Stochastic reduced order model for real-time unsteady flow estimation

Valentin Resseguier, Matheus Ladvig, Agustin M Picard, Etienne Mémin, Reda Bouaida, Bertrand Chapron

STOCHASTIC REDUCED ORDER MODEL FOR REAL-TIME UNSTEADY FLOW ESTIMATION

Valentin Resseguier,

Matheus Ladvig, Agustin M Picard

Etienne Mémin, Reda Bouaida, Bertrand Chapron

INVENTORS FOR THE DIGITAL WORLD

IRMAR

SCALIAN

irstea

Ifremer

CONTENT

1. Context
2. Physics + data = reduced order model (ROM)
3. Simulation + measurements = data assimilation
4. Results

PART I

CONTEXT

SCALIAN

Lab
(~ 15 peoples)

Research, R&T, R&D

Expertise:

- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

Other Business Units

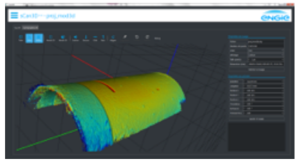
~ 2400 people

CEN « Simulation » (~ 70 people)

R&D and engineering

Expertise:

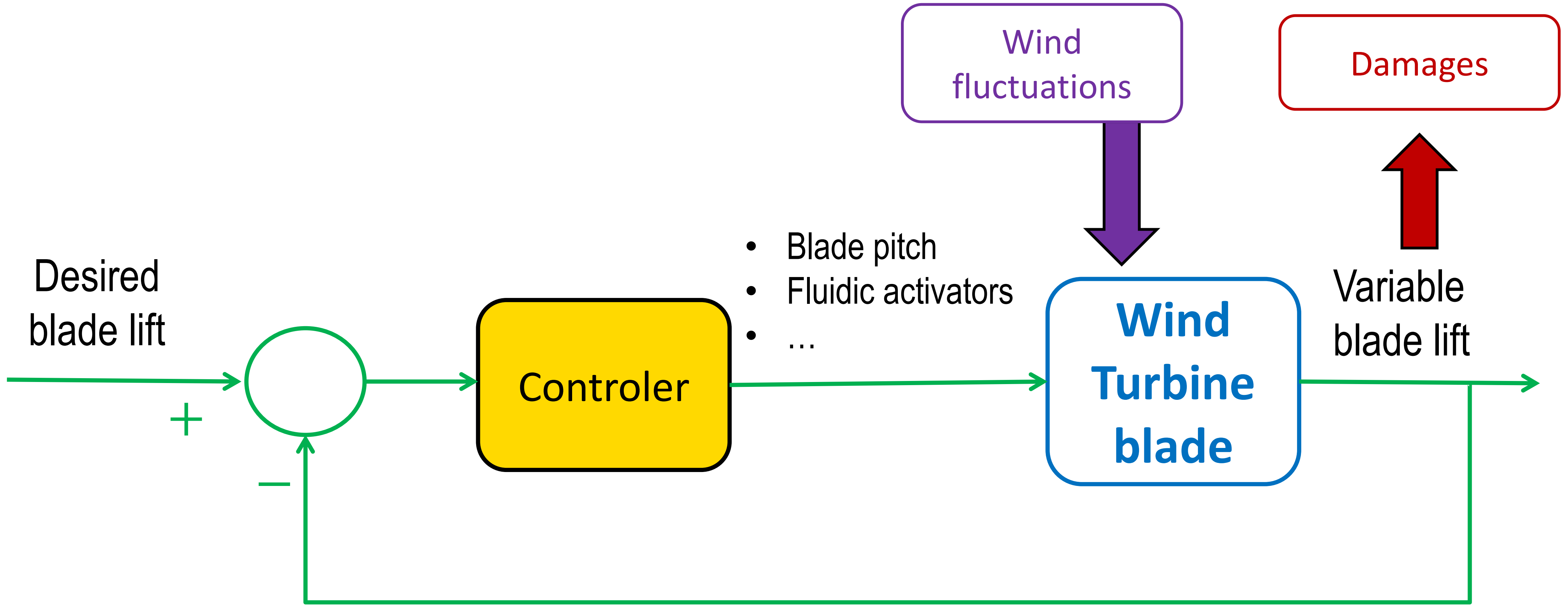
- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal



Business:

- Scientific softwares
- Simulations, HPC
- VR & AR

BLADE LIFT CONTROL



OBSERVEUR & CONTROL

- <u>Which simple model?</u>
- <u>How to combine model & measurements?</u>

Controler

Simple model

- Blade pitch
- Fluidic activators
- …

Estimation and prediction:

- Flow
- Lift
- …

Observer

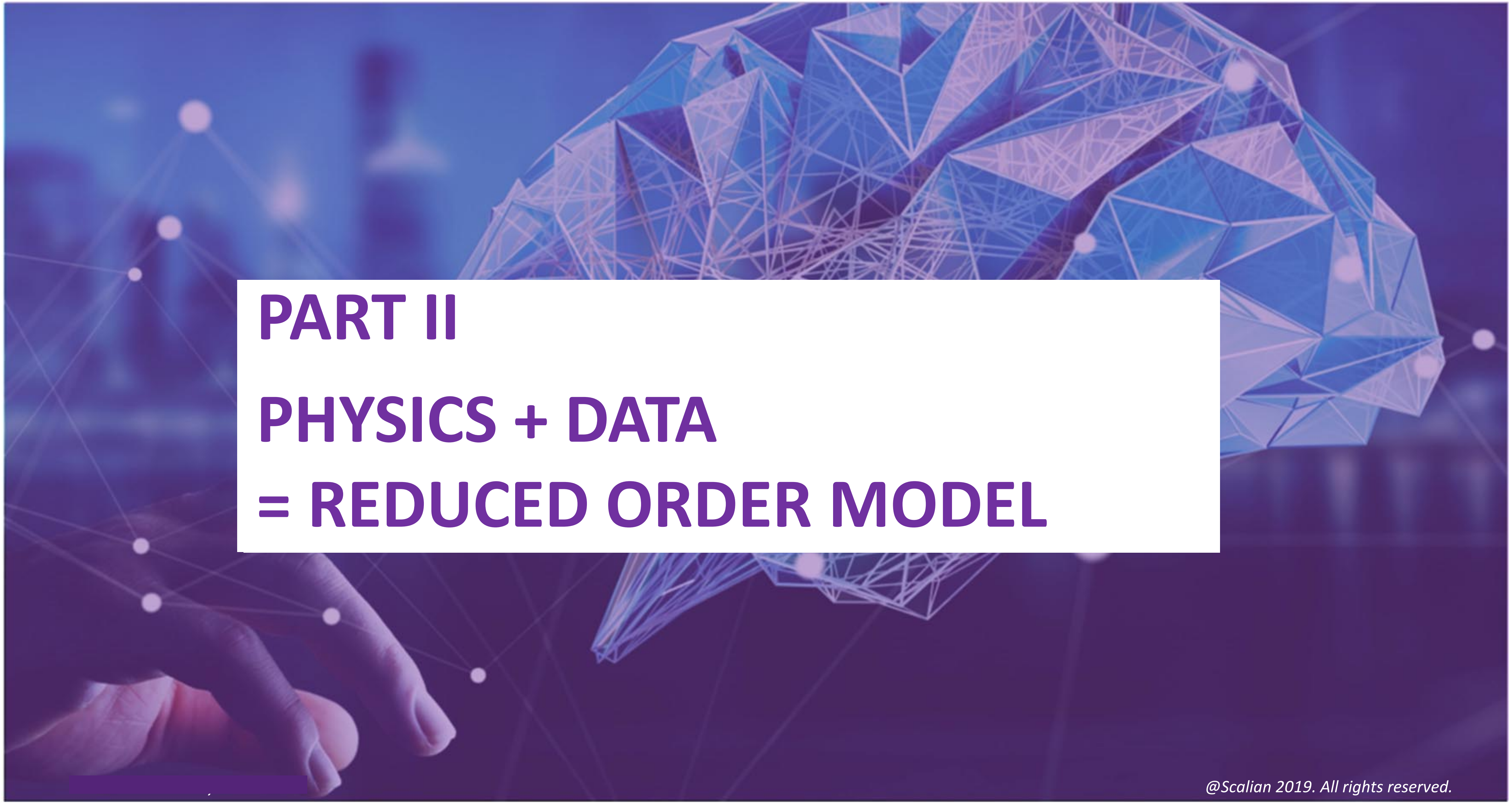
Simple model

Wind turbine blade

<u>Incomplete measurements :</u>

- TrimControl
- LIDAR
- …

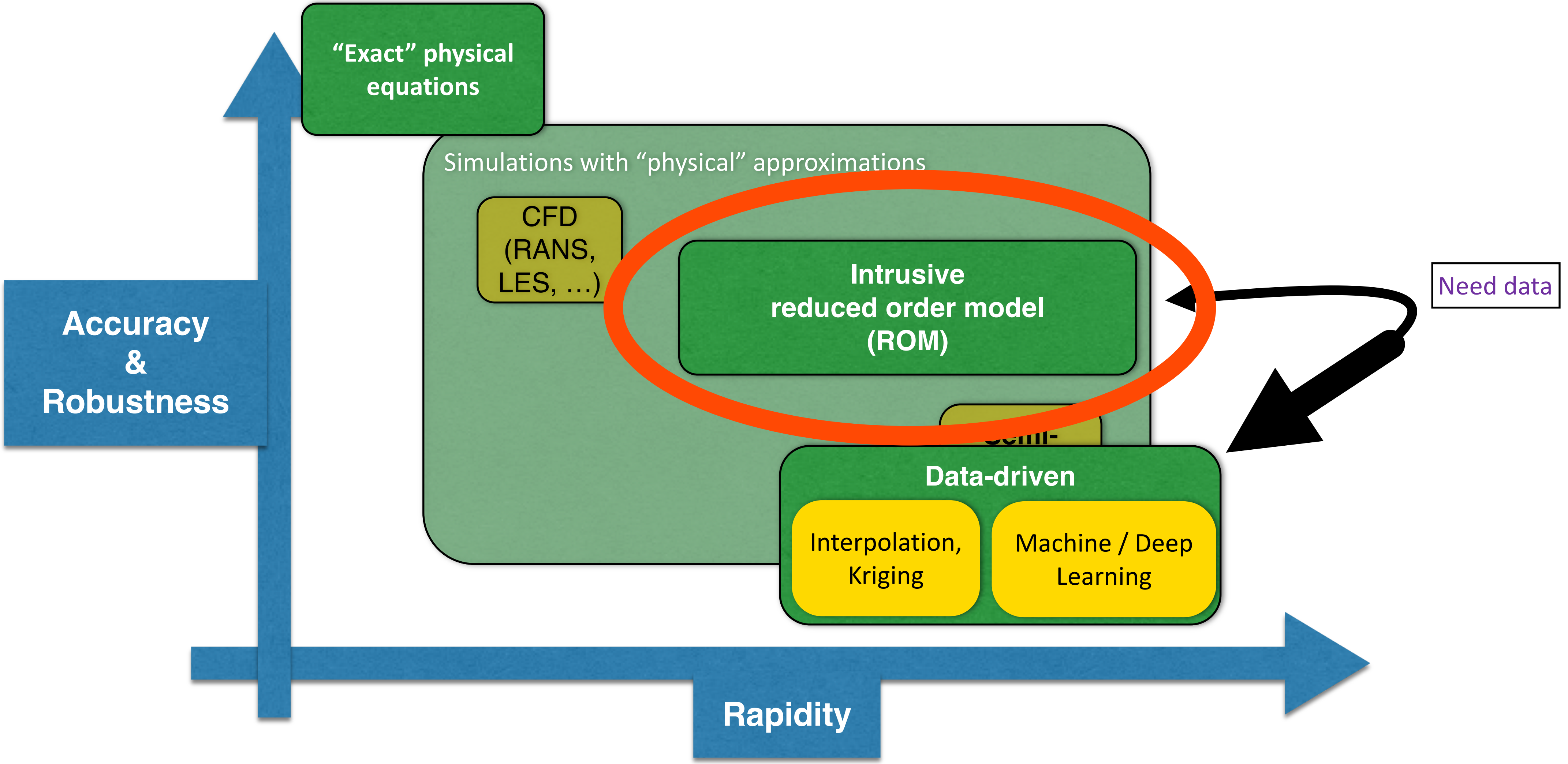
TrimControl



PART II

PHYSICS + DATA
= REDUCED ORDER MODEL

TRADEOFF ACCURACY / RAPIDITY



REDUCED ORDER MODEL (ROM)

