Comment on ”Gain-assisted superluminal propagation and rotary drag of photon and surface polaritons”

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In their study of superluminal propagation, rotary drag and surface polaritons [Phys. Rev. A **96**, 013848 and 049906(E) (2017)], Khan *et al.* consider a four-level atomic arrangement with transitions in the optical domain. In fact, the values they give to the parameters lead to a probe wavelength lying in the decimeter band and we point out that, in such conditions, all their results are irrelevant..

In their study of superluminal propagation, rotary drag and surface polaritons [1, 2], Khan *et al.* consider a four-level atomic arrangement with transitions in the optical domain. See Fig. 1(a) in [1]. On the other hand, they specify in [2] that all the (angular) frequencies are given in units of $\gamma = (2\pi)\times 1\,\mathrm{MHz}$ and that the probe frequency $\nu_p = 1000\gamma$. The corresponding wavelength is thus $\lambda_p = 30\,\mathrm{cm}$ (in the decimeter band!). As shown in the following this invalidates all the results given in [1, 2].

As correctly given in [1], the electric susceptibility for the probe reads, in SI units,:

$$\chi = \frac{2N\left|\wp_{ac}\right|^2 \rho_{ac}}{\varepsilon_0 \hbar \Omega_p} \tag{1}$$

where N is the atomic number density, a (c) is the upper (lower) level of the probe transition, $\wp_{ac}$ (ρ_{ac}) is the corresponding matrix element of the dipole moment (of the density operator) and Ω_p is the Rabi (angular) frequency of the probe. Expressing the susceptibility as a function of the probe wavelength as made to obtain Eq. (5) in [1, 2] can be achieved by introducing the Einstein's coefficient A_{ac} associated with the transition $a \to c$. From its expression given in [3], we get:

$$\left|\wp_{ac}\right|^2 = \left(\frac{3\lambda_p^3}{8\pi^2}\right)\hbar\varepsilon_0 A_{ac} \tag{2}$$

and finally

$$\chi = \left(\frac{3N\lambda_p^3}{32\pi^3}\right)\left(\frac{8\pi A_{ac}}{\Omega_p}\right)\rho_{ac}. \tag{3}$$

The expression $\chi = \left(\frac{3N\lambda_p^3}{32\pi^3\Omega_p}\right)\rho_{ac}$ given by Eq. (5) in [2] thus holds only if Ω_p is expressed in units of $8\pi A_{ac}$. According to the above choice of γ as unit of (angular) frequency, this implies that $8\pi A_{ac} = \gamma$.

It is specified in [2] that "the susceptibility and group index plotted versus probe detuning have units of $2N\left|\wp_{ac}\right|^2/(\varepsilon_0\hbar)$". As shown in Eq.(1), this quantity has the dimension of an angular frequency and, for consistency, it should also be expressed in units of γ. It then reads $u_\chi = 2N\left|\wp_{ac}\right|^2/(\varepsilon_0\hbar\gamma)$ and, taking into account the above relations,

$$u_\chi = \frac{3N\lambda_p^3}{32\pi^3} \tag{4}$$

For wavelengths λ_p in the visible domain and typical values of the atomic number density N, the susceptibility unit u_χ given by Eq. (4) is in the order of 3×10^{-3}. On the other hand, for $\lambda_p = 30\,\mathrm{cm}$ with $N = 5\times10^{12}\,\mathrm{cm}^{-3}$ as considered in [2], this unit rises to $u_\chi \approx 4\times10^{14}$. Figure 2 in [2] shows that the peak value of the *relative* susceptibility χ/u_χ can exceed 5×10^{-3}. The corresponding *absolute* susceptibility χ is then in the order of 10^{12}. *Such values are meaningless.*

Although this point is less important, we note that, in SI units, the refractive index reads $n = \sqrt{1+\chi}$ and not $n = \sqrt{1+4\pi\chi}$ as used in [1] to determine the group index. Anyway the approximation $n \approx 1+2\pi\chi$ also made to obtain Eq. (6) in [1] fails when $|\chi| \gg 1$.

Without examining in detail the parts of [1, 2] devoted to rotary drag and surface polaritons, we remark that these phenomena occur when the sample thickness L is large compared to the probe wavelength λ_p . According to [2], $L = 10\,\mathrm{cm}$ and this condition is far from being fulfilled since this thickness is only one third of the probe wavelength. By the way, we also note the incompatibility of the figures 3(b) and 4 in [2] which show rotary drags, respectively, in the order of 10^{-2} and 10^{-7} rad.

Khan *et al.* support their choice of the ratio $\nu_p/\gamma = 1000$ by referring to a paper on the phase control of light velocity [4]. The same ratio was actually considered in this paper but without specifying the absolute value of the frequencies. We, however, point out that, for a probe frequency in the visible domain, this ratio leads to lifetimes of the excited atomic levels which are fully unrealistic (in the subpicosecond domain).

Independently of the above criticisms, we remark that, quite generally, large negative group delays are not a sufficient condition to observe visible effects of superluminal propagation. A convincing demonstration of such effects would have required a comparison of the transmitted and incident pulses, which is not made in [1, 2].

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