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Abstract—**In this paper, the maximum energy rate that can be achieved by a given code designed for simultaneously transmitting information to an information receiver (IR) and energy to an energy harvester (EH) through the binary symmetric channel is studied. The energy transmission rate is measured at the EH that is not necessarily colocated with the IR. Moreover, the energy transmission rate is calculated taking into account a reliability constraint and a latency constraint. The reliability is expressed in terms of the energy shortage probability, whereas the the latency is expressed in terms of the duration of the communication in channel uses. The exact maximum of the energy rate is presented. Nonetheless, it is not given in closed-form. To remediate this, approximations of the ESP are explored using large deviations and Gaussian approximations. These approximations lead to close form bounds on the energy transmission rate.**

Index Terms—**Simultaneous Information and Energy Transmission (SIET), Information-Energy Capacity Region, Finite Block-Length Regime.**

I. Introduction

Simultaneous information and energy transmission (SIET) refers to systems in which at least one transmitter aims to simultaneously send information to a set of information receivers and energy to a set of energy harvesters. This idea, initially proposed by Nikola Tesla in 1914 [1], is one of the central ideas in modern communications systems to wirelessly power up devices with low-energy consumption, such as sensors and wearable electronic devices [2]. Within this context, there are two fundamental considerations to be taken into account: the information and energy transmission must be both ultra reliable and exhibit a low latency. This translates into two simple requirements. First, the probability that the energy transmission rate falls below the targeted rate, i.e., the energy shortage probability (ESP), must be bounded above by a given value. Second, the communication must occur within a fixed number of channel uses. These two requirements break away from the classical analysis of SIET, which is typically performed under the assumption that the duration of the communication is sufficiently long and that the ESP can be made arbitrarily close to zero. See for instance [3], [4], [5], [6], [7], and [8].

This paper builds upon existing results in which the fundamental limits of SIET are studied in the context of strictly positive ESP, positive decoding error probability (DEP) and finite block length, c.f., [9] and [10]. The focus of this paper is on a system in which a transmitter simultaneously sends information to an information receiver (IR) and energy to an energy harvester (EH) through memoryless binary symmetric channels. The main contribution consists in characterizing the exact maximum of the energy rate that can be achieved by any given code. Nonetheless, such exact bound is not provided in closed-form. To remediate this, approximations of the ESP are explored using large deviations and Gaussian approximations. These approximations lead to close form expressions bounds on the energy transmission rate, which are the desired results.

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II. Notation

Throughout this paper, sets are denoted with uppercase calligraphic letters, e.g., $\mathcal{X}$. Random variables are denoted by uppercase letters, e.g., X, and their realizations are denoted by lower case letters, e.g., x. The probability distribution of X is denoted by P_X. Whenever a second random variable Y is involved, P_{XY} and $P_{Y|X}$ denote, respectively, the joint probability distribution of (X, Y) and the conditional probability distribution of Y given X. Let n be a fixed natural number. An n-dimensional vector of random variables is denoted by bold upper case letters, e.g., $\boldsymbol{X} \triangleq (X_1, X_2, \ldots, X_n)^{\mathsf{T}}$, and its corresponding realization by bold lower case letters, e.g., $\boldsymbol{x} \triangleq (x_1, x_2, \ldots, x_n)^{\mathsf{T}}$. Let $\boldsymbol{x}$ be a binary vector. Then, the number of zeros and ones in $\boldsymbol{x}$ are denoted by $N(0|\boldsymbol{x})$ and $N(1|\boldsymbol{x})$, respectively. The notation $\mathbb{E}_X[\cdot]$ is for the expected value of the random variable X. The complementary cumulative distribution function $Q : \mathbb{R} \to [0, 1]$ of the standard Gaussian distribution is

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x, \qquad (1)$$

and the functional inverse of Q is $Q^{-1} : [0, 1] \to \mathbb{R}$.

