cannot be a waiting period: $p_T = 0$. Preferences are defined over dated consumptions given a sequence of waiting $(x, t; p_0, p_1, ..., p_{t-1})$ where $x \in X = (0, \bar{x}]$ is the quantity consumed and t the consumption date. Strict preference relations $\succ$, inverse and indifference relations $\prec$ and $\sim$ are expressed at time 0 over dated consumptions.[6] They are complete, transitive, and satisfy three axioms:

Axiom 1 (*Monotonicity*) $\forall x, x' \in X$ *satisfying* $x' > x$, $(x', t; p_0, ..., p_{t-1}) \succ (x, t; p_0, ..., p_{t-1})$ $\forall p_s \in [0, 1]$, $\forall s < t$ *and* $\forall t = 0, 1, ..., T$.

Axiom 2 (*Waiting aversion*) $\forall x \in X$, $(x, t; p_0, ..., p_j, ..., p_{t-1}) \succ (x, t; p_0, ..., p'_j, ..., p_{t-1})$ $\forall p_s \in [0, 1]$ *satisfying* $p_j < p'_j$, $\forall s < t$ *and* $\forall t = 0, 1, ..., T$.

Axiom 3 (*Temporal indifference*) $\forall x \in X$, $(x, t; p_0, ..., p_{t-1}) \sim (x, t{+}1; p_0, ..., p_{t-1}, p_t)$ *if* $p_t = 0$, $\forall p_s \in [0, 1]$, $\forall s < t$ *and* $\forall t = 0, 1, ..., T$.

[6] The issue of time-consistency is sidestepped by focusing on time preferences from date 0 perspective, as if the DM could commit to them.

Axiom 1 ensures that the good is valuable to the DM for any sequence of probabilities. Axiom 2 states that the DM dislikes waiting. She prefers waiting to be less likely all else equal. Axiom 3 states that if date t is not possibly a reminding period ($p_t = 0$), the DM is indifferent between consuming at this period or next period. It formalizes the intuition that people may delay consumption effortlessly if they are distracted away. For instance the DM may be willing to postpone watching the last James Bond until evening if she expects no to remind the movie during the day.

Axioms 2 and 3 define a weak form of impatience (see Proof in Appendix):

Proposition 1 (*Weak impatience*) *Under Axioms 2 and 3,* $\forall x \in X$*,* $(x, t; p_0, ..., p_{t-1}) \succ (x, t+1; p_0, ..., p_{t-1}, p_t)$ *if* $p_t > 0$*,* $\forall p_s \in [0, 1]$*,* $\forall s < t$ *and* $\forall t = 0, 1, ..., T$*.*

As in classical models of time discounting, an impatient DM prefers consuming the earliest period. Impatience is weak in the sense that it only happens if the good is recalled to mind with a strictly positive probability. For example, the DM may be indifferent between eating a piece of chocolate at 10:00 or 11:00 A.M. as she expects to be absorbed by her work during this time. She however strictly prefers eating the chocolate at 01:00 P.M. instead of one hour later since she anticipates drinking her coffee, which will reminds her the pleasure of eating a piece of chocolate.

Axioms 1, 2 and 3 are model's core axioms. Additional axioms will be necessary for some additional results presented. First, the DM prefers experiencing waiting as late as possible:

Axiom 4 (*Preference for late waiting*) $\forall x \in X$, $\forall j = 0, 1, ..., t-2$, $(x, t; p_0, ..., p_j, p_{j+1}, ..., p_{t-1}) \succ (x, t; p_0, ..., p'_j, p'_{j+1}, ..., p_{t-1})$ $\forall p_s \in [0, 1]$ *satisfying* $p_j = p'_{j+1} < p_{j+1} = p'_j$, $\forall s < t$ *and* $\forall t = 0, 1, ..., T$*.*

The DM prefers to swap two temporally adjacent reminding probabilities if it results in delaying the higher probability. The assumption that people prefer to wait lately than early is supported by the common observation that people tend to postpone unpleasant feelings or tasks.[7] Second, her preference relative to the timing of waiting evolves smoothly with delay:

Axiom 5 (*Preference smoothness*) $\forall x \in X$, $\exists \beta > 0$ *such that* $(x, t; p_0, ..., p_j, p_{j+1}, ..., p_{t-1}) \sim (x, t; p_0, ..., p_j + \Delta, p_{j+1} - \frac{\Delta}{\beta}, ..., p_{t-1})$ $\forall \Delta \in (0, min(1 - p_j, \beta p_{j+1}))$, $\forall p_s \in [0, 1]$, $\forall s < t$, $\forall j = 0, 1, ..., t - 2$ *and* $\forall t = 1, 2, ..., T$.

An increase of the waiting probability at date j by the margin Δ leaves the DM indifferent if one period later, the waiting probability is decreased by the margin Δ/β, where β is a common coefficient for all dates. A value of β smaller than 1 denotes a preference for late waiting in the sense of Axiom 5.

3.2 Expected discounted utility

I consider a setting in which the DM maximizes a time additive expected discounted utility function which comprises two types of utility flows: a period utility $u(x)$ from consuming x, increasing and twice continuously differentiable, and a disutility from waiting. When the DM delays consumption from the present to date t, she may remind the reward with probability p_s every period $s = 0, 1, ..., t - 1$ before it is consumed. If the reward is recalled to mind at date s, she incurs the waiting costs $\delta(s, t)u(x)$. The disutility is proportional to deferred utility $u(x)$, assuming that the more pleasurable the outcome, the more unpleasant the waiting. Date 0 expected intertemporal utility is the sum

[7] The alternative assumption would be consistent with the DM experiencing craving for the good. Early waiting is preferred in this case as the feeling of deprivation is building up.

of expected waiting costs accumulated until $t-1$ and discounted utility:

$$-p_0\delta(0,t)u(x) - p_1\delta(1,t)u(x) - ... - p_{t-1}\delta(t-1,t)u(x) + \gamma(t)u(x) \quad (1)$$

where $p_s\delta(s,t)u(x)$ is expected disutility incurred at date s of delaying consumption until $t > s$. $\gamma(t)$ is the discount applied to utility when the good is eventually consumed. How future utility is discounted depends on the whole expected sequence of waiting.

