Characterization of cracks of damaged concrete structures in dynamics

Rana Akiki, Cédric Giry, Fabrice Gatuingt

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R. Akiki & C. Giry & F. Gatuingt
LMT, ENS Cachan, CNRS, Université Paris-Saclay, 94235 Cachan, France

ABSTRACT: The analysis and prediction of the degradation process and cracking of concrete structures with numerical models is an important issue in the field of civil engineering. In order to describe the global behavior of a structure composed of quasi-brittle material as well as local fields, a continuous approach using nonlinear constitutive law (e.g. damage, plasticity, ...) remains the most efficient one regarding the computational time. However, one has to consider additional tools to extract discrete information about cracks like spacing and openings from these computations. The objective of this research is to propose tools capable of extracting local information such as cracking using two post-treatment methods of a global finite element analysis.
These methods are applied to a dynamic one-point bend test case and results show the capability of both methods to give a good estimation of crack path and openings.

1 INTRODUCTION

1.1 *Context*

This research work is part of the safety of packaging used to confine radioactive wastes. In order to assess the capacity of a package made of concrete to avoid the leakage of waste contained in it, one has to quantify its properties of permeability and more particularly the cracking in case of accident. Among the possible loading inducing damage and cracking in the context of waste containment, the falling of a package is a critical situation. The current methodologies for qualifying packages are based entirely on experimental analyzes carried out on structures at full scale. As a consequence, associating the experimental procedure with a step of simulation using a numerical tool is an interesting alternative.

1.2 *Modeling strategies of cracking in quasi-brittle media*

The modeling of the cracking in quasi-brittle materials can be tackled through different strategies. A first approach consists in a diffuse description of the cracking with a continuous description of the displacement field. Then the progressive degradation is described through a field of internal variable like damage. Another approach consists in an explicit description of the cracking with the use of a discontinuous displacement field through a kinematic enrichment. This enrichment can be introduced at the finite element scale like E-FEM, for instance, or as additional degrees of freedom like X-FEM, for instance. Despite the advantage of getting a direct information on the crack opening and crack location with the second approach, the continuous description of degradation remains the most efficient technic to deal with the response of a full scale structure.

In order to quantify cracking (location, opening) from a continuous field, an additional post-treatment needs to be introduced. In this work, two post-treatment methods have been considered.

The first method combines a topological search method used to locate cracks developed by (Bottoni et al. 2015) and a continuous/discontinuous approach used to compute the crack opening initially proposed by (Dufour et al. 2008). It consists in comparing the computed strain fields, with the analytical one derived from the displacement profile described as a strong discontinuity. Both strain fields are regularized using a Gaussian function. The crack opening can then be adjusted so as to reduce the gap between the regularized strain field and the regularized strong discontinuity strain field. Furthermore, a general formulation of these tools for 2D and 3D problem is proposed in this work. The direction of the mode I crack at a point is determined as the one which maximizes the crack opening along the associated 1D profile.

The second method is a non-intrusive re-analysis at the local scale performed with a discrete model in order to extract fine information about crack opening. A region of interest (ROI) corresponding to the damaged area obtained from the global analysis is defined. Then, the loading steps corresponding to the steps of re-analysis are determined, where boundary conditions are extracted from the continuous displacement field and applied on the non-free surfaces of the ROI. The material is described with a discrete element approach based on an assembly of polyhedral particles linked by Euler-Bernoulli beams with brittle behavior. Further details concerning the discrete model used at the local scale can be found in (Oliver-Leblond et al. 2013).

2 EXPERIMENTAL CAMPAIGN

In order to understand the mechanisms of deformation and cracking of fiber concrete (the quasi-brittle material considered in this study) under mechanical stresses close to those encountered by the packages during the impact and in order to identify the most appropriate behavior law to use in the simulations as well as its parameters, an experimental campaign has been performed. The main results obtained are summed up in this part.

2.1 *Quasi-static loading*

Three different types of test have been performed under quasi-static conditions : compression test, splitting test and 3-point bending test. Two types of fibers were considered for the industrial context : FRC1 (amorphous cast iron flat fiber) and FRC2 (steel fiber). The mean mechanical properties obtained are given for each concrete in Tab. 1.

