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Bringing present bias back to the present

Alexis Direr *

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Abstract

A focused definition of present bias is proposed which takes preferences as primitives. A present biased individual over-weights immediate costs and benefits relative to those occurring at any point in the future. The definition allows to sort out previous confounds, such as decreasing impatience, choice reversal or short-term impatience. It intuitively connects to usual utility representations of present bias like the quasi-hyperbolic model of Laibson (1997) or the fixed cost model of Benhabib, Bisin and Schotter (2010).

J.E.L. codes: D8, E21

Keywords : time preferences, decreasing impatience, present bias

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1 Introduction

The last two decades have witnessed a rapid growth of theoretical and empirical works on present bias. It is now considered an important determinant of many intertemporal decisions related to saving or borrowing (Meier and Sprenger, 2010), retirement timing (Diamond and Koszegi, 2003), addiction (Laibson, 2001, Bernheim et Rangel, 2004), health (Loewenstein et al. 2012), bargaining (Schweighofer-Kodritsch, 2018), or job search (DellaVigna and Paserman, 2005). It helps explain why individuals have self-control problems, procrastinate, or do not stick to the plans they have made earlier (O'Donoghue and Rabin, 2015, Bisin and Hyndman, 2014).

It is fair however to recognize that the very idea of present bias still lacks a formal definition. It has been used so far as a label for addressing various behavioral properties like short-term impatience, decreasing impatience, choice reversal toward early outcomes, and more generally, procrastination, self-control problems, demand for commitment, high required rates of return and naiveté (when people underestimate their own procrastination or present bias). The profusion of concepts surrounding the term raises questions. Given the importance ascribed to the concept in the behavioral literature, a precise and well founded definition is of prime importance.

The aim of this article is to propose a simple and focused definition of present bias, which closely relates to, but remains distinct from choice reversal, decreasing or increasing impatience and short-term impatience. A present biased individual values more an immediate outcome than one postponed in the near future, where the near future can be arbitrarily soon. The definition makes a natural distinction between present bias, present neutrality and future bias. It takes a simple axiomatic form, which, once mixed with other axioms of time preference has the property of turning present neutral preferences into present or future

biased ones. Two well known models of present bias, the quasi-hyperbolic model of Laibson (1997) and the fixed cost model of Benhabib, Bisin, and Schotter (2010) conforms in an intuitive way to the definition.

The expression "present bias" is relatively recent. It can be traced back to the article by O'Donoghue and Rabin (1999) who present it as "a more descriptive term for the underlying human characteristic that hyperbolic discounting represents". Hyperbolic discounting is taken as an equivalent expression for decreasing impatience, which means that when considering trade-offs between two future periods, individuals give stronger relative weight to the earlier moment as it gets closer. Decreasing impatience has since then served as a testable implication of present bias (e.g. Andreoni and Sprenger, 2012). Subsequent studies (Hayashi, 2003; Benhabib, Bisin and Schotter, 2010) have extended the meaning to the preference reversal property, which is the tendency of reversing one's choice from late to early outcome once a trade-off is moved closer to the present (Thaler, 1981; Read and van Leeuwen, 1998). More recently, O'Donoghue and Rabin (2015) proposed as an alternative test of present bias the detection of non negligible impatience over short delays.

Experimental studies show that a significant fraction of subjects exhibit opposite tendencies, like increasing impatience (Attema et al., 2010) or choice reversals toward the late outcome (Read, Frederick and Airoldi, 2012; Sayman and Öncüler, 2009; Takeuchi, 2011), especially when the outcomes are money. While it seems natural to refer to those properties as future bias, the same remark about the need for a unified definition applies.