3.3 Restrictions on temporal weights

Time preferences satisfy Axioms 1, 2 and 3. The consequences of Axiom 3 are first derived (see Proof in Appendix):

Proposition 2 *Under Axiom 3, temporal weights $\gamma(t)$ and $\delta(s,t)$ satisfy:*

1. $\gamma(t) = \gamma(t+1)$, $\forall t = 0, ..., T-1$,

2. $\delta(s, s+1) = \delta(s, s+2) = ... = \delta(s, T)$, $\forall s = 0, ..., T-1$.

The first part of the proposition means equal valuation of present and future utility. The second part states that waiting costs depend on the date s at which they are incurred in the present or the future, but not on the remaining delay until consumption. Both results stem from Axiom 3, which states that the waiting costs are the only reason why future utility is discounted. For instance, the two sequences $(x, T-1; p_0, ..., p_{T-1})$ and $(x, T; p_0, ..., p_{T-1})$ are identically valued by the DM if $p_{T-1} = 0$ according to Axiom 3. The second sequence does not entail additional waiting costs and is therefore equivalent to the first sequence despite varying delays between the present and the consumption date or between the waiting periods and the consumption date.

In accordance with Proposition 2 and from now on, all $\gamma(t)$, $t = 0, 1, ..., T$, are normalized to 1 and waiting costs simplified to $\delta(s,t) = \delta_s$ for consumption dates $t = 0, 1, ..., T$. Let us define expected utility as the product of utility and the discount factor: $D(t)u(x)$. $D(t)$ is the sum of weights attached to utility until date $t > 0$:

$$D(t) = 1 - p_0\delta_0 - p_1\delta_1 - p_2\delta_2 - ... - p_{t-1}\delta_{t-1} \tag{2}$$

and $D(0) = 1$. The constraints imposed on temporal weights by Axioms 1 and 2 are derived in Proposition 3 (see Proof in Appendix):

Proposition 3 *Under Axioms 1, 2, and 3, temporal weights δ_s, $s = 0, 1, ..., T - 1$, satisfy* $1 > 1 - \delta_0 > 1 - \delta_0 - \delta_1 > ... > 1 - \delta_0 - \delta_1 - ... - \delta_{T-1} > 0$.

Axiom 1 requires that consumption is valuable at every horizon, i.e. $D(t) > 0$ $\forall t = 0, 1, ..., T$, even in the less favorable environment in which the DM waits every period before consuming, that is when all probabilities are equal to 1. Proposition 3 implies that the longer the delay until consumption, the smaller the sum of temporal weights attached to utility: $D(0) \geq D(1) \geq D(2) \geq ... \geq D(T) \geq 0$, whatever the sequence of reminding probabilities $p_s \in [0, 1]$, $s = 1, ..., T - 1$.

The decrease of discount factors with delay is the classical definition of impatience. Here, since utility is not time discounted per se, impatience entirely rests on anticipated waiting costs. The further away consumption is delayed, the greater number of periods during which the DM may remind future consumption and the less expected utility. The decrease is non-linear however, as she may expect periods during which consumption is not recalled.

Axioms 4 and 5 impose additional restrictions on waiting costs (see Proof in Appendix):

Proposition 4 *Under Axioms 1, 2, 3, 4 and 5,* $\exists \beta \in (0,1)$ *such that period* t *waiting cost is* $\delta_t = \beta^t \delta_0$*, with* $\delta_0 < \dfrac{1-\beta}{1-\beta^T}$.

Future waiting costs are discounted by a factor β^t which takes an exponential form. Preference for late waiting (Axiom 4) implies $\beta < 1$. Condition $\delta_0 < \dfrac{1-\beta}{1-\beta^T}$ ensures that utility delayed arbitrarily far in the future, net of waiting costs, remains positive.

3.4 Asymptotic properties

Propositions 2 and 3 should be valid for an arbitrarily large number of periods, especially when the unit of time is short, like a day or an hour. Proposition 3 tells us that utility postponed to a finite date is non-negative for any sequence of probabilities. Let us define the asymptotic minimal utility $D_{min}u(x)$ as the infinitely postponed discounted utility with maximal waiting costs, that is when all probabilities are set to 1:

$$D_{min}u(x) = \lim_{T\to\infty} (1 - \delta_0 - \delta_1 - ... - \delta_{T-1})u(x) \tag{3}$$

Condition $D_{min}u(x) \geq 0$ extends Proposition 3 to the infinite horizon case. The condition implies that temporal weights δ_s become arbitrarily close to each other as the sequence progresses.[8] It will be convenient in the next sections to go a step further and normalize minimal utility to zero:

$$D_{min}u(x) = 0$$

so that the discount function may conventionally vary between 0 and 1. The convention is consistent with the requirement that infinitely delayed utility is useless

[8] Using the fact that any convergent sequence is a Cauchy sequence, for any given $\varepsilon > 0$, there exists a date T_0 such that for any pair of dates (s,t) satisfying $T_0 < s < t$, we have $|D(t) - D(s)| < \varepsilon$ or $\delta_s + \delta_{s+1} + ... + \delta_{t-1} < \varepsilon$.

whatever the finite reward x.[9] Together with axioms 4 and 5, the normalization implies the following restriction on the parameters:[10]

$$\delta_0 = 1 - \beta \tag{4}$$

The higher the immediate waiting costs δ_0, the heavier future waiting costs must be discounted so as discounted utility remains non-negative.

