Stochastic reduced order model for real-time unsteady flow estimation
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STOCHASTIC REDUCED ORDER MODEL FOR REAL-TIME UNSTEADY FLOW ESTIMATION

Valentin Resseguier,
Matheus Ladvig, Agustin M Picard
Etienne Mémin, Reda Bouaida, Bertrand Chapron
1. Context
2. Physics + data = reduced order model (ROM)
3. Simulation + measurements = data assimilation
4. Results
PART I
CONTEXT
Lab (~ 15 peoples)

Research, R&T, R&D

**Expertise:**
- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

CEN « Simulation » (~ 70 people)

**R&D and engineering**

**Expertise:**
- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

**Business:**
- Scientific softwares
- Simulations, HPC
- VR & AR

Other Business Units ~ 2400 people
BLADE LIFT CONTROL

- Desired blade lift
- Controller
- Wind Turbine blade
- Wind fluctuations
- Damages
- Blade pitch
- Fluidic activators
- ...
- Variable blade lift

Desired blade lift + Controller Wind Turbine blade Wind fluctuations Damages
Variable blade lift

\[ + \rightarrow \text{Controller} \rightarrow \text{Wind Turbine blade} \rightarrow \text{Wind fluctuations} \rightarrow \text{Damages} \rightarrow \text{Variable blade lift} \rightarrow \text{Controller} \rightarrow + \]
**OBSERVEUR & CONTROL**

Estimation and prediction:
- Flow
- Lift
- ...

Simple model

Controller
- Blade pitch
- Fluidic activators
- ...

Incomplete measurements:
- TrimControl
- LIDAR
- ...

Wind turbine blade
PART II

PHYSICS + DATA

= REDUCED ORDER MODEL
Simulations with “physical” approximations

**CFD** (RANS, LES, …)

**Semi-analytic** formula

**“Exact” physical equations**

Simulations with “physical” approximations

**Intrusive reduced order model (ROM)**

**Data-driven**

Interpolation, Kriging

Machine / Deep Learning

**TRADEOFF ACCURACY / RAPIDITY**

Accuracy & Robustness

Rapidity

Need data
**REDUCED ORDER MODEL (ROM)**

Solution of an PDE with the form:

\[ v(x, t, \alpha) \approx \sum_{i=0}^{n} b_i(t) \phi_i(x) \gamma_i(t) \]

<table>
<thead>
<tr>
<th>Full space</th>
<th>Reduced space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution coordinates</td>
<td>( v_q(x_i, t))q_i )</td>
</tr>
<tr>
<td>Dimension</td>
<td>( M \times d \sim 10^7 )</td>
</tr>
</tbody>
</table>
POD (PROPER ORTHOGONAL DECOMPOSITION)

- Principal Component Analysis (PCA) on a dataset to reduce the dimensionality:

![Diagram showing the process of POD](image)

- Approximation:

\[ v(x, t) \approx \sum_{i=0}^{n} b_i(t) \phi_i(x) \]

- Projection of the “physics” onto the spatial modes:

\[ \int_{\Omega} dx \phi_i(x) \cdot ( \text{Physical equation} (\text{e.g. Navier-Stokes})) \]

\[ \rightarrow \text{ROM for very fast simulation of temporal modes} \]
PART III

SIMULATION + MEASUREMENTS = DATA ASSIMILATION
COMBINING SIMULATIONS AND MEASUREMENTS

**Numerical Simulation (ROM)** → erroneous

**Data assimilation (particle filtering)**

**On-line measurements**
- incomplete
- possibly noisy

Need for uncertainty / errors quantification → Random dynamics

More accurate estimation globally in space

Velocity

3 m.s\(^{-1}\)  5 m.s\(^{-1}\)
LOCATED UNCERTAINTY MODELS (LUM)

\[ \nu = \sum_{i=0}^{n} b_i \phi_i + \text{Residual} \]

Assumed time-uncorrelated

Randomized Navier-Stokes model
- Good closure
- Good model error quantification for data assimilation