<u>Solution of an PDE with the form:</u>

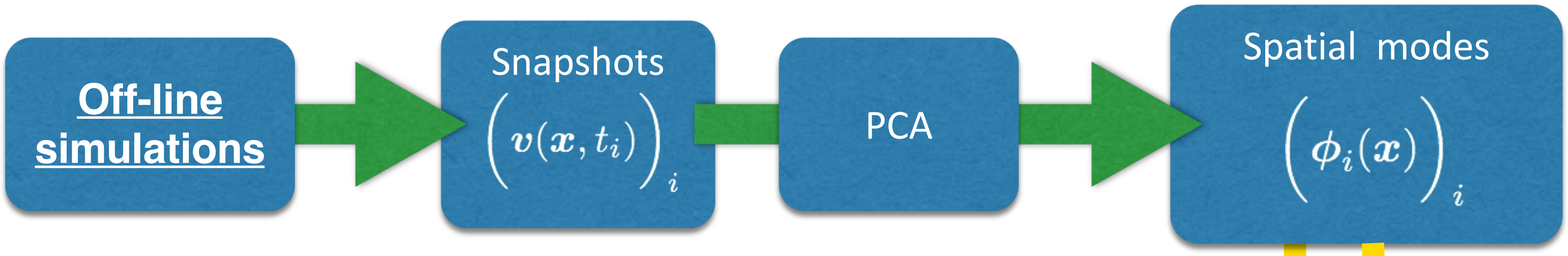
$$v(x,t,\alpha) \approx \sum_{i=o}^{n} \overset{\text{Time}}{\boxed{b_i(t)}}\,\overset{\text{Space}}{\boxed{\phi_i(x)}}\,\cancel{\overset{\text{Parameters (if any)}}{\boxed{\gamma_i(\alpha)}}}$$

<u>Order of magnitude examples in CFD</u>

	Full space	Reduced space
Solution coordinates	$v_q(x_i,t))_{qi}$	$(b_i(t))_i$
Dimension	$M \times d \sim 10^7$	$n \sim 10 - 100$

POD (PROPER ORTHOGONAL DECOMPOSITION)

- Principal Component Analysis (PCA) on a *dataset* to reduce the dimensionality:



- Approximation:

$$v(x,t) \approx \sum_{i=o}^{n} \underbrace{b_i(t)}_{\text{Resolved modes}} \phi_i(x)$$

- Projection of the "physics" onto the spatial modes :

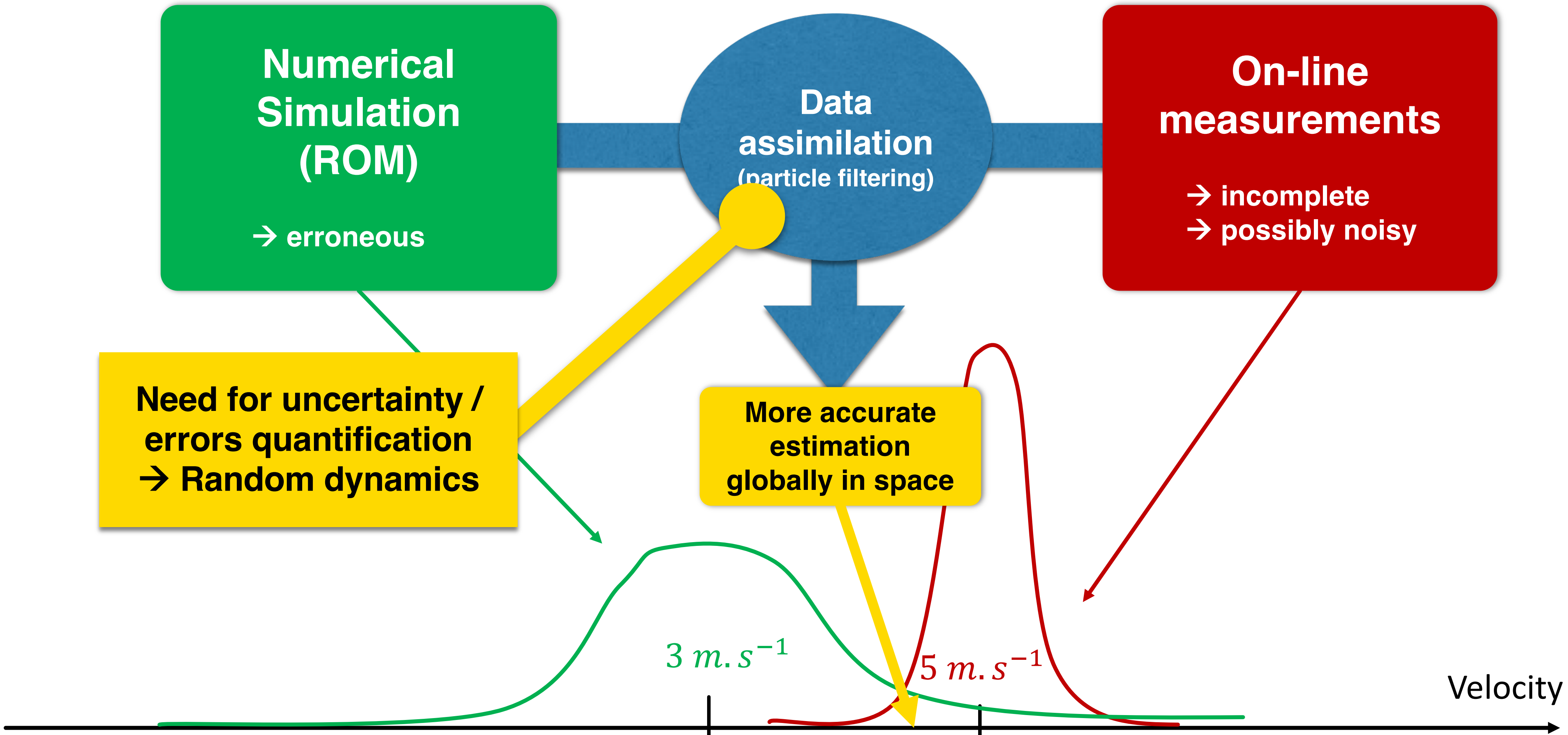
$\int_\Omega dx\ \phi_i(x) \cdot$ (*Physical equation* (e.g. Navier-Stokes))