3.5 Reminding probabilities

When reminding probabilities are time varying, not only time relative to the evaluation period matters but also events occurring in calendar time. This makes difficult disentangling in decisions what comes from time preferences per se and time-varying probabilities. In the next sections, I will assume that all periods in which waiting is uncertain have a common probability of reminding.

Reminding will be certain in two types of period. First, the present is a special date as it is either a waiting period with certainty or a forgetful period. Examples of present reminding periods are decision or planning dates. Even if consumption is not yet available, choosing between alternative plans may act as a cue to consume and trigger waiting costs. The rest of the analysis will concentrate on environments in which the present is a decision or planning date and therefore a reminding period.

Second, in trade-offs involving two future dates, the first date of the trade-off stands out. Since the DM has to choose between consuming in this period or delaying consumption further, it is expected to be a reminding period. I

[9]The normalization is adopted by all common models of intertemporal choice, including the exponential, hyperbolic and quasi-hyperbolic models. With generalized hyperbolic preferences: $\lim_{t\to\infty}(1 + ht)^{-r/h}u(x) = 0$, with $h, r > 0$. With quasi-hyperbolic discounting: $\lim_{t\to\infty} \alpha\beta^t u(x) = 0$, given $0 < \alpha, \beta < 1$.

[10] Since $\lim_{t\to\infty} D(t) = 1 - \delta_0 - \beta\delta_0 - \beta^2\delta_0 - ... - \beta^{t-1}\delta_0 = 1 - \delta_0/(1 - \beta) = 0$.

assume that reminding is expected with certainty in those periods. The three assumptions are gathered in Assumption 1.

Assumption 1 $p_s = p \in [0,1]$, $\forall s = 0, 1, ..., T-1$, *except in the present if it is a reminding period:* $p_0 = 1$, *and in postponed trade-offs in which* $t > 0$ *is the first date of the trade-off:* $p_t = 1$.

Assumption 1 makes preferences time invariant i.e. immune to calendar effects. The ranking at time 0 of two dated payments does not change when both the evaluation period (date 0) and the consumption dates are postponed by a common delay (Fishburn and Rubistein, 1982; Halevy, 2015).

Having laid the foundations of the wait-based model, we now turn to its behavioral implications.

4 The two dimensions of time discounting

The wait-based model stresses the key role of two conceptually distinct factors behind discounting: the waiting costs and the frequency with which consumption is reminded. Axioms 1 to 5 and restriction parameter (4) lead to a much simpler discount function (see Proof in Appendix):

Proposition 5 *Under Axioms 1, 2, 3, 4, 5, Assumption 1, and restriction parameter (4), the discount function (2) becomes:*

$$D(t) = p\beta^t + (1-p)\beta \tag{5}$$

with $\beta \in (0,1)$ *the rate used to discount future waiting costs, and* p *the probability of reminding the good every future period.*

The parameter β is also an inverse measure of the waiting costs: $\beta = 1 - \delta_0$ according to restriction (4). The discount factor is a probability-weighted mean of two discount functions in which the reminding frequency p plays a key role.[11] The smaller p, the more patient the DM. Patience is maximal if the DM does not expect to remind the reward in the future ($p = 0$), implying $D(t) = \beta = 1 - \delta_0$.

To the opposite, impatience is maximal if the DM expects to remind the reward every period ($p = 1$). Discounting becomes exponential under this scenario: $D(t) = \beta^t$. Preferences inherit the normative features of the exponential model: constant impatience and present neutrality. Interestingly, the usual interpretation according to which the exponential model is normatively appealing is challenged when waiting costs are introduced. From a welfare perspective, the DM would like to minimize waiting costs by avoiding reminding future consumption, which an exponential discounter fails to do.

5 Impatience and salience

Intertemporal choices are affected by whether DM's attention is directed to the reward. It will be the case if the DM is physically proximate to the reward, watches someone else consuming it, contemplates it in a store, or is exposed to sensory cues related to it. A main finding of the "marshmallow experiment" discussed in the related literature, is that attending to the reward makes people more impatient. The degree of salience of the reward during the wait is a key factor modulating impatience (Mischel and Ebbesen, 1970). Children were more

[11] The duality may also be interpreted as reflecting the conflict of two selves or systems. One self is impatient and discounts exponentially. The second is more patient and equally discounts all future utilities. The higher the reminding probability, the greater the weight given to the impatient self. McClure et al. (2007) offer a similar interpretation for the quasi-hyperbolic model. See also Ainslie (1992) and Metcalfe and Mischel (1999).

impatient when they were directly exposed to the treat than when they saw an image of it (Mischel, Shoda and Rodriguez, 1989).

The effect of salience on impatience arises naturally in the wait-based model. Suppose the DM has to choose between x at $t = 0$ or $y > x$ at $t = 2$. The physical availability of the good in the present attracts DM's attention ($p_0 = 1$), and puts her in a decision state. Moreover, the DM's continued exposition to the reward at date 1 maintains her attention ($p_1 = 1$) and strengthens her impatience at date 0. To the contrary, if the reward is not physically or immediately available, or if its presence is obscured, she may recall the reward only with some probabilities p_0 and p_1. Future utility is more discounted if the DM directly attends to the reward and expects to think about it next period:

$$1 - \delta_0 - \delta_1 \leq 1 - \delta_0 - p_1\delta_1 \leq 1 - p_0\delta_0 - p_1\delta_1$$

It follows that the DM may be willing to endure the wait if it is transient ($p_0 = 1$ and $p_1 \leq 1$), but may immediately yield to temptation if it is expected to last. The result that impatience can be modulated by manipulating individuals' expectations regarding not only present but also future recalls, is a distinctive and testable implication of the model.