III. System Model

Consider a three-party communication system in which a transmitter aims at simultaneously sending information to an IR and energy to an EH through a binary symmetric channel. Such a system can be modeled by a random transformation

$$(\{0,1\}^n, \{0,1\}^n \times \{0,1\}^n, P_{\boldsymbol{Y}\boldsymbol{Z}|\boldsymbol{X}}), \qquad (2)$$

where $n \in \mathbb{N}$ is the block length. Given an input $\boldsymbol{x} \triangleq (x_1, x_2, \ldots, x_n) \in \{0,1\}^n$, the outputs $\boldsymbol{y} \triangleq (y_1, y_2, \ldots, y_n) \in \{0,1\}^n$ and $\boldsymbol{z} \triangleq (z_1, z_2, \ldots, z_n) \in \{0,1\}^n$ are observed at the IR and at the EH, respectively, with probability

$$P_{\boldsymbol{Y}\boldsymbol{Z}|\boldsymbol{X}}(\boldsymbol{y}, \boldsymbol{z}|\boldsymbol{x}) = \prod_{t=1}^{n} P_{Y|X}(y_t|x_t) P_{Z|X}(z_t|x_t), \qquad (3)$$

where for all $(x, y, z) \in \{0,1\}^3$,

$$P_{Y|X}(y|x) = \alpha_1 \mathbb{1}_{\{x \neq y\}} + (1-\alpha_1)\mathbb{1}_{\{x=y\}}, \qquad (4)$$

$$P_{Z|X}(z|x) = \alpha_2 \mathbb{1}_{\{x \neq z\}} + (1-\alpha_2)\mathbb{1}_{\{x=z\}}, \qquad (5)$$

and $\alpha_1 \in [0, \frac{1}{2})$ and $\alpha_2 \in (0, \frac{1}{2})$. In this context, two tasks are carried out by the transmitter: (a) the information transmission task; and (b) the energy transmission task.

A. Information Transmission Task

The purpose of this task is to send a message from the transmitter to the IR. The message index is a realization of a random variable uniformly distributed in $\{1, 2, \ldots, M\}$, with $M \in \mathbb{N}$. To carry out this task within n channel uses, the transmitter uses an (n, M)-code.

Definition 1 ((n, M)-code): An (n, M)-code for the random transformation in (2) *is a system*

$$\{(\boldsymbol{u}(1), \mathcal{D}_1), (\boldsymbol{u}(2), \mathcal{D}_2), \ldots, (\boldsymbol{u}(M), \mathcal{D}_M)\}, \tag{6}$$

where for all $(i, j) \in \{1, 2, \ldots, M\}^2$, with $i \neq j$,

$$\boldsymbol{u}(i) \triangleq (u_1(i), u_2(i), \ldots, u_n(i)) \in \{0, 1\}^n, \tag{7a}$$
$$\mathcal{D}_i \cap \mathcal{D}_j = \emptyset, \text{ and} \tag{7b}$$
$$\bigcup_{i=1}^{M} \mathcal{D}_i \subseteq \{0, 1\}^n. \tag{7c}$$

Given the system in (6), for all $i \in \{1, 2, \ldots, M\}$, to transmit the message with index i, the transmitter inputs the symbol $u_t(i)$ to the channel at time $t \in \{1, 2, \ldots, n\}$. The IR observes the output y_t at the end of channel use t. At the end of n channel uses, the IR decides that the symbol i was transmitted if it satisfies the rule

$$(y_1, y_2, \ldots, y_n) \in \mathcal{D}_i. \tag{8}$$

The decoding error probability associated with the transmission of message index i, denoted by $\lambda_i \in [0, 1]$, is

$$\lambda_i \triangleq \Pr\left[\boldsymbol{Y} \in \mathcal{D}_i^{\mathsf{c}} \mid \boldsymbol{X} = \boldsymbol{u}(i)\right], \tag{9}$$

where the probability is with respect to the marginal $P_{\boldsymbol{Y}|\boldsymbol{X}}$, and $\mathcal{D}_i^{\mathsf{c}}$ represents the complement of $\mathcal{D}_i$ with respect to $\{0, 1\}^n$. The average probability of error, denoted by λ, is

$$\lambda \triangleq \frac{1}{M} \sum_{m=1}^{M} \lambda_m. \tag{10}$$

Information transmission is said to be reliable if the average or maximum DEP is controlled. This leads to the following refinements of Definition 1.

Definition 2 ((n, M, ϵ)-code with maximum DEP): Let $\epsilon \in [0, 1]$ be fixed. An (n, M)-code that satisfies $\lambda_i < \epsilon$, for all $i \in \{1, 2, \ldots, M\}$, is said to be an (n, M, ϵ)-code with maximum DEP.