→ **ROM for very fast simulation of temporal modes**

PART III

SIMULATION + MEASUREMENTS = DATA ASSIMILATION

COMBINING SIMULATIONS AND MEASUREMENTS



LOCATION UNCERTAINTY MODELS (LUM)

$$v = \underbrace{\sum_{i=0}^{n} b_i \phi_i}_{\text{Resolved modes}} + \text{Residual}$$

Resolved modes

Assumed time-uncorrelated

Residual

Randomized Navier-Stokes model

- Good closure
- Good model error quantification

for data assimilation

Randomized ROM

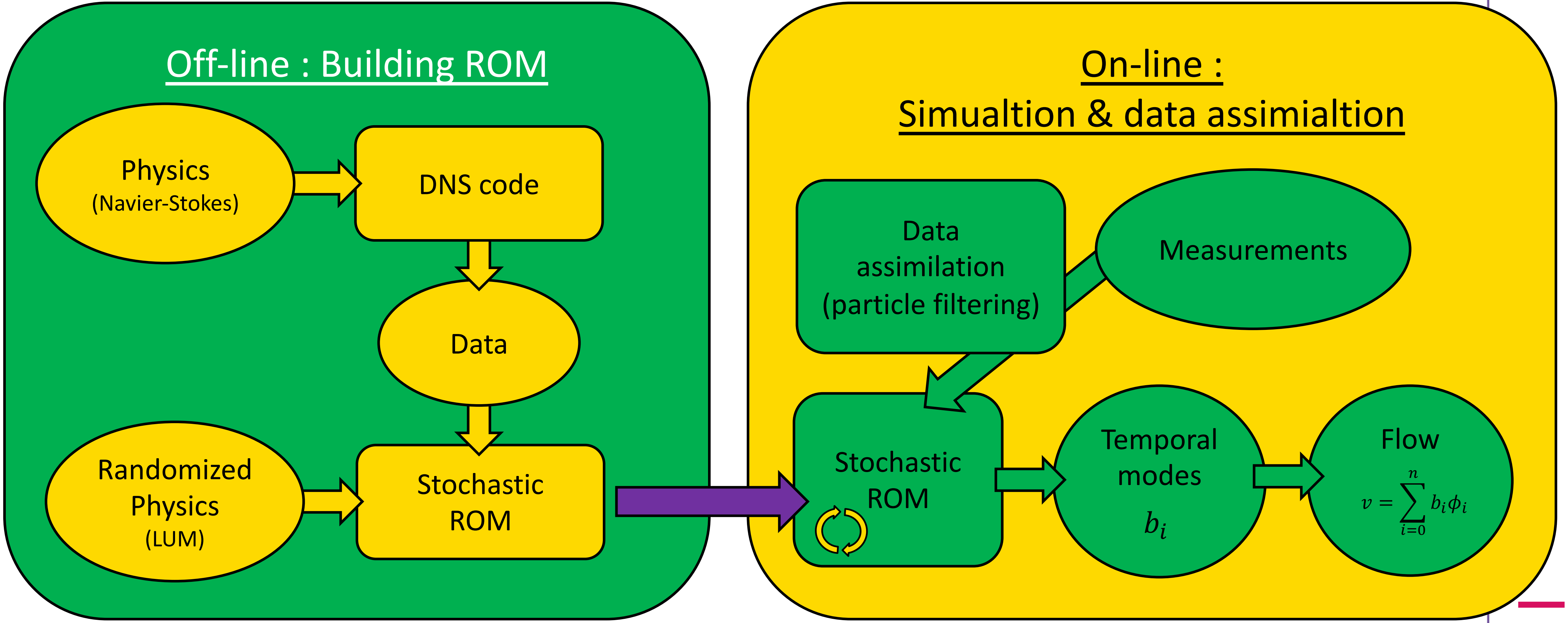
References :