6 Decreasing impatience

Psychological discount rates tend to decline as people consider their preferences for longer time periods. Thaler (1981) found that to delay a \$15 lottery winning for 3 months, people required an extra \$15 (277% annual discount rate); but to delay the same amount for 1 year, they required only an extra \$45 (139% annual discount rate). In addition, as both early and late consumptions get closer to the present, people tend to assign progressively greater weight to early consumption relative to late consumption.[12]

Decreasing impatience, also called hyperbolic discounting, is supported by most experimental studies (e.g. Benzion, Rapoport and Yagil, 1989; Green, Myerson and Mcfadden, 1997; Kirby, 1997; Benhabib, Bisin and Schotter, 2010; and Bleichrodt, Gao and Rohde 2016).[13] Decreasing impatience has also been observed for substance abusers (Kirby, Petry and Bickel, 1999). Based on neuroimagining, Kable and Glimcher (2007) find that hyperbolic discount functions fit behavior better than the exponential discounting function.

In the wait-based model, impatience is decreasing if for any couple of dated consumptions (x, t) and $(x', t+1)$ such that the DM is indifferent between them, she prefers delaying consumption when the two dates are shifted forward by one period.

Definition 1 *(decreasing impatience)* $\forall x, x' \in X$ *such that* $(x, t-1; p_0, ..., p_{t-2}) \sim (x', t; p_0, ..., p_{t-1})$, *impatience is decreasing if* $(x, t; p_0, ..., p_{t-1}) \prec (x', t+1; p_0, ..., p_t)$ $\forall t \in \{1, ..., T-1\}$.

Beside temporal weights, the sequence of reminding probabilities is an important driver of the evolution of impatience. Recall that the DM is perfectly patient between dates $t-1$ and t if $p_{t-1} = 0$ (Axiom 3), and impatient between dates t and $t+1$ if $p_t > 0$. It follows for instance that the sequence of reminding probabilities $(1, 0, 1, 0, ..., 1)$ would be consistent with a DM being cyclically decreasingly and increasingly impatient. The very property of decreasing impa-

[12]A prominent model of decreasing impatience is Loewenstein and Prelec (1992) in which future utility is discounted by $(1 + ht)^{-r/h}$ with $h \geq 0$ and $r > 0$. Two special cases are proportional discounting (Mazur, 1987) when $h = r$ and power discounting (Harvey, 1986) when $h = 1$. Bleichrodt, Rohde and Wakker (2009) and Ebert and Prelec (2007) introduce discount functions which are the intertemporal analogues of constant absolute risk aversion and constant relative risk aversion utility.

[13] Attema et al. (2010) and Takeuchi (2011) find non-decreasing impatience.

tience can still be studied in environments in which the evolution of reminding probabilities is sufficiently smooth. This is done thanks to Assumption 1 in which all reminding probabilities are equal to p, except in the present and the first date of a trade-off during which reminding is certain. With exponentially discounting waiting costs ($\delta_s = \delta_0 \beta^s$) and parameter restriction (4), intermittent waiting exhibits decreasing impatience (see Proof in Appendix):

Proposition 6 *Under Axioms 1, 2, 3, 4, 5, Assumption 1 and parameter restriction (4), impatience is decreasing if* $p < 1$.

To understand the result, recall that the discounting function is exponential ($D(t) = \beta^t$) when the temporal weights are exponentially discounted (Axioms 4 and 5) and reminding repeats every period ($p = 1$). In this case, impatience is constant at all dates. If reminding is intermittent ($p < 1$), time intervals appear during which the DM does not incur waiting costs. Therefore the pace at which the good is discounted slows down compared to permanent waiting, which makes the consumer decreasingly impatient.

7 Present bias

Present bias is the propensity of overvaluing immediate rewards at the expense of futures ones. It has been shown to be relevant for saving or borrowing decisions (Meier and Sprenger, 2010), retirement timing (Diamond and Koszegi, 2003), addiction (Laibson, 2001, Bernheim et Rangel, 2004), health (Loewenstein et al. 2012), bargaining (Schweighofer-Kodritsch, 2018), or job search (DellaVigna and Paserman, 2005). It helps explain why individuals have self-control problems, procrastinate, or do not stick to the plans they have made earlier (O'Donoghue and Rabin, 2015, Bisin and Hyndman, 2014).

Experimental evidence in favor of decreasing impatience[14] and short-run impatience are both consistent with the assumption of a strong weight given to present utility.[15] The property of decreasing impatience has already been investigated. I focus in this section on the second manifestation of present bias, short-run impatience.

Impatience over short-delay intervals, a property commonly observed in experiments (Frederick, Loewenstein and O'Donoghue, 2002), is interpreted as a unequivocal consequence and a test of present bias (Rabin, 2002; Shapiro, 2005; O'Donoghue and Rabin, 2006, 2015). To see why, consider a present-neutral exponential discounter whose discount rate and discount factor over a short period of time (e.g. a day or a week) are $\rho \geq 0$ and $D(1) = (1+\rho)^{-1}$ respectively. Compounded over a full year, the psychological long-run rate is $R = (1+\rho)^{-t} - 1$, with t the number of unit periods in a year. The exponential relation leads small levels of short-term impatience translate into potentially extreme degrees of impatience. For instance, a tiny discount rate of $\rho = 0.1$ percent over one day leads to an already strong annualized discount rate of 44 percent. Such value seems incompatible with individuals engaging in profitable long-term investments like saving for their long term standard of living. We conclude that present-neutral exponential discounting cannot plausibly account for short-term impatience.