Definition 3 ((n, M, ϵ)-code with average DEP): Let $\epsilon \in [0, 1]$ be fixed. An (n, M)-code that satisfies $\lambda < \epsilon$ is said to be an (n, M, ϵ)-code with average DEP.

Note that any (n, M, ϵ)*-code with maximum DEP* is also a (n, M, ϵ)*-code with average DEP*. Nonetheless, the converse is not necessarily true.

B. Energy Transmission Task

Let $g : \{0, 1\} \rightarrow \mathbb{R}_+$ be a positive real-valued function that determines the energy harvested from the channel output symbols. Let

$$b_0 \triangleq g(0), \quad \text{and } b_1 \triangleq g(1) \tag{11a}$$

be the energy harvested when the channel outputs at the EH are 0 and 1, respectively. At the end of n channel uses, the average energy delivered to the EH by the channel outputs $\boldsymbol{z} = (z_1, z_2, \ldots, z_n)$ is given by the function $B_n : \{0, 1\}^n \rightarrow \mathbb{R}_+$, with

$$B_n(\boldsymbol{z}) \triangleq \frac{1}{n} \sum_{t=1}^{n} g(z_t) = (b_0 - b_1) \frac{N(0|\boldsymbol{z})}{n} + b_1. \tag{12}$$

The objective of the transmitter is to ensure that energy is harvested at the EH at a rate not smaller than b energy units per channel use, with $b \geqslant 0$. An energy-shortage event occurs when the energy harvested at the EH is less than b at the end of the transmission. The case in which $b_0 = b_1$ is trivial, since for all channel outputs $\boldsymbol{z} \in \{0, 1\}^n$, it holds that $B_n(\boldsymbol{z}) = b_0 = b_1$. That is, the average energy rate at the input of the EH is independent of the codebook, and either an energy shortage is never observed if $b \geqslant b_0 = b_1$; or the the system is always under energy shortage if $b < b_0 = b_1$. Hence, to avoid these trivial cases, the following assumption is adopted without loss of generality:

$$b_1 < b_0. \tag{13}$$

The probability of energy-shortage when transmitting the message with index $i \in \{1, 2, \ldots, M\}$ is

$$\theta_i \triangleq \Pr\left[B_n(\boldsymbol{Z}) < b \mid \boldsymbol{X} = \boldsymbol{u}(i)\right] \tag{14}$$
$$= \Pr\left[\sum_{t=1}^{n} \mathbb{1}_{\{Z_t = 0\}} < \left(\frac{n(b - b_1)}{b_0 - b_1}\right) \middle| \boldsymbol{X} = \boldsymbol{u}(i)\right], \tag{15}$$

where the probability is with respect to the marginal $P_{\boldsymbol{Z}|\boldsymbol{X}}$. The average probability of energy-shortage, denoted by θ, is

$$\theta \triangleq \frac{1}{M} \sum_{i=1}^{M} \theta_i. \tag{16}$$

Note that for all $\boldsymbol{z} \in \mathcal{Z}^n$, $B_n(\boldsymbol{z})$ is bounded according to

$$b_1 \leqslant B_n(\boldsymbol{z}) \leqslant b_0. \tag{17}$$

The inequalities in (17) imply that there exists a case in which energy transmission might occur with zero (maximal or average) ESP for all energy transmission rates $b \leqslant b_1$. This is because the event $B_n(\boldsymbol{Z}) < b_1$ is observed with zero probability. Alternatively, any energy transmission rate $b > b_0$ cannot be achieved with an average or maximal energy-shortage probability strictly smaller than one.

Energy transmission is said to be reliable if the average or maximum ESP is controlled. This leads to the following refinements of Definition 1.

Definition 4 ($(n, M, \epsilon, \delta, b)$-code with maximum ESP): Let $\delta \in [0, 1]$ and $b \geq 0$ be fixed. An (n, M, ϵ)-code that satisfies $\theta_i < \delta$, for all $i \in \{1, 2, \ldots, M\}$, is said to be an $(n, M, \epsilon, \delta, b)$-code with maximum ESP.

Definition 5 (($n, M, \epsilon, \delta, b$)-code with average ESP): Let $\delta \in [0, 1]$ and $b \geq 0$ be fixed. An (n, M, ϵ)-code that satisfies $\theta < \delta$ is said to be an $(n, M, \epsilon, \delta, b)$-code with average ESP. Note that any $(n, M, \epsilon, \delta, b)$*-code with maximum ESP* is also a $(n, M, \epsilon, \delta, b)$*-code with average ESP*. Nonetheless, the converse is not necessarily true.