Table 1: Mechanical properties of the fiber reinforced concretes

Parameter	Unit	FRC1	FRC2
Tensile strength (splitting test)	MPa	6.9	7.4
Tensile strength (flexural test)	MPa	11.3	12.9
Compressive strength	MPa	75	87.4
Fracture energy	N/m	900	9000
Young modulus	GPa	42	40

A noticeable gain for the fracture energy is observed with the steel fibers. Indeed, the hooks of these fibers highly improve the ductile behavior of the concrete. With the flat fibers, only the yield strength is increased compared to standard concrete but only a small improvement of the ductile behavior is observed.

2.2 *Dynamic loading*

In order to quantify the sensitivity of our fiber reinforced concrete to the strain rate, several dynamic loadings have been performed thanks to a split Hopkinson bar device. Compressive and splitting loading have been applied in order to identify the compressive and tensile behavior of the material and a one-point bending test has been developed (Akiki, Gatuingt, Giry, Schmitt, & Stéfan 2016) to identify the flexural behavior and the dynamic fracture energy. Classical post-treatment thanks to strain gauges placed along the bars has been considered as well as Digital Image Correlation (DIC) to quantify cracking at the surface of the specimen tested. These last results are used later for comparison with the results obtained numerically.

Fig. 1 shows the classical evolution of the forces at the input and output bar/specimen interfaces during the compressive test.

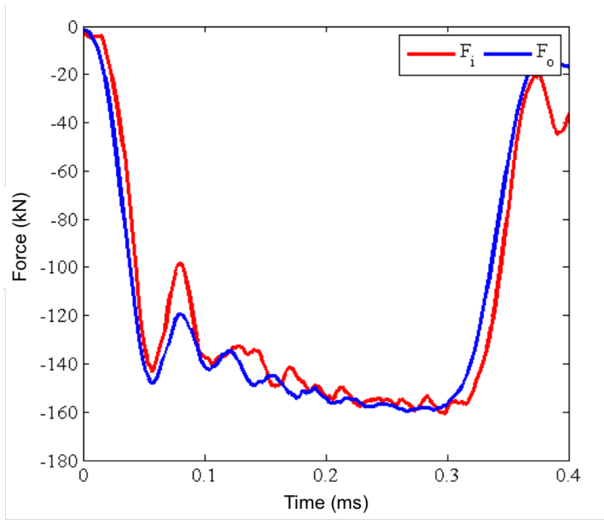

Figure 1: Evolution of the forces at the input and output bar/specimen interfaces during the compressive test

One can see for the compressive test that the equilibrium is reached in the sample as the evolution of the force obtained at the input and the output faces of the specimen are superposed.

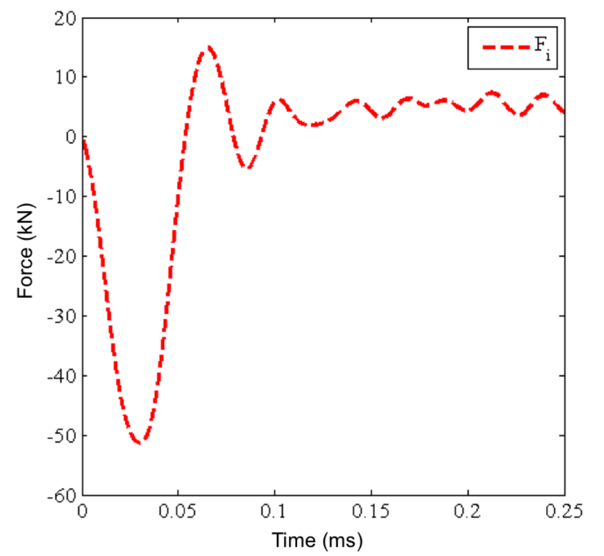

Figure 2: Experimental evolution of the force at the input interface bar/specimen during the one-point bending test.