Many studies referring to present bias have worked with functional representations, usually the quasi-hyperbolic model of Laibson (1997), also called the $(\beta - \delta)$) model, where, compared to the exponential model of Samuelson (1937), an extra weight $1/\beta$ on present utility is added. In his seminal article, Laibson did not propose a specific theory of present bias, whose expression appeared later

in his work (Laibson et al., 2003). His objective was rather to propose a time inconsistent discounting model which exhibits decreasing impatience while being closer to the canonical model of Samuelson (1937) than the hyperbolic formulation used by psychologists (Ainslie, 1992). Since then, the model has served as a workhorse to investigate the consequences of many behaviors like decreasing impatience, choice reversal, naiveté or short-term impatience. Although functional forms offer tractable ways for analyzing a wide range of issues, they may lack scope and generality (Spiegler, 2019).

A few articles give a conceptual or axiomatic definition of present bias. Chakraborty (2017) weakens the stationarity axiom to allow possible choice reversals in a way that does not contradict a preference for the present. Montiel Olea and Strzalecki (2014) provide an axiomatic characterization of quasi-hyperbolic discounting and a more general class of semi-hyperbolic preferences. Both articles elaborate on present bias defined as a deviation from stationarity and constant impatience.

The remainder of the paper is organized as follows. Section 2 lays out existing definitions of present bias and proposes a new definition. Section 3 presents axiomatic foundations of the definition. Section 4 concludes.

2 Present bias: definitions

2.1 Previous definitions

Present bias is loosely defined as the propensity of overvaluing immediate rewards at the expense of futures ones. The tendency is often phrased as an extreme form of impatience or a strong preference for early rather than late outcomes. A variety of psychological drivers have been mentioned, such as impulsivity, deprivation,

addiction, or transient visceral factors such as hunger or thirst.

More formally, let us consider a decision maker (henceforth DM) whose time preferences are defined over the set of possible positive or negative outcomes $X \subseteq \mathbb{R} - \{0\}$. $T = \{0, 1, 2, ..., \bar{t}\}$ is the set of times at which an outcome may occur, with $t = 0$ the present. A time-dependent plan (x, t) delivers the outcome $x \in X$ at date $t \in T$. Let the relations $\succ$, $\prec$ and $\sim$ define complete and transitive preference ordering on $X \times T$ expressed at time 0. Positive outcomes (consumption in a broad sense) are defined by $(x, t) \succ (y, t)$, $\forall x, y \in X$, $x > y$, and $\forall t \in T$, and negative outcomes (unpleasant experience or tasks) by the reverse preference. In the following, the set of dated outcomes is restricted to either positive outcomes or negative ones. The issue of time-consistency is sidestepped by focusing on time preferences from date 0 perspective, as if the DM could commit to them.

Before presenting an axiom-based definition of present bias, I first recall the definitions of decreasing impatience (Prelec, 2004) and choice reversal.

Definition 1 *(decreasing impatience)* $\forall\, x, y \in X$, $\forall\, s, t \in T$, $s < t$, *such that* $(x, s) \sim (y, t)$, *impatience is decreasing if* $(x, s + \Delta) \prec (y, t + \Delta)$, *for all* $\Delta > 0$ *and* $t + \Delta \in T$.

Impatience is decreasing (or discounting is hyperbolic) if for any couple of equivalent dated outcomes, the DM prefers the delayed option when the two dates are shifted forward by the same time interval. Increasing impatience is defined the same way with a reversed preference relation between the early and late outcomes.

Most experimental studies have found decreasing impatience (Thaler, 1981; Benzion, Rapoport and Yagil, 1989; Green, Myerson and Mcfadden, 1997; Kirby, 1997; Kirby, Petry, and Bickel, 1999; Kable and Glimcher, 2007; Benhabib, Bisin, and Schotter, 2010; Bleichrodt, Gao, and Rohde, 2016). Some studies have found

increasing impatience for money (Attema et al., 2010; Sayman and Öncüler, 2009; Scholten and Read 2006; Loewenstein, 1987 and Takeuchi, 2011).