IV. Bounds on the Energy Rate

Assume that the transmitter uses an $(n, M, \epsilon, \delta, b)$-code and it aims at sending the message index $i \in \{1, 2, \ldots, M\}$. Then, for all $t \in \{1, 2, \ldots, n\}$, the random variable $\mathbb{1}_{\{Z_t=0\}}$ in (15) follows a distribution in which the probability of a "one" is

$$P_{Z|X}(0|u_t(i)) = \begin{cases} \alpha_2 & \text{if} \quad u_t(i) = 1 \\ 1 - \alpha_2 & \text{if} \quad u_t(i) = 0 \end{cases} . \quad (18)$$

Therefore, the random variable $\sum_{t=1}^{n} \mathbb{1}_{\{Z_t=0\}}$ can be expressed as follows

$$\sum_{t=1}^{n} \mathbb{1}_{\{Z_t=0\}} = \sum_{t \in \{m : u_m(i)=0\}} \mathbb{1}_{\{Z_t=0\}} + \sum_{t \in \{m : u_m(i)=1\}} \mathbb{1}_{\{Z_t=0\}}, \quad (19)$$

which corresponds to the sum of two random variables with binomial distributions $\mathcal{B}\left(N(0|\boldsymbol{u}(i)), 1-\alpha_2\right)$ and $\mathcal{B}\left(N(1|\boldsymbol{u}(i)), \alpha_2\right)$, respectively. This implies that for all $i \in \{1, 2, \ldots, M\}$:

$$\theta_i = \sum_{m=0}^{\left\lfloor \frac{n(b-b_1)}{b_0-b_1} \right\rfloor} \Pr\Bigg[\sum_{t \in \{m : u_m(i)=0\}} \mathbb{1}_{\{Z_t=0\}} + \sum_{t \in \{m : u_m(i)=1\}} \mathbb{1}_{\{Z_t=0\}} = m \Bigg| \boldsymbol{X} = \boldsymbol{u}(i) \Bigg] \quad (20)$$

$$= \sum_{k=0}^{\left\lfloor \frac{n(b-b_1)}{b_0-b_1} \right\rfloor} \sum_{s=0}^{k} \Pr\Bigg[\sum_{t \in \{m : u_m(i)=0\}} \mathbb{1}_{\{Z_t=0\}} = s \Bigg| \boldsymbol{X} = \boldsymbol{u}(i) \Bigg] \Pr\Bigg[\sum_{t \in \{m : u_m(i)=1\}} \mathbb{1}_{\{Z_t=0\}} = k-s \Bigg| \boldsymbol{X} = \boldsymbol{u}(i) \Bigg] \quad (21)$$

$$= \sum_{k=0}^{\left\lfloor \frac{n(b-b_1)}{b_0-b_1} \right\rfloor} \sum_{s=0}^{k} \binom{N\left(0|\boldsymbol{u}(i)\right)}{s} \binom{N\left(1|\boldsymbol{u}(i)\right)}{k-s} (1-\alpha_2)^{N(1|\boldsymbol{u}(i))-k+2s} \alpha_2^{N(0|\boldsymbol{u}(i))+k-2s}. \quad (22)$$

The equation in (22) is the ground truth individual ESP. Hence, from Definition 4 and Definition 5 the following holds.

Proposition 1 (Ground-Truth Bound): Consider an $(n, M, \epsilon, \delta, b)$-code described by the system in (6) for the random transformation in (2) satisfying (13). Then, subject to a maximal ESP constraint, it holds that,

$$b < \tilde{B} \quad (23)$$

where $\tilde{B}$ is the largest real that satisfies for all $i \in \{1, 2, \ldots, M\}$,

$$\sum_{k=0}^{\left\lfloor \frac{n(\tilde{B}-b_1)}{b_0-b_1} \right\rfloor} \sum_{s=0}^{k} \binom{N\left(0|\boldsymbol{u}(i)\right)}{s} \binom{N\left(1|\boldsymbol{u}(i)\right)}{k-s} (1-\alpha_2)^{N(1|\boldsymbol{u}(i))-k+2s} \alpha_2^{N(0|\boldsymbol{u}(i))+k-2s} < \delta, \quad (24)$$