2.3 *Identification of the parameters of the continuous model*

In order to identify the parameters of the material model driving the linear and nonlinear behavior, an inverse analysis including the global response measured experimentally is considered. A minimization process of an error fonction comparing the experimental results and the one obtained from the FE modeling of the test is used. For this identification, the MATLAB® function *lsqnonlin* (i.e. nonlinear least-square solver) is considered. The error function r is defined according to Eq. 1 and includes the response of the compressive test under quasi-static loading $f^{i,c}$, the tensile strength y^t and the evolution of the force at the input interface bar/specimen during the one-point bending test $F^{i,f}$ (see Fig. 2). The indice $._i$ stands for the time step i.

$$r=\sqrt{\sum_{i=1}^{n}\left(\frac{f_s^{i,c}(x)-f_e^{i,c}}{\sum_{i=1}^{n} f_e^{i,c}}\right)^2+\left(\frac{y_s^t(x)-y_e^t}{y_e^t}\right)^2+\sum_{i=1}^{n}\left(\frac{F_s^{i,f}(x)-F_e^{i,f}}{\sum_{i=1}^{n} F_e^{i,f}}\right)^2}, \quad (1)$$

Table 2: Characteristics of the mesh for the one-point bending test

Entity	Number of nodes	Number of elements
Impactor	4329	3680
Incoming bar	22126	20700
Specimen	115816	107100

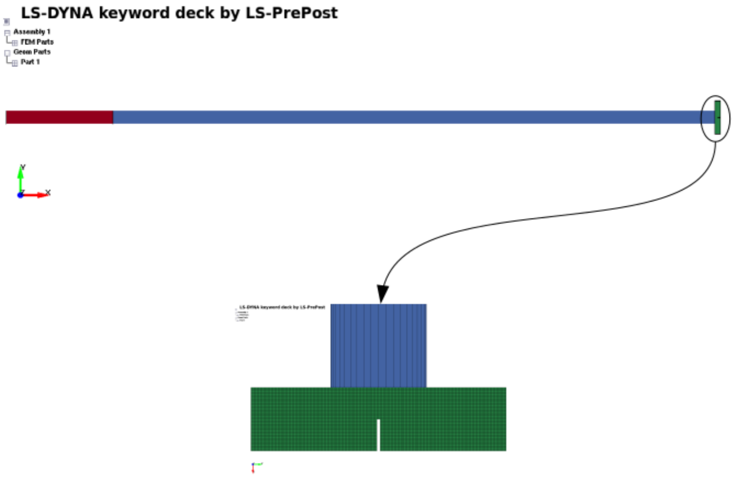

Figure 3: FE mesh of the one-point bending test: global view and zoom over the contact between the bar and the specimen

3 CONTINUOUS DESCRIPTION OF THE DEGRADATION

The modeling of the degradation of the structure is managed thanks to a continuous approach. The material's behavior used in our FE simulations is described by an elasto-plastic constitutive law including damage and rate effect: model MAT-159 in LS-DYNA (Murray 2007).

The parameters of the model have been identified according to the method exposed in the previous section. In order to test the method exposed in the next section regarding the cracking quantification, a finite element model of the full Hopkinson bar device for 1-point bending test has been used. The main characteristics

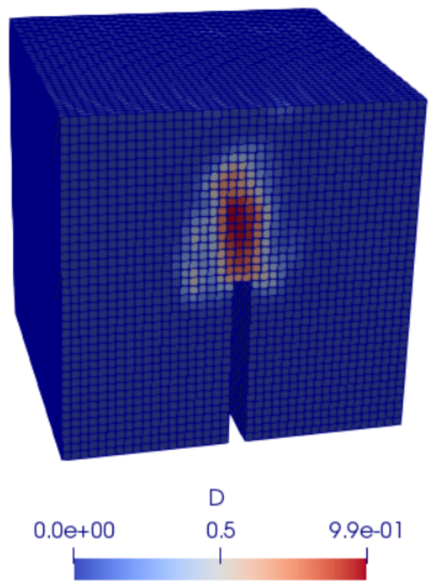

Figure 4: Damage field obtained during the failure of the specimen

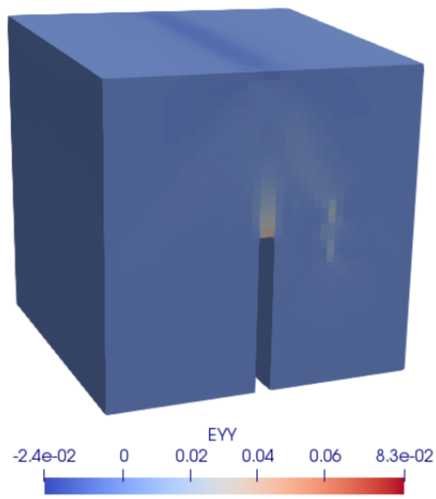

Figure 5: Strain component ε_{yy} field obtained during the failure of the specimen

of the mesh are given in Tab. 2 and Fig. 3 gives the an overview of the modeling.

