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STOCHASTIC REDUCED ORDER MODEL FOR REAL-TIME UNSTEADY FLOW ESTIMATION

Valentin Resseguier,
Matheus Ladvig, Agustin M Picard
Etienne Mémin, Reda Bouaida, Bertrand Chapron
1. Context
2. Physics + data = reduced order model (ROM)
3. Simulation + measurements = data assimilation
4. Results
PART I
CONTEXT
CEN «Simulation» (~70 people)

R&D and engineering

Expertise:
- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

Business:
- Scientific softwares
- Simulations, HPC
- VR & AR

Lab (~15 people)

Research, R&T, R&D

Expertise:
- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

Other Business Units
~ 2400 people
BLADE LIFT CONTROL

Desired blade lift → Controller

- Blade pitch
- Fluidic activators
- ...

Wind Turbine blade

Wind fluctuations → Damages

Variable blade lift
OBSERVER & CONTROL

Estimation and prediction:
- Flow
- Lift
- ...

Simple model

Controller
- Blade pitch
- Fluidic activators
- ...

Observer

Simple model

Wind turbine blade

Incomplete measurements:
- TrimControl
- LIDAR
- ...

Which simple model?
How to combine model & measurements?
PART II

PHYSICS + DATA

= REDUCED ORDER MODEL
Simulations with “physical” approximations

CFD (RANS, LES, …)

Semi-analytic formula

“Exact” physical equations

TRADEOFF ACCURACY / RAPIDITY

Rapidity

Accuracy & Robustness

Intrusive reduced order model (ROM)

Need data

Data-driven

Interpolation, Kriging

Machine / Deep Learning
REduced Order Model (ROM)

Solution of an PDE with the form:

\[ v(x, t, \alpha) \approx \sum_{i=0}^{n} b_i(t) \phi_i(x) \gamma_i(t) \]

<table>
<thead>
<tr>
<th></th>
<th>Full space</th>
<th>Reduced space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution coordinates</td>
<td>( v_q(x_i, t) ) ( q_i )</td>
<td>( (b_i(t))_i )</td>
</tr>
<tr>
<td>Dimension</td>
<td>( M \times d \sim 10^7 )</td>
<td>( n \sim 10 - 100 )</td>
</tr>
</tbody>
</table>
POD (PROPER ORTHOGONAL DECOMPOSITION)

- Principal Component Analysis (PCA) on a dataset to reduce the dimensionality:
  - Off-line simulations
  - Snapshots \( \left( v(x, t_i) \right) \)
  - PCA
  - Spatial modes \( \left( \phi_i(x) \right) \)

- Approximation:
  \[
v(x, t) \approx \sum_{i=0}^{n} b_i(t) \phi_i(x)
\]

- Projection of the “physics” onto the spatial modes:
  - \( \int_{\Omega} dx \, \phi_i(x) \cdot (\text{Physical equation} \text{ (e.g. Navier-Stokes)}) \)
  - ROM for very fast simulation of temporal modes
PART III

SIMULATION + MEASUREMENTS = DATA ASSIMILATION
COMBINING SIMULATIONS AND MEASUREMENTS

Numerical Simulation (ROM) → erroneous

Data assimilation (particle filtering)

On-line measurements
→ incomplete
→ possibly noisy

Need for uncertainty / errors quantification → Random dynamics

More accurate estimation globally in space

3 m/s⁻¹

5 m/s⁻¹
LOCATION UNCERTAINTY MODELS (LUM)

\[ v = \sum_{i=0}^{n} b_i \phi_i + \text{Residual} \]

- Randomized Navier-Stokes model
  - Good closure
  - Good model error quantification for data assimilation

References:
- LUM: Memin, 2014
  - Resseguier et al. 2017 a, b, c, d
  - Cai et al. 2017
  - Chapron et al. 2018
  - Yang & Memin 2019
- SALT: Holm, 2015
  - Holm and Tyranowski, 2016
  - Arnaudon et al. 2017
  - Crisan et al., 2017
  - Gay-Balmaz & Holm 2017
  - Cotter and al. 2018 a, b
  - Cotter and al. 2019

Resolved modes
- Assumed time-uncorrelated

SALT
- Crisan et al., 2017
- Gay-Balmaz & Holm 2017
- Cotter and al. 2018 a, b
- Cotter and al. 2019
**SUMMARY**

**Off-line : Building ROM**
- Physics (Navier-Stokes)
- Randomized Physics (LUM)
- DNS code
- Data
- Stochastic ROM

**On-line : Simulation & data assimilation**
- Measurements
- Data assimilation (particle filtering)
- Temporal modes $b_i$
- Flow $v = \sum_{i=0}^{n} b_i \phi_i$

Stochastic ROM
PART IV
RESULTS :
FAST OBSERVER OF THE FLOW
1ST RESULTS: WAKE AT RE 100

Reference
(DNS)
$10^4$ degrees of freedom

Our method
(Red-LUM-based data-assimilation)
6 degrees of freedom

Reduced order models with $n = 6$
and 2dB-SNR obs. assimilated every 5 sec

Theoretical bound
(Optimal from 6-d.o.f. linear decomposition)
6 degrees of freedom

Benchmark
(POD-ROM (with eddy viscosity) + init. by obs.)
6 degrees of freedom
1ST RESULTS: WAKE AT RE 300

Reduced order models with $n = 6$ and 2dB-SNR obs. assimilated every 5 sec

Reference (DNS) $10^7$ degrees of freedom

Our method (Red-LUM-based data-assimilation) 6 degrees of freedom

Theoretical bound (optimal from 6-d.o.f. linear decomposition) 6 degrees of freedom

Benchmark (POD-ROM (with eddy viscosity) + init. by obs.) 6 degrees of freedom
CONCLUSION
CONCLUSION

- Reduced order model (ROM) : for very fast and robust CFD (10⁷ → 6 degrees of freedom.)
  - Combine data & physics (built off-line)
  - Closure problem handled by LUM
- Data assimilation : to correct the fast simulation on-line by incomplete/noisy measurements
  - Model error quantification handled by LUM
- First results
  - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
  - Robust far outside the learning period

NEXT STEPS

- Real measurements (PIV, TrimControl, ...)
- Increasing the degrees of freedom (n)
- Increasing Reynolds (reduced DNS → reduced LES)
- Blade geometry
LUM: ADVECTION OF TRACER $\Theta$

$$\nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right) = 0$$

$$\nabla \cdot (\sigma \dot{B})$$

$$v = w + \sigma \dot{B}$$

Drift correction

Multiplicative random forcing

Balanced energy exchanges
GALERKIN PROJECTION GIVES SDES FOR RESOLVED MODES

\[ \int_{\Omega} \phi_i \cdot (\text{stochastic Navier-Stokes}) \]

\[ db_i = F_i(b)dt + \left( \alpha_i \cdot dB_t \right)^T b + \left( \theta_i \cdot dB_t \right) \]

multiplicative noise
additive noise

Correlations to estimate

2\text{nd} order polynomial: coefficients given by physics,

\[ \left( \phi_j \right)_j \quad a(x, x) = \frac{1}{t} < (\sigma(x)B)_{\text{obs}}, (\sigma(x)B)_{\text{obs}}^T >_t \]