More reasonable levels of long-term impatience consistent with short-term impatience are obtained once a bias for the present is introduced. This is classically done with the two-parameter model of Laibson (1997) where future utility is discounted exponentially ($d(t) = (1+\rho)^{-t}$) and an extra weight $d(0) = 1/\alpha > 1$ applies to present utility.

[14] See e.g. Thaler (1981), Benzion, Rapoport and Yagil (1989), Green, Myerson and Mcfadden, (1997), Kable and Glimcher (2007), Benhabib, Bisin, and Schotter (2010) or Bleichrodt, Gao, and Rohde (2016).

[15] See Direr (2019) for a synthetic definition of present bias distinct from decreasing impatience.

The wait-based model of discounting is also consistent with present bias and proposes a behaviorally founded interpretation of the phenomenon. Under Axioms 1, 2, and 3, DM's short-term preferences are given by $D(1) = 1 - p_0\delta_0$, whereas the long-term discount function is:

$$D(t) = 1 - p_0\delta_0 - p_1\delta_1 ... - p_{t-1}\delta_{t-1}$$

Significant impatience over short-delays (as measured by $D(1)^{-1} - 1$) and moderate impatience over long delays (as measured by $D(t)^{-1} - 1$) can be jointly obtained if (i) the present is a reminding period ($p_0 = 1$) and (ii) subsequent episodes of reminding are infrequent, i.e. p_s are small for all $s > 0$. Condition (i) is consistent with Assumption 1 and can be motivated by the immediate availability of the reward. Condition (ii) is realistic given that most individuals spend only a small fraction of their time thinking about future consumption. Hence, long-term impatience may potentially remain bounded even though present waiting costs δ_0 are large.

The relation between short-run and long-run discounting can be investigated further by using the variant (5) of the model with exponentially discounted waiting costs and the parameter restriction (4) (see Proposition 5): $D(t) = p\beta^t + (1 - p)\beta$. If $p = 1$, reminding repeats over and over, with the consequence that present and future periods look alike. The symmetry leads to the exponential model and its inability to plausibly account for both short-run and long-run impatience. However the more infrequent reminding is expected to be, the closer the discount factor to the constant β and the more patient the DM in the long-run. The actual frequency of reminding is therefore a key factor determining to what extent short-run impatience translates into long-run impatience.

To get a quantitative assessment of the relationship between short- and long-run impatience, let us assume that the short delay is one day. With costly waiting, the implicit psychological short-term rate ρ is defined by $(1+\rho)^{-1} = D(1) = 1-\delta_0$. The implicit long-run rate R is defined by $(1+R)^{-1} = D(t) = p(1-\delta_0)^{365} + (1-p)(1-\delta_0)$. Table 1 shows long-run rates R for various values of short-run rates ρ and reminding probabilities p.

Table 1: Implicit long-run rate R (in percent) in function of the short-run rate ρ (in percent) and reminding probability p

	$p = 1$	0.5	0.3	0.1	0.05	0.01	0
$\rho = 0.1$	44	18.1	10.2	3.2	1.65	0.41	0.10
1	3,678	96.7	42.7	11.9	6.17	1.99	1
2	1.37×10^5	103.8	45.7	13.3	7.36	3.03	2
5	5.42×10^9	110.0	50.0	16.7	10.5	6.06	5
10	1.28×10^{17}	120.0	57.1	22.2	15.8	11.1	10

The lower the reminding probability, the closer the long-run rate to the short-run rate. The limit case $p = 1$ corresponds to the present-neutral exponential model in which long-run rates take implausibly high values. To the contrary, the intermittent wait-based model is able to account for both non-trivial short-term impatience and reasonable long-run impatience. Even for daily short-run rates as large as 10%, the long-run rate is only 5 percentage points higher when the DM reminds the future reward 5% of the time.

The wait-based model puts forth a fundamental reason why long-run rates do not reduce to a compound of short-run rates. The latter are generally elicited with subject's attention caught and oriented toward a concrete choice in which immediate consumption is feasible. It is therefore not surprising that short-run impatience is non trivial. Yet, extrapolating long-run rates by repeatedly compounding the obtained short-run rate is like presuming that the DM is placed in the same short-term decision situation over and over. Instead, it is more realistic to expect that she forgets the reward most of the time. This provides a strong intuition of why impatience over long-delay trade-offs is likely to remain in a reasonable range.

The wait-based model also predicts new patterns not predicted by the present bias model of Laibson (1997). In Laibson's model, the DM behaves like an exponential discounter in intertemporal trade-offs which do not involve an immediate consumption. Hence any departure from perfect patience in delayed trade-offs over short time intervals like a day or a week leads to implausible long-term impatience, as in the exponential model. For instance, assume the DM prefers x at date 1 to $y > x$ at date 2. With quasi-hyperbolic preferences: $\alpha(1+\rho)^{-1}u(x) > \alpha(1+\rho)^{-2}u(y)$, which is equivalent to preferring x now to y next period in the exponential model: $u(x) > (1+\rho)^{-1}u(y)$. We are back to the quantitative impossibility encountered by the model.