Mikulevicius & Rozovskii, 2004
Flandoli, 2011

LUM	SALT	
Memin, 2014 Resseguier et al. 2017 a, b, c, d Cai et al. 2017 Chapron et al. 2018 Yang & Memin 2019	Holm, 2015 Holm and Tyranowski, 2016 Arnaudon et al. 2017	Crisan et al., 2017 Gay-Balmaz & Holm 2017 Cotter and al. 2018 a, b Cotter and al. 2019

Cotter and al. 2017 Resseguier et al. 2019 a, b

SUMMARY



Off-line : Building ROM
Physics
(Navier-Stokes)
DNS code
Data
Randomized
Physics
(LUM)
Stochastic
ROM
On-line :
Simualtion & data assimialtion
Data
assimilation
(particle filtering)
Measurements
Stochastic
ROM
Temporal
modes
b_i
Flow
$v = \sum_{i=0}^{n} b_i \phi_i$

PART IV

RESULTS :

FAST OBSERVER OF THE FLOW

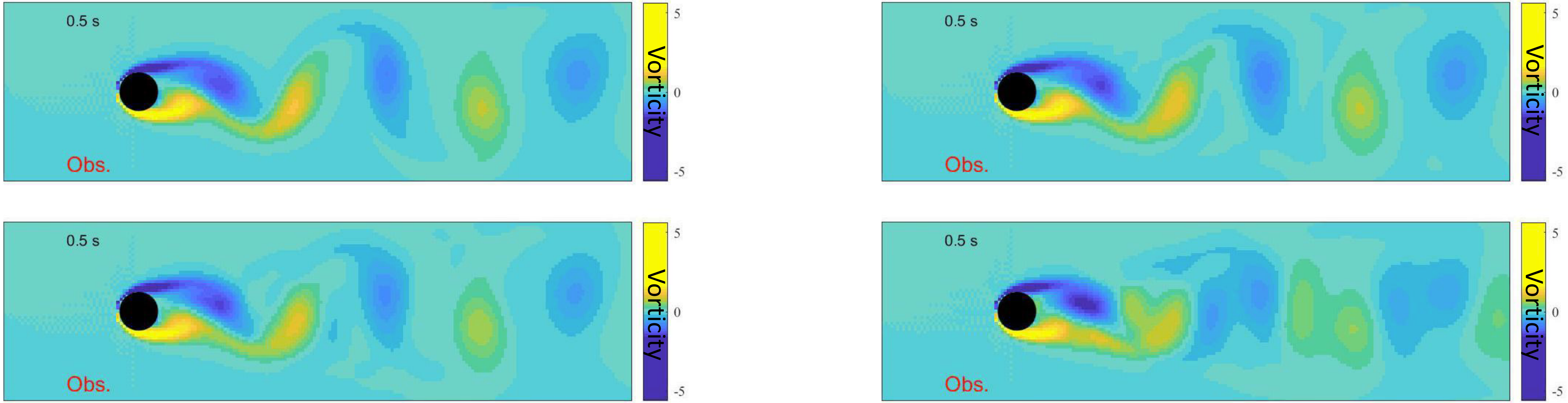
SCALIAN

1^{ST} RESULTS: WAKE AT RE 100

Reduced order models with $n = 6$ and 2dB-SNR obs. assimilated every 5 sec

Reference
(DNS)
10^4 degrees of freedom

Theoretical bound
(Optimal from 6-d.o.f. linear decomposition)
6 degrees of freedom