Fig. 4 and 5 show the damage field D and the strain component ε_{yy} during the failure process of the beam. One can see classical fracture process zone propagating from the notch up to the side of the beam in contact with the impactor. These fields allow to characterize the global degradation of the specimen and provide diffuse information at the local scale.

4 EXTRACTION OF CRACKING FROM CONTINUOUS COMPUTATION

The continuous description of degradation based on damage and/or plasticity model allow to describe the global behavior but does not provide information on crack. As a consequence, additional tools need to be introduced in order to get discontinuous information like crack location and opening. In this work, two approaches have been considered to post-treat the continuous fields.

4.1 *Topological search and continuous-discontinuous approach for crack opening*

Let's consider an isotropic material with a nonlinear behavior which state is characterized by a strain tensor $\boldsymbol{\varepsilon}$ and a corresponding stress tensor $\boldsymbol{\sigma}$. By considering that the linear part of the material keeps the same properties as the ones of the virgin material, one can express an anelastic strain $\boldsymbol{\varepsilon}^{an}$according to Eq. 2.

$$\boldsymbol{\varepsilon}^{an} = \boldsymbol{\varepsilon} - \mathbf{C}^{-1} : \boldsymbol{\sigma}, \tag{2}$$

with **C** the elasticity tensor.

In the analysis of quasi-brittle materials, by considering that the nonlinearities fully participate to the cracking process, one can associate the anelastic strain tensor to the cracking. As a consequence, the crack location and crack opening are obtained from the post-treatment of this field.

This first method proposed is dedicated to the quantification of mode I crack opening. The main idea behind locating the direction of the crack and evaluating its opening is to consider that the crack opens in mode I in the direction that maximizes the crack opening. In order to quantify the crack opening at a point in a given direction from a continuous field, the method proposed by (Dufour et al. 2008) is used. The main equations are recalled hereafter.

For a given direction **n**, the projection of the strain tensor obtained from the finite element computation ε_N^{FE} is defined by:

$$\varepsilon_N^{FE} = \mathbf{n}^T.\varepsilon.\mathbf{n} \tag{3}$$

One can get a scalar field $\varepsilon_N^{FE}(s)$ over a 1D profil parameterized by the direction **n** and a distance to the center s. (Dufour et al. 2008) propose to quantify the crack opening from this scalar field by comparing it with the 1D strain profil corresponding to a strong discontinuity $\varepsilon_N^{SD}(s)$. As this last field is a Dirac like function, a direct comparison is not possible. As a consequence, the authors propose to compare the convolution product of both profils with a filter function $\phi(x_0 - s)$ (x_0 defining the crack location on the 1D profil). By considering that the convoluted strain field are equal at the crack location (i.e. location of the maximum of $\phi \star \varepsilon_N^{FE}(x_0)$), the crack opening $\|u\|$ writes:

$$\|u\|(x_0) = \int_\ell \varepsilon_N^{FE}(s).\phi(x_0 - s)\mathrm{d}s \tag{4}$$

From this equation, the location x_0 and the crack opening $\|u\|$ are obtained.

From this quantification of the crack opening, the identification of the direction of mode I opening at a given point along the crack is obtained by maximimizing $\|u\|$ according to **n**. This direction is parameterized in 2D by an angle θ. Finally, the maximization problem is defined by Eq.5.

$$\theta_{crack} = \max_\theta \{\theta \in [0, \pi[; \|u\|(\theta, \boldsymbol{\varepsilon}^{an})\} \tag{5}$$

The formulation of the problem can easily be generalized to 3D field by defining the orientation of **n** with two angles: θ and φ.