and subject to an average ESP constraint, the energy rate b satisfies

$$b < \breve{B} \quad (25)$$

where $\breve{B}$ is the biggest positive real that satisfies

$$\frac{1}{M} \sum_{i=1}^{M} \sum_{k=0}^{\left\lfloor \frac{n(\breve{B}-b_1)}{b_0-b_1} \right\rfloor} \sum_{s=0}^{k} \binom{N\left(0|\boldsymbol{u}(i)\right)}{s} \binom{N\left(1|\boldsymbol{u}(i)\right)}{k-s} (1-\alpha_2)^{N(1|\boldsymbol{u}(i))-k+2s} \alpha_2^{N(0|\boldsymbol{u}(i))+k-2s} < \delta. \quad (26)$$

The bounds in Proposition 1 are not in closed-form and thus, are difficult to calculate. Moreover, they bring very little insight about the maximum energy rate at which an (n, M, ϵ) code can transmit energy. Therefore, it would be desirable to approximate the individual ESP in order to obtain an upper bound on the energy transmission rate in a closed form expression, of course, at the expense of precision. The following lemma, proved in [11], provides some bounds on the ESP that are instrumental in obtaining some bounds on the energy transmission rates.

Lemma 1: Consider the random variable $\sum_{t=1}^{n} \mathbb{1}_{\{Z_t=0\}}$ in (14) for a fixed $i \in \{1, 2, \ldots, M\}$. Then,

$$\Pr\Bigg[\sum_{t=1}^{n} \mathbb{1}_{\{Z_t=0\}} \geqslant \left(\frac{n(b-b_1)}{b_0-b_1} \right) \Bigg| \boldsymbol{X} = \boldsymbol{u}(i) \Bigg] < \exp\left(-n \frac{\left(\frac{b-b_1}{b_0-b_1} - \left((1-2\alpha_2)\, \bar{P}_X^{(i)}(0) + \alpha_2 \right) \right)^2}{\frac{b-b_1}{b_0-b_1} + \left((1-2\alpha_2)\, \bar{P}_X^{(i)}(0) + \alpha_2 \right)} \right). \quad (27)$$

Solving for b in (27) and after some algebraic manipulations shown in [11], leads to the following proposition.

Proposition 2 (Large Deviation Bound): Consider an $(n, M, \epsilon, \delta, b)$-code described by the system in (6) for the random transformation in (2) satisfying (13). Then, subject to a maximal ESP constraint, it holds that for all $i \in \{1, 2, \ldots, M\}$,

$$b < (b_0 - b_1)\left((1-2\alpha_2)\, \bar{P}_X^{(i)}(0) + \alpha_2 \right) + b_1 + \frac{b_0 - b_1}{\sqrt{n}} \sqrt{-2\left((1-2\alpha_2)\, \bar{P}_X^{(i)}(0) + \alpha_2 \right) \log(1-\delta)} - \frac{b_0 - b_1}{n} \log(1-\delta). \quad (28)$$

and subject to an average ESP constraint, the energy rate b satisfies

$$b < \hat{B}, \quad (29)$$

where $\hat{B}$ is the biggest positive real that satisfies

$$1 - \delta < \frac{1}{M} \sum_{i=1}^{M} \exp\left(-n \frac{\left(\frac{b-b_1}{b_0-b_1} - \left((1-2\alpha_2)\, \bar{P}_X^{(i)}(0) + \alpha_2 \right) \right)^2}{\frac{b-b_1}{b_0-b_1} + \left((1-2\alpha_2)\, \bar{P}_X^{(i)}(0) + \alpha_2 \right)} \right). \quad (30)$$

Another bound that follows immediately from the Berry-Esseen theorem [12] is presented hereunder.