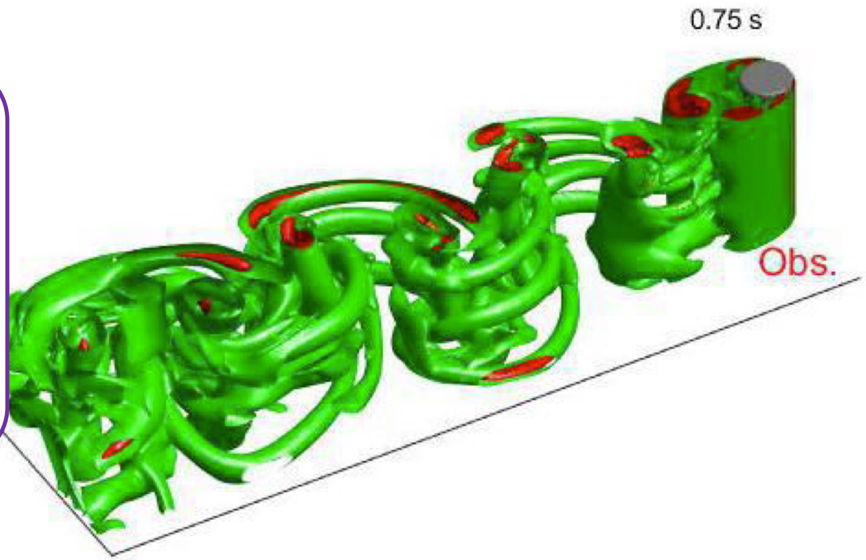
Our method
(Red-LUM-based data-assimilation)
6 degrees of freedom

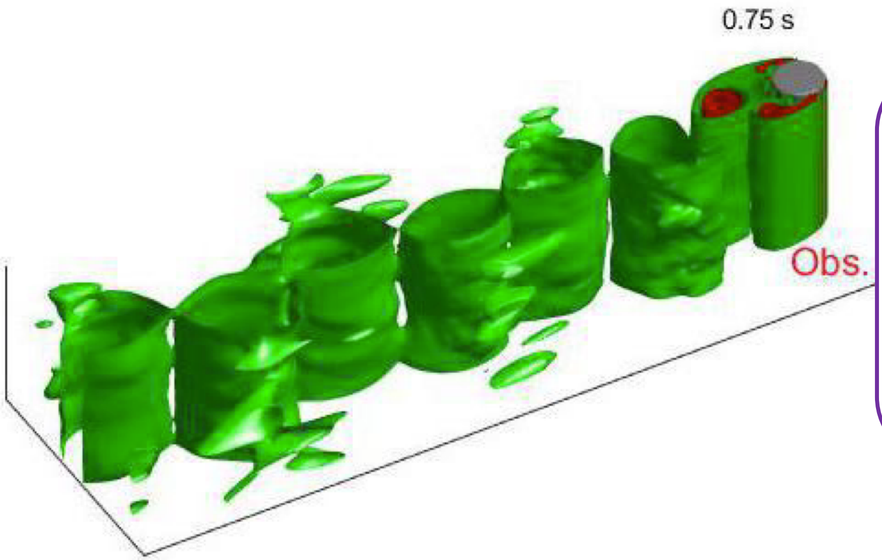
Benchmark
(POD-ROM (with eddy viscosity) + init. by obs.)
6 degrees of freedom