Randomized ROM

References:
- Memin, 2014
- Ressiguier et al. 2017 a, b, c, d
- Cai et al. 2017
- Chapron et al. 2018
- Yang & Memin 2019
- Holm, 2015
- Holm and Tyranowski, 2016
- Arnaudon et al. 2017
- Crisan et al., 2017
- Gay-Balmaz & Holm 2017
- Cotter and al. 2018 a, b
- Cotter and al. 2019

LUM
- Mikulevicius & Rozovskii, 2004
- Flandoli, 2011

SALT
- Cotter and al. 2017
- Resseguier et al. 2019 a, b
SUMMARY

Off-line: Building ROM

Physics (Navier-Stokes) → DNS code → Data → Randomized Physics (LUM) → Stochastic ROM

On-line: Simulation & data assimilation

Data assimilation (particle filtering) → Measurements → Temporal modes $b_i$ → Flow $v = \sum_{i=0}^{n} b_i \phi_i$
PART IV
RESULTS :
FAST OBSERVER OF THE FLOW
1ST RESULTS: WAKE AT RE 100

Reference (DNS)
$10^4$ degrees of freedom

Our method (Red-LUM-based data-assimilation)
6 degrees of freedom

Theoretical bound (Optimal from 6-d.o.f. linear decomposition)
6 degrees of freedom

Benchmark (POD-ROM (with eddy viscosity) + init. by obs.)
6 degrees of freedom

Reduced order models with $n = 6$ and 2dB-SNR obs. assimilated every 5 sec
**1ST RESULTS: WAKE AT RE 300**

Reduction order models with $n = 6$ and 2dB-SNR obs. assimilated every 5 sec

Reference (DNS) $10^7$ degrees of freedom

Our method (Red-LUM-based data-assimilation) 6 degrees of freedom

Theoretical bound (optimal from 6-d.o.f. linear decomposition) 6 degrees of freedom

Benchmark (POD-ROM (with eddy viscosity) + init. by obs.) 6 degrees of freedom
CONCLUSION
CONCLUSION

- Reduced order model (ROM): for very fast and robust CFD ($10^7 \rightarrow 6$ degrees of freedom.)
  - Combine data & physics (built off-line)
  - Closure problem handled by LUM
- Data assimilation: to correct the fast simulation on-line by incomplete/noisy measurements
  - Model error quantification handled by LUM
- First results
  - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
  - Robust far outside the learning period

NEXT STEPS

- Real measurements (PIV, TrimControl, ...)
- Increasing the degrees of freedom ($n$)
- Increasing Reynolds (reduced DNS $\rightarrow$ reduced LES)
- Blade geometry
LUM: ADVECTION OF TRACER $\Theta$

\[
\frac{\partial \Theta}{\partial t} + (w^* \cdot \nabla \Theta + \Theta \cdot \dot{B} \cdot \nabla \Theta) = 0
\]

\[
v = w + \sigma \dot{B}
\]

- Drift correction
- Multiplicative random forcing
- Balanced energy exchanges
GALERKIN PROJECTION GIVES SDES FOR RESOLVED MODES

\[
\int_{\Omega} \phi_i \cdot \text{(stochastic Navier-Stokes)}
\]

\[
\begin{align*}
\frac{db_i}{dt} &= F_i(b) + \langle \alpha, dB_t \rangle^T b + \langle \theta, dB_t \rangle \\
&= \begin{bmatrix} n \times M \end{bmatrix} \begin{bmatrix} M \times 1 \end{bmatrix} \begin{bmatrix} n \times 1 \end{bmatrix} \begin{bmatrix} 1 \times M \end{bmatrix} \begin{bmatrix} M \times 1 \end{bmatrix}
\end{align*}
\]

Correlations to estimate

2\textsuperscript{nd} order polynomial:
coefficients given by physics,

and \[
\begin{align*}
\langle \phi_j, \phi_i \rangle &= \frac{1}{t} < (\sigma(x)B)_{obs}, (\sigma(x)B)_{obs}^T > \\
\end{align*}
\]