To the contrary, short-term impatience over postponed short delays is consistent with plausible long-term impatience in the wait-based model. x is preferred to y if $(1-p_0\delta_0)u(x) > (1-p_0\delta_0-p_1\delta_1)u(y)$ where the date 1 reminding probability p_1 is high since it is the first date of the trade-off (see discussion of Assumption 1). The long-term discount function $D(t) = 1 - p_0\delta_0 - p_1\delta_1 ... - p_{t-1}\delta_{t-1}$ may remain in a plausible range of values if other reminding probabilities are small.

8 Sub-additive discounting

Consider a DM who is indifferent between consuming x immediately or y in two periods: $(x, 0) \sim (y, 2)$. By transitivity of indifference, we can find a payoff z such that the DM is both indifferent between consuming x immediately and z in period 1, and consuming z in period 1 and y in period 2: $(x, 0) \sim (z, 1) \sim (y, 2)$. With a discounted utility formulation: $D(0)u(x) = D(1)u(z) = D(2)u(y)$. Alternatively, the two-period discount factor can be decomposed into a product of two one-period discount factors:

$$\frac{D(0)}{D(2)} = \frac{D(0)}{D(1)} \frac{D(1)}{D(2)} \tag{6}$$

As in financial computations, a long-period interest rate can be expressed as the compounding of sub-period interest rates. The relation holds for all usual time-separable discount functions $D(t)$. In a series of experiments, Read (2001), Read and Roelofsma (2003) and Scholten and Read (2006) find however that, on average, people tend to be more impatient when confronted with multiple short-delay trade-offs in a sub-divided interval than with a single trade-off over the whole interval:

$$\frac{D(0)}{D(2)} \leq \frac{D(0)}{D(1)} \frac{D(1)}{D(2)} \tag{7}$$

The pattern has also been reported in German representative samples (Dohmen et al., 2012; Dohmen et al. 2017). Dohmen et al. (2017) find, after controlling for several potential confoundings, that a large majority of respondents have preferences consistent with sub-additivity.

The mathematical equality (6) may not hold empirically if the discount factors $D(t)$, $t = 0, 1, 2$, which enter twice the equation are elicited from trade-offs with different alternative dates. Let us denote $d(t, t')$ the generalized discount rate applied to date t utility for a trade-off between t and t'. Condition (7) of sub-additivity is verified if:

$$\frac{d(0,2)}{d(2,0)} \leq \frac{d(0,1)}{d(1,0)} \frac{d(1,2)}{d(2,1)} \tag{8}$$

The property and its opposite, super-additivity, are defined the following way:

Definition 2 *(sub-additivity)* $\forall x, z, y, y' \in X$ *and* $\forall s, t,\ 0 < s < t$, *such that* $(x, 0) \sim (y, t)$, $(x, 0) \sim (z, s)$ *and* $(z, s) \sim (y', t)$. *Preferences are additive if* $y = y'$, *sub-additive if* $y < y'$ *and super-additive* $y > y'$.

Preferences are sub-additive if the latest payoff which makes the DM indifferent with immediate consumption is higher when the trade-off is broken down into two shorter trade-offs. It reflects more impatience over repeated short delays than over long horizons. Definition 2 of sub-additivity imposes a restriction on discount factors which generalizes example (8):

$$\frac{d(0,t)}{d(t,0)} \leq \frac{d(0,s)}{d(s,0)} \frac{d(s,t)}{d(t,s)} \tag{9}$$

The formulation makes discount rates a function of how far outcomes are removed from the present as in standard models of intertemporal choice (the first product of the discount function), but also of the alternative date (the second second).[16] Usual discounting models assume $d(t, s) = D(t)$ and cannot account for sub-additivity.

Sub-additivity has profound implications for the way people discount future utility flows. The convenient parallel often made between psychological and market rates falls short.[17] Familiar methods in financial planning like continuous short-term rates compounding or annualization of discount rates estimated over different horizons may not apply to subjective discount rates. In experiments, preferences over long delays cannot be inferred from preference elicited over shorter delays. The unit of time over which choices are made becomes important. Transitivity, a cornerstone of rational choices, may be violated. For

[16] The discount factor has been interpreted as a function of t and trade-off's interval $|t - s|$ (Read, 2001; or Scholten and Read, 2006). The "discounting by interval" formulation has the disadvantage of disregarding whether the alternative date is before or after date t.

[17] The parallel was made as early as Samuelson (1937, p.156) who remarked that the subjective discount rate "bears the (...) familiar relationship to the rate of discount."

instance, assume the DM prefers the early outcome (z, s) to the late one (y, t). If a third dominated choice $(x, 0)$ is introduced, she may reverse her choice if (z, s) is first compared to $(x, 0)$. The next subsection proposes a rationale for this class of discount factors based on waiting costs.

With intermittent waiting, the valuation of a quantity consumed in t periods may depend on the alternative consumption date if the expected sequence of waiting varies with the trade-off at hand. Condition (9) of sub-additivity expresses as:

$$\frac{1}{1-\delta_0-p\delta_1} \leq \frac{1}{1-\delta_0}\,\frac{1-\delta_0}{1-\delta_0-\delta_1}$$

which is true if $p \leq 1$. Consistent with Assumption 1, the present is a reminding period, but future periods may or may not be reminding episodes. In the first trade-off, the DM compares an immediate consumption and one in date 2. Nothing special happens in period 1 during which the DM might be distracted by other occupations, hence $p \leq 1$. To the contrary, in the second trade-off, the DM compares consumption at dates 1 and 2. The very possibility of consuming the good makes period 1 a reminding period.

Inequality $d(2, 0) \geq d(2, 1)$ implies in turn sub-additivity.[18] Wait-based preferences are sub-additive according to Definition 2 (see Proof in Appendix):

Proposition 7 *Under Axioms 1, 2, 3, and Assumption 1, preferences are sub-additive if* $p < 1$.