1^{ST} RESULTS: WAKE AT RE 300

Reduced order models with $n = 6$
and 2dB-SNR obs. assimilated every 5 sec

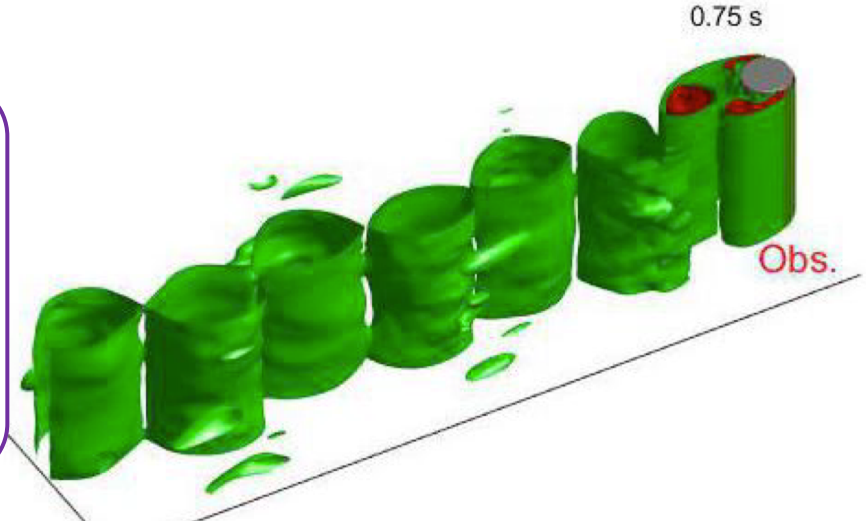
Reference (DNS)
10^7 degrees of freedom

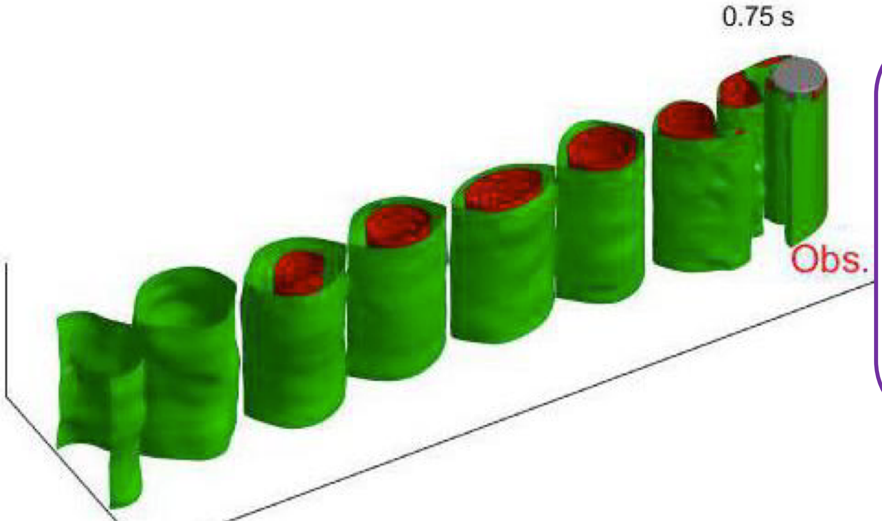




Theoretical bound (optimal from 6-d.o.f. linear decomposition)
6 degrees of freedom

Our method (Red-LUM-based data-assimilation)
6 degrees of freedom





Benchmark (POD-ROM (with eddy viscosity) + init. by obs.)
6 degrees of freedom

valentin.resseguier@scalian.com

CONCLUSION

- Reduced order model (ROM) : for very fast and robust CFD $(10^7 \rightarrow 6$ degrees of freedom.)
  - Combine data & physics (built off-line)
  - Closure problem handled by LUM
- Data assimilation : to correct the fast simulation on-line by incomplete/noisy measurements
  - Model error quantification handled by LUM
- First results
  - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
  - Robust far outside the learning period

NEXT STEPS

- Real measurements (PIV, TrimControl, …)
- Increasing the degrees of freedom (n)
- Increasing Reynolds (reduced DNS → reduced LES)
- Blade geometry

valentin.resseguier@scalian.com

valentin.resseguier@scalian.com

LUM: ADVECTION OF TRACER Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^{\star} \cdot \boldsymbol{\nabla} \Theta + \boldsymbol{\sigma} \dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla} \Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla} \Theta \right)$$

Drift correction

Multiplicative random forcing

Balanced energy exchanges

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma} \dot{\boldsymbol{B}}$$

GALERKIN PROJECTION GIVES SDES FOR RESOLVED MODES

$\int_{\Omega} \phi_i \cdot$ (stochastic Navier-Stokes)

n x M M x 1 n x 1 1 x M M x 1

$$db_i = \boxed{F_i(\boldsymbol{b})}\mathrm{d}t + \boxed{(\boldsymbol{\alpha}_{\bullet i \bullet} \mathrm{d}\boldsymbol{B}_t)^T \boldsymbol{b}} + \boxed{(\boldsymbol{\theta}_{i\bullet} \mathrm{d}\boldsymbol{B}_t)}$$

multiplicative noise additive noise

Correlations to estimate

2nd order polynomial:
coefficients given by physics,

and $\left(\phi_j\right)_j$ $\boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{1}{t} < (\boldsymbol{\sigma}(\boldsymbol{x})\boldsymbol{B})_{obs}, (\boldsymbol{\sigma}(\boldsymbol{x})\boldsymbol{B})^T_{obs} >_t$