Lemma 2: Consider the random variable $\sum_{t=1}^{n} \mathbb{1}_{\{Z_t=0\}}$ in (14) for a fixed $i \in \{1, 2, \ldots, M\}$. Then,

$$\Pr\left[\sum_{t=1}^{n} \mathbb{1}_{\{Z_t=0\}} < \left(\frac{n(b-b_1)}{b_0 - b_1} \right) \middle| \boldsymbol{X} = \boldsymbol{u}(i) \right] \geq Q\left(\frac{n\left((1-2\alpha_2)\bar{P}_X^{(i)}(0) + \alpha_2 - \frac{b-b_1}{b_0-b_1} \right)}{\sqrt{n\alpha_2(1-\alpha_2)}} \right) - \frac{n\left[\alpha_2(1-\alpha_2)^3 + (1-\alpha_2)\alpha_2^3\right]}{2\left(n\alpha_2(1-\alpha_2)\right)^{3/2}} \quad (31)$$

Solving for b in (31) and after some algebraic manipulations shown in [11], leads to the following proposition.

Proposition 3 (Gaussian Approximation Bound [10]): Consider an $(n, M, \epsilon, \delta, b)$-code described by the system in (6) for the random transformation in (2) satisfying (13). Then, subject to a maximal ESP constraint, it holds that for all $i \in \{1, 2, \ldots, M\}$,

$$b < (b_0 - b_1)\Big((1-2\alpha_2)\,\bar{P}_X^{(i)}(0) + \alpha_2 \Big) + b_1 - \sqrt{\frac{(b_0-b_1)^2\alpha_2(1-\alpha_2)}{n}}\, Q^{-1}\left(\delta + \frac{(1-\alpha_2)^2 + \alpha_2^2}{2\sqrt{n\alpha_2(1-\alpha_2)}} \right), \quad (32)$$

and subject to an average ESP constraint, the energy rate b satisfies

$$b < \hat{B}, \quad (33)$$

where $\hat{B}$ is the biggest positive real that satisfies

$$\frac{1}{M}\sum_{i=1}^{M} Q\left(\frac{n\left((1-2\alpha_2)\bar{P}_X^{(i)}(0) + \alpha_2 - \frac{\hat{B}-b_1}{b_0-b_1} \right)}{\sqrt{n\alpha_2(1-\alpha_2)}} \right) - \frac{(1-\alpha_2)^2 + \alpha_2^2}{2\sqrt{n\alpha_2(1-\alpha_2)}} < \delta. \quad (34)$$

Proof: The proof of Proposition 3 is presented in [11]. ■

Note that the upper bound in (32), is valid when the following condition is satisfied

$$0 < \delta + \frac{(1-\alpha_2)^2 + \alpha_2^2}{2\sqrt{n\alpha_2(1-\alpha_2)^2}} < 1, \quad (35)$$

given that the domain of the function Q^{-1} is $(0, 1)$.

V. Homogeneous Codes

The class of homogeneous codes is of particular interest in this study due to the fact that an average or maximum constraint on the ESP leads to the same bound on the energy transmission rate. A formal definition of these codes is hereunder.

Definition 6 (Homogeneous Codes): A code $\mathcal{C}$ described by the system in (6) is said to be homogeneous if the following conditions hold:

$$N(0|\boldsymbol{u}(1)) = N(0|\boldsymbol{u}(2)) = \ldots = N(0|\boldsymbol{u}(M)) \text{ and} \quad (36)$$

$$N(1|\boldsymbol{u}(1)) = N(1|\boldsymbol{u}(2)) = \ldots = N(1|\boldsymbol{u}(M)). \quad (37)$$

The corollary hereunder follows immediately from Proposition 1 and Proposition 3.

Corollary 1: Consider an $(n, M, \epsilon, \delta, b)$-code described by the system in (6) *for the random transformation in* (2) *satisfying* (13)*, and assume it is a homogeneous code. Then, the bounds on the energy rate b subject to a maximum ESP and average ESP are identical. That is, the bound in* (23) *is identical to* (25)*; the one in* (28) *is identical to* (29)*; and the one in* (32) *is identical to* (33)*.*

VI. Conclusions

In this paper, the exact ESP achieved by each of the codewords of a given code has been calculated. Using this result, the exact maximum energy transmission rate for a given code has been calculated. Nonetheless, the expression is cumbersome and brings very little light into the understanding of this problem. Therefore, two lower-bounds on the ESP achieved by each of the codewords have been reported. Each lower-bound has been transformed into an upper-bound on the energy transmission rate of the given code, which is the desired result. In general, the bounds presented in Proposition 3 are tighter than those presented in Proposition 2 for small values of the block length n. Nonetheless, the bounds in Proposition 2 are easier to calculate and perform equally well for large n.

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