Compared to the date t discount $d(t, 0)$ in the trade-off between now and date t, the discount factor involved in the postponed trade-off $d(t, s)$ is smaller since the date s is a reminding period with certainty. It follows that the DM is more

[18] The interpretation of sub-additivity based on waiting costs is reminiscent of the intuition given by Read (2001): "The imagined pain of two days waiting, for instance, might be increased if the days are contemplated separately than together."

patient in the long-term trade-off than in a sequence of short-term trade-offs. The result only holds if other periods outside the trade-off are not reminding episode with certainty ($p < 1$). Otherwise, a long-term trade-off would have the same number of waiting episodes as a sequence of short-term trade-offs has.

9 Conclusion

A novel theory of time discounting is proposed in which a single psychological mechanism based on the disutility of waiting, accounts for important properties of time preferences like present bias, decreasing impatience, non-additive discounting or savoring. While other economic models of discounting treat time as a continuous flow, the model adopts a nonlinear approach, more familiar to psychologists, in which experienced time elapses only when attention to future gratifications is paid. As stressed by Stout (1932): "In general, temporal perception is bound up with the process of attention... What measures the lapse of time is the cumulative effect of the process of attending". The reasoning transposed to an intertemporal model of choice implies that future utility is progressively but not regularly discounted with the passing of time.

Researchers have documented sharp differences in intertemporal choice across individuals, groups and tasks (Frederick, Loewenstein and O'Donoghue, 2002), which are primarily interpreted as variability in discount rates. A significant source of heterogeneity may however come from the frequency with which individuals remind future gratifications. Smokers (Baker, Johnson and Bickel, 2003), alcoholics (Vuchinich and Simpson, 1998) or substance-dependent individuals (Kirby, Petry, and Bickel, 1999) show large discounting of delayed rewards but are also presumed to remind the addictive substance many times a day. The paper does not investigate why some people experience more temptation episodes than others. Addiction, weak habits, or genetic predispositions may be part of the story. Some people seem better able to avoid potential conflicts, e.g. by installing adaptive routines (Gillebaart and de Ridder, 2015). To this regard, it would be interesting to design experiments which would separately estimate pure discount rates and propensity of forgetting future rewards.

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Appendix Proofs of Propositions

Proposition 1. *Under Axioms 2 and 3,* $\forall x \in X$ *and* $\forall t = 0, 1, ..., T$, $(x, t; p_0, ..., p_{T-1}) \succ (x, t+1; p_0, ..., p_{T-1})$ *if* $p_t > 0$, $\forall p_s \in [0, 1]$, $s \neq t$.

Proof. $(x, t; p_0, ..., p'_t, ..., p_{T-1}) \sim (x, t+1; p_0, ..., p'_t, ..., p_{T-1})$ if $p'_t = 0$ (Axiom 3) and $(x, t+1; p_0, ..., p'_t, ..., p_{T-1}) \succ (x, t+1; p_0, ..., p_t, ..., p_{T-1})$ if $p_t > p'_t = 0$ (Axiom 2). □

Proposition 2. *Under Axiom 3, temporal weights in (1) satisfy: (i)* $\gamma(t+1) = \gamma(t)$ $\forall t = 0, ..., T-1$, *and (ii)* $\delta(s, s+1) = \delta(s, s+2) = ... = \delta(s, T)$, $\forall s = 0, ..., T-1$.

Proof. Consumptions x at date 0 or 1 are equivalent if $\gamma(0)u(x) = -p_0\delta(0,1)u(x) + \gamma(1)u(x)$ and $p_0 = 0$, hence $\gamma(0) = \gamma(1)$. Consuming x at date 1 or 2 are equivalent if

$$-p_0\delta(0,1)u(x) + \gamma(1)u(x) = -p_0\delta(0,2)u(x) - p_1\delta(1,2)u(x) + \gamma(2)u(x)$$

With $p_1 = 0$, $\gamma(2) - \gamma(1) - p_0\big(\delta(0,2) - \delta(0,1)\big) = 0$. The equality holds for any $p_0 \in [0,1]$, hence $\delta(0,1) = \delta(0,2)$ and $\gamma(1) = \gamma(2)$. Likewise, indifference between dates $t = 2, ..., T-1$ and $t+1$, with $p_t = 0$, implies:

$$\gamma(t+1) - \gamma(t) - p_0\big(\delta(0,t+1) - \delta(0,t)\big) - p_1\big(\delta(1,t+1) - \delta(1,t)\big)$$

$$- ... - p_{t-1}\big(\delta(t-1,t+1) - \delta(t-1,t)\big) = 0$$

The equality holds for any $\{p_0, p_1, ..., p_{t-1}\} \in [0,1]^t$. Hence, $\forall t = 1, ..., T-1$, $\gamma(t) = \gamma(t+1)$, and $\delta(s,t) = \delta(s,t+1)$, $\forall s = 0, ..., t-1$. Blocking the reminding date $s = 0, ..., T-1$ and varying the consumption date $t = 1, ..., T$ gives $\delta(s, s+1) = \delta(s, s+2) = ... = \delta(s, T)$. □

Proposition 3. *Under Axioms 1, 2, and 3, temporal weights* δ_s*,* $s = 0, 1, ..., T-1$*, satisfy* $1 > 1 - \delta_0 > 1 - \delta_0 - \delta_1 > ... > 1 - \delta_0 - \delta_1 - ... - \delta_{T-1} > 0$.