Many articles assimilate present bias and decreasing impatience (O'Donoghue and Rabin, 1999; Halevy, 2008; Benhabib, Bisin, and Schotter, 2010; Andreoni and Sprenger, 2012). Present bias can be viewed as a particular instance of decreasing impatience in which the early date is the present one ($t = 0$). However, decreasing impatience, which characterizes the evolution of impatience at every date, does not stress what is special about the present.

Decreasing impatience is a cause of time inconsistency and choice reversal. For this reason, present bias is also sometimes assimilated to choice reversal (Manzini and Mariotti, 2009; Chakraborty, 2017). Several experimental studies use it as a test of present bias (Read and van Leeuwen, 1998, Takeuchi, 2011, Andreoni and Sprenger, 2012, Augenblick et al., 2015). As a future trade-off gets closer to the present, preferences are increasingly biased toward the early option.

Definition 2 *(choice reversal)* $\forall\, x, y \in X$ *and* $\forall\, s, t \in T$, $t < s$, *such that* $(x, t) \sim (y, s)$, *there is choice reversal toward the early outcome if,* t *periods later,* $(x, 0) \succ (y, s - t)$, *and toward the late outcome if* $(x, 0) \prec (y, s - t)$.

The definition of choice reversal toward the early outcome is close to the definition of decreasing impatience, but requires the passage of time. Decreasing impatience entails choice reversal towards the present outcome if preferences are stable across periods (Halevy, 2015). Experiments usually find that choices are reversed in favor of the present option for positive outcomes. Since choice reversal toward the present option often reveals self-control problems and toward the late outcome procrastination, present bias is also sometimes inferred from demand for precommitment devices that restricts the set of future consumption or actions (Bryan, Karlan and Nelson, 2010). The test relies on the joint assumption of sophistication where the DM is sufficiently aware of her self control problem.

Impatience over short-term trade-offs, a property commonly observed in experiments, is also interpreted as signaling present bias (Rabin, 2002; Shapiro, 2005; O'Donoghue and Rabin, 2006). O'Donoghue and Rabin (2015) argue that short-term discounting is a test of present bias, even better than choice reversal. To see why, consider an exponential discounter whose discount rate over a short period of time is $\rho \geq 0$ and her short term discount factor is $d(1) = (1+\rho)^{-1}$. Her utility in n years is discounted by $d(nm) = (1+\rho)^{-nm}$, with m the number of unit periods in a year. Her annualized discount factor ρ_a is equal to $1 + \rho_a = (1+\rho)^{-m}$. Even a small level of short-term impatience may translate into a potentially extreme degree of impatience once compounded over long periods of time. For instance, a tiny discount rate of $\rho = 0.1$ percent over one day leads to a strong annualized discount rate of 44 percent. Such value seems incompatible with individuals engaging in profitable long-term investments like saving for their long term standard of living. More reasonable long-term impatience is consistent with short-term impatience once a bias for the present is accounted for. This can be done with the two-parameter $(\beta - \delta)$ model of Laibson (1997) where future utility is discounted exponentially $(d(t) = (1+\rho)^{-t})$ and an extra weight $d(0) = 1/\beta > 1$ applies to present utility.

Although short-term impatience may reveal a bias toward the present, it is not a clear-cut criterion. In particular, the issue of what constitutes excessive impatience in a quantitative sense is not immune from judgment. O'Donoghue and Rabin (2015) claim that any noticeable short-term discounting is evidence of present bias. It may be true for very short delays like a day or a week, but is less compelling for medium delays like a month. Also, the boundary between excessive and plausible short-term impatience is a moving one, as it depends on the time discounting model and the functional form at hand. For example, if exponential discounting is replaced by constant absolute decreasing impatience discounting (Bleichrodt, Rohde and Wakker, 2009): $d(t) = exp(exp(-ct) - 1)$,

the previous calibration exercise yields annualized long-term discount rates of 36, 30 and 18 percent at the horizons of 1, 2 and 5 years respectively, compared to the fixed annual discount rate of 44 percent in the exponential model. They are still high numbers, but less so than in the exponential case.