From a given point and a direction of opening, the definition of the next searching point along the crack is defined according to the topological method proposed by (Bottoni et al. 2015). A new point of search P_{search}^{i+1} is defined by Eq.6.

$$\begin{aligned} \|P_{crack}^{i} P_{search}^{i+1}\| &= h, \\ \left(\overline{P_{crack}^{i} P_{search}^{i+1}}\right).\mathbf{n} &= 0, \\ \left(\overline{P_{crack}^{i-1} P_{crack}^{i}}\right) \cdot \left(\overline{P_{crack}^{i} P_{search}^{i+1}}\right) &> 0 \end{aligned} \tag{6}$$

4.2 *Global/local approach for the description of cracking*

In this section the global/local method used in a reanalysis of the continuous calculation is described. This approach allows to give local information relative to cracking (e.g. location, opening, ...) by reanalyzing the damaged areas of the structure studied in a continuous framework (Oliver-Leblond et al. 2013). Fig. 6 gives the evolution process of analysis of the method. At the global scale, the nonlinear behavior of the structure is described thanks to the continuous model. The obtained damage pattern is studied and several Regions of Interest (ROIs) can be defined corresponding to distinct areas of damage if necessary. Then, the loading steps for the extraction of crack features are determined and will correspond to the steps of re-analysis. For those loading steps, boundary conditions are extracted from the continuous displacement field and applied on the non-free surfaces of the ROIs – namely the ones which cut the whole domain. The natural way to transfer the displacement field from the global scale to the local scale is to use the shape functions of the finite elements used for the global computation. Then, the displacement $u_L(x_L)$ applied at a local node x_L of a non-free surface of a ROI is directly obtained with Eq. 7.

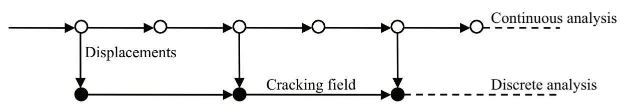

Figure 6: Global/Local sequential analysis.

$$u_L(x_L) = \sum_{j=1}^{N_{FE}} N_j(x_L)u_j, \tag{7}$$

where N_j are the shape functions of the finite element model, u_j is the displacement vector computed at the global scale and N_{FE} is the number of finite element nodes. The cracking pattern obtained at the previous step will be retrieved at the current step of the local re-analysis in order to follow the crack propagation accurately. The discrete computation of the chosen ROIs can be parallelised.

4.3 *Discrete local model*

The discrete model used at the local scale has been proposed by (Delaplace 2005) and offers a reliable description of concrete behaviour for tensile loadings. The material is described as an assembly of polyhedral particles linked by Euler-Bernoulli beams. The quasi-brittle behaviour of the material is obtained through a brittle behaviour for the beams. The breaking threshold P_{ab} of $a-b$, the beam linking the particles a and b, not only depends on the beam extension ε_{ab} but also on the rotations of the two particles θ_a and θ_b (Eq.8).

$$P_{ab} = \left(\frac{\varepsilon_{ab}}{\varepsilon^{cr}}, \frac{|\theta_b - \theta_a|}{\theta^{cr}} \right) > 1, \tag{8}$$

The six "material" parameters of the beam $a-b$ need to be calibrated. First, the length and the area are imposed by the geometry. Then, the inertia and the elastic modulus are identified in order to retrieve the elastic behaviour of the global computation. Finally, the calibration of the breaking thresholds ε^{cr} and θ^{cr} of the beam allow us to fit the peak and post-peak behaviour of the global model. The calibration is performed on an independent case study (Oliver-Leblond et al. 2013). Our study focuses on a fine description of crack pattern and on the measurement of the crack opening. The crack pattern is defined as the common side of the particles initially linked by the breaking beams. The opening of the crack is computed by considering the relative displacement $u_b - u_a$ of the unlinked particles a and b. The measure of the opening between those particles e_{ab} is projected on the normal n_{ab} of the local discontinuity (Eq.9).

$$e_{ab} = \langle (u_b - u_a).n_{ab} \rangle_+, \tag{9}$$

4.4 *Quantification of cracks properties*

In order to get information on cracking from the discrete element re-analysis, additional tools have been considered. The micro cracked domain obtained from the discrete element re-analysis is described in graph theory (Fig.7).

Using the Bellman-Ford algorithm (Moore 1959) in this framework, an identification of the macro-crack in the domain reanalyzed is obtained. In order to choose the right crack path at a crossroad, the crack opening associated to each discrete element is used as a weight.