Proof. Under Axiom 1, $(x', t; p_0, ..., p_{t-1}) \succ (x, t; p_0, ..., p_{t-1})$ if $\forall t = 0, 1, ..., T$ and any vector of probabilities $(p_0, p_1, ..., p_{t-1}) \in [0, 1]^t$:

$$(1 - p_0\delta_0 - ... - p_{t-1}\delta_{t-1})(u(x') - u(x)) > 0$$

This is satisfied if all probabilities are set to 1:

$$1 - \delta_0 - \delta_1 - ... - \delta_{t-1} > 0 \tag{10}$$

Under Axiom 2, $(x, t; p_0, ..., p_j, ..., p_{T-1}) \succ (x, t; p_0, ..., p'_j, ..., p_{T-1})$ with $p_j < p'_j$ if $(1 - p_0\delta_0 - ... - p_j\delta_j - ... - p_{t-1}\delta_{t-1})u(x) > (1 - p_0\delta_0 - ... - p'_j\delta_j - ... - p_t\delta_t)u(x)$, or $p_j\delta_j < p'_j\delta_j$, which is true if $\delta_j > 0$. Together with inequality (10), they prove Proposition 3. □

Proposition 4. *Under Axioms 1, 2, 3, 4 and 5,* $\exists \beta \in (0, 1)$*, such that period t waiting cost is* $\delta_t = \beta^t\delta_0$*, with* $\delta_0 < \dfrac{1-\beta}{1-\beta^T}$.

Proof. Proposition 3 implies $D(t) = 1 - p_0\delta_0 - p\delta_1 - ... - p\delta_{t-1}$. Axiom 5 implies $p_j\delta_j + p_{j+1}\delta_{j+1} = (p_j + \Delta)\delta_j + (p_{j+1} - \dfrac{\Delta}{\beta})\delta_{j+1}$, $\forall j \in \{0, 1, ..., t-2\}$ and $\forall \Delta \in (0, min(1 - p_j, \beta p_{j+1}))$. Hence $\delta_{j+1} = \beta\delta_j$. The equivalence is valid $\forall j = 0, 1, ..., t-2$, hence $\delta_{j+1} = \beta^{j+1}\delta_0$. Axiom 4 implies $\beta \in (0, 1)$. Last, $D(t) > 0$ $\forall t \leq T$ and $\forall p_s$, $s = 0, 1, ..., t-1$ (Axiom 1). $D(t)$ is minimal for $t = T$ and all $p_s = 1$, hence $\delta_0 < \dfrac{1-\beta}{1-\beta^T}$. □

Proposition 5. *Under Axioms 1, 2, 3, 4, 5, Assumption 1, and restriction parameter (4), the discount function (2) becomes:* $D(t) = p\beta^t + (1-p)\beta$.

Proof. Applying Proposition 4 with all reminding probabilities equal to p, except $p_0 = 1$ (Assumption 1), the discount function (2) becomes:

$$D(t) = 1 - \delta_0 - p\beta\delta_0 - p\beta^2\delta_0 - ... - p\beta^{t-1}\delta_0$$
$$= 1 - (1-p)\delta_0 - p\delta_0(1 + \beta + \beta^2 + \beta^{t-1})$$
$$= 1 - (1-p)\delta_0 - p\delta_0\frac{1-\beta^t}{1-\beta}$$

With restriction (4) $\delta_0 = 1-\beta$: $D(t) = 1-(1-p)\delta_0 - p(1-\beta^t) = p\beta^t + (1-p)\beta$.

□

Proposition 6. *Under Axioms 1, 2, 3, 4, 5, Assumption 1, and restriction parameter (4), impatience is decreasing if* $p < 1$.

Proof. The DM is decreasingly impatient according to Definition 1 if, $\forall t = 0, 1, ..., T-2$:

$$\frac{D(t) - p_t\delta_t}{D(t)} < \frac{D(t) - p\delta_t - p_{t+1}\delta_{t+1}}{D(t) - p\delta_t}$$

with $D(t) = 1 - \delta_0 - p\delta_1 - ... - p\delta_{t-1}$ (Assumption 1). In definition 1 of decreasing impatience, Assumption 1 implies that all $p_s = p \in [0,1]$, except $p_0 = 1$ and $p_t = 1$ in the left-hand side trade-off between t and $t+1$, and $p_{t+1} = 1$ in the right-hand side trade-off between $t+1$ and $t+2$. The condition simplifies to

$$\frac{\delta_t - \delta_{t+1}}{\delta_t} > \frac{p\delta_t}{D(t)} \tag{11}$$

With $\delta_t = \beta^t\delta_0$ (Prop. 4), $D(t) = p\beta^t + (1-p)\beta$ (Prop. 5). Using parameter restriction (4), the condition simplifies to $(1-p)\beta > 0$. □

Proposition 7. *Under Axioms 1, 2, 3, and Assumption 1, preferences are subadditive if* $p < 1$.

Proof. The indifference conditions $(x, 0) \sim (y, t)$, $(x, 0) \sim (z, s)$ and $(z, s) \sim (y', t)$ express as:

$$u(x) = (1 - \delta_0 - p\delta_1 - ... - p\delta_s - ... - p\delta_{t-1})u(y)$$

$$u(x) = (1 - \delta_0 - p\delta_1 - ... - p\delta_{s-1})u(z)$$

$$(1 - \delta_0 - p\delta_1 - ... - p\delta_{s-1})u(z) = (1 - \delta_0 - p\delta_1 - ... - p\delta_{s-1} - \delta_s - p\delta_{s+1} - ... - p\delta_{t-1})u(y')$$

The equalities simplify to $(1 - \delta_0 - ... - p\delta_s - ... - p\delta_{t-1})u(y) = (1 - \delta_0 - ... - \delta_s - ... - p\delta_{t-1})u(y')$. Then $y' > y$ If $p < 1$. □