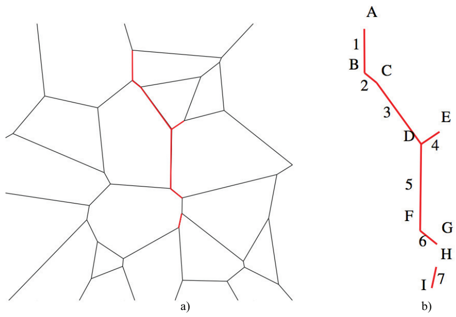

Figure 7: Discrete elements description of the media with micro cracks a) Representation of microcracks in graph theory b).

5 DYNAMIC ONE-POINT BENDING TEST

In this last part, the methods of quantification of the crack opening exposed in the previous sections are applied to the case of a dynamic one-point bending test with a split Hopkinson bar apparatus.
The specimen is a notched beam with a square section of size d=4 cm and length L=4d. The height of the notch is equal to 0.5d. The characteristics of the fiber reinforced concrete are : E = 41 GPa, f'_c = 75 MPa and ν = 0.23. The speed of the impactor of the input bar of the Hopkinson bar apparatus is limited to 8 m/s.
The results obtained numerically for cracking are compared to the experimental ones obtained by Digital Image Correlation (DIC) thanks a high-speed camera and the software CORRELI RT3 (Tomicevic et al. 2013). For the first approach (i.e. Topological search and continuous-discontinuous approach for crack opening), the anelastic strain ε^{an} is directly used to identify the crack location and crack opening obtained on the surface of the specimen. For the second approach (i.e. global/local approach for the description of cracking) an area of post-treatment associated to the location of the FPZ is defined. Fig. 8 gives the ROI and the discrete element mesh used for the re-analysis. 3500 particles are used to describe the area of re-analysis. The red lines defined the boundary where displacements are imposed. As the area of interest is sufficiently small, the kinetic energy is considered as homogeneous over the region of interest and an equivalent quasi-static analysis introducing only the displacement at the boundaries seems to be reasonable.

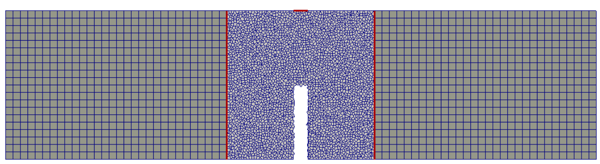

Figure 8: Discrete element mesh used for the re-analysis

Fig. 9 shows the residual field of the DIC which is a good indicator of the crack path obtained experimentally.

Fig. 10 shows the crack pattern obtained with the first approach. One can see a good prediction of the

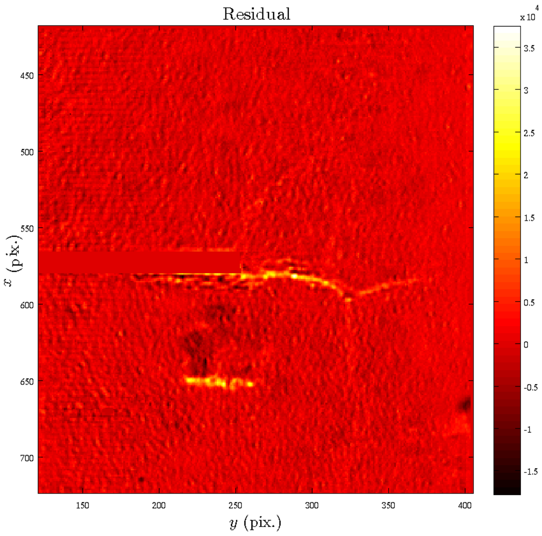

Figure 9: Experimental crack path obtained by DIC

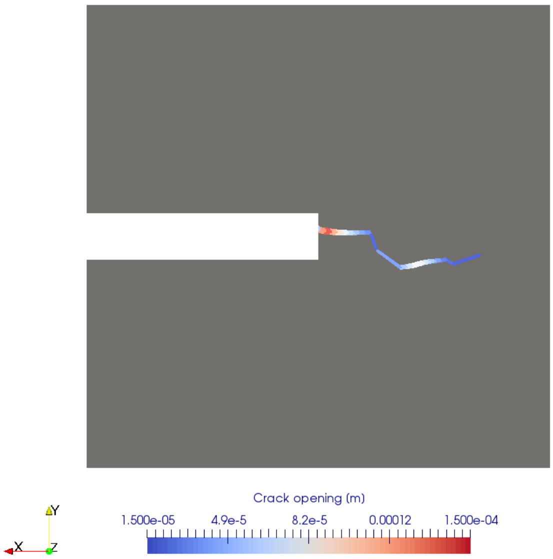

Figure 10: Crack pattern obtained with the second approach

crack path compared to the one experimentally obtained.

Fig. 11 shows the crack pattern obtained with the second approach. One can see that the second approach also gives in average a good prediction of the crack path compared to the one obtained experimentally
Furthermore, the crack opening for both methods is in the same order of magnitude and corresponds to the experimental data. In particular, the maximum crack opening along the crack path is around 150 μm for both modeling.
Regarding the definition of the crack tip, it may be not well capture with the first approach but as this data is not relevant for the final application and as this information still is a tricky experimental measurement (e.g. order of magnitude of the opening versus noise).
In conclusion, both approaches show good prediction of crack pattern from a continuous computation.

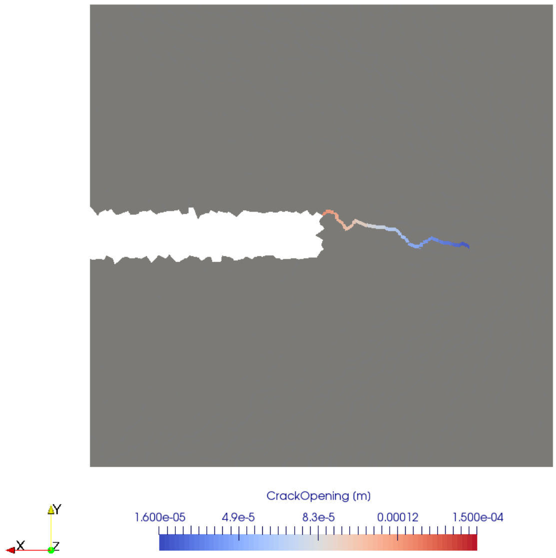

Figure 11: Crack pattern obtained with the second approach

6 CONCLUSION

Two different methods have been proposed in this work to characterize the crack pattern from a continuous computation. The first one exploits directly the fiels obtained from a FE simulation (stress and strain) to locate the crack and quantify the opening. The second one considers a re-analysis of the FPZ through a discrete element modeling. Both methods are not dependent of the model used for the continuous modeling and can be considered in a general purpose to quantify cracking. In the work presented, this is a first application of these methods to quantify cracking in the case of dynamic loading. It has been shown that both methods are capable of quantifying precisely the location and the opening of the crack for a dynamic one-point bending test on a fiber-reinforced concrete beam with a split Hopkinson bar apparatus. Further investigation and validation of the methods, more particularly for 3D post-treatments are in progress.

7 ACKNOWLEDGMENTS

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REFERENCES

Akiki, R., F. Gatuingt, C. Giry, N. Schmitt, & L. Stéfan (2016). Modeling and simulation for an optimized design of a dynamic bending test. In *ECCOMAS Congress 2016 - Proceedings of the 7th European Congress on Computational Methods in Applied Sciences and Engineering*, Volume 1, pp. 242–253.

Bottoni, M., F. Dufour, & C. Giry (2015). Topological search of the crack pattern from a continuum mechanical computation. *Eng. Struc. 99*, 346–359.

Delaplace, A. (2005). Fine description of fracture by using discrete particle model. In *11th International Conference on Fracture*.

Dufour, F., G. Pijaudier-Cabot, M. Choinska, & A. Huerta (2008). Extraction of a crack opening from a continuous approach using regularized damage models. *Computers and Concrete 5(4)*, 375–388.

Moore, E. F. (1959). The shortest path through a maze. In *Proc. Internat. Sympos. Switching Theory*, Volume 2, pp. 285–292.

Murray, Y. (2007). Users manual for ls-dyna concrete material model 159. Technical report.

Oliver-Leblond, C., A. Delaplace, F. Ragueneau, & B. Richard (2013). Non-intrusive global/local analysis for the study of fine cracking. *Int. J. for Numer. Anal. Meth. Geomech. 37(8)*, 973–992.

Tomicevic, Z., F. Hild, & S. Roux (2013). Mechanics- aided digital image correlation. *The Journal of Strain Analysis for Engineering Design. 48(5)*, 330–343.