To sum up, short-term impatience, decreasing impatience and choice reversal are all intuitively related to the notion of present bias. The link is however informally established. The next subsection proposes a different approach based on a novel definition of present bias.

2.2 Definition

Present bias is defined by including in the time set an additional period of duration μ distinct from the present, but asymptotically close to it. The DM is present biased if her preference satisfies Axiom 1.

Axiom 1 *(present bias)* $(x, 0) \succ (x, 0^+)$, $\forall x \in X$.

with the short notation $0^+ = \lim_{\mu \to 0^+} \mu$. A present biased DM values more an immediate outcome than one postponed in the near future, where the near future can be as soon as desired. The definition applies both to positive and negative outcomes. The Axiom stresses the specificity of the present, a point on a timeline, and is consistent with the claim that present bias "is about now" (O'Donoghue and Rabin, 2015). It is also close to the description of Volpp and Loewenstein (2015) as the "tendency to over-weight immediate costs and benefits relative to those occurring at any point in the future". It does not necessarily imply a strong appeal toward the immediate outcome relative to a postponed outcome. The important ingredient is the temporal discontinuity between the present and the future.

Future bias can be symmetrically defined as a preference for the late outcome in the near future, a special case of patience.

Axiom 2 *(future bias)* $(x, 0^+) \succ (x, 0)$, $\forall x \in X$.

Future bias may occur when the DM faces negatively valued outcomes like unpleasant tasks, or when individuals derive a positive utility of anticipating a pleasant consumption experience (Loewenstein, 1987, Shu and Gneezy, 2010). The DM may also be willing to postpone a positive outcome to create an improving sequence (Loewenstein and Prelec, 1991). The expression future bias has been used in the literature with a different. It stands either for increasing impatience (Attema et al., 2010) or choice reversals toward the late outcome (Read, Frederick and Airoldi, 2012; Sayman and Öncüler, 2009; Takeuchi, 2011).

Last, a DM is present neutral if she is time neutral around the present moment.

Axiom 3 *(present neutrality)* $(x, 0) \sim (x, 0^+)$, $\forall x \in X$.

All individuals are characterized by one of the three attitudes toward immediate outcomes. They can be used to classify usual models of discounting as present biased or time neutral, and to provide axiomatic foundations to present biased models.

3 Implications

3.1 Utility representation

Present bias is defined in a narrow sense by focusing on some minimal necessary ingredients. Consequently, it does not impose a complete preference ordering of

intertemporal plans. One may wonder how the definition fits with full-fledged models of intertemporal choices. We have first to express Axiom 1 as a restriction on discounted utilities. Let $u(x,t)$ be the utility of x in t periods. The DM is present biased if

$$u(x,0) > u(x,0^+) \tag{1}$$

where $u(x,0^+)$ stands for $\lim_{\mu\to 0^+} u(x,\mu)$. The DM is future biased if the inequality is reversed, and present neutral if the condition is an equality.

All discounting models which do not give an extra weight to the immediate outcome are present neutral. To illustrate, the exponential discount function (Samuelson, 1937) $d(t) = (1+\rho)^{-t}$ with $\rho \in (0,1)$ a subjective discount rate defined over a unit period of time, is present neutral:

$$u(x,0) - u(x,0^+) = u(x)\Big(1 - \lim_{\mu\to 0^+}(1+\mu\rho)^{-1}\Big) = 0$$

While the result is well known in the literature, the interpretation is novel. Exponential discounting is present neutral because it is continuous in the neighborhood of the present. The same analysis applies to the generalized model of hyperbolic discounting of Loewenstein and Prelec (1992), which nests as special cases the models of Harvey (1986) and Mazur (1987).[1] Although those models satisfy the property of decreasing impatience, they are present neutral since the discounting functions are smooth around the present. Discount functions in Bleichrodt, Rohde and Wakker (2009) and Ebert and Prelec (2007) are also present neutral for the same reason.

The discontinuity between the present and the future in (1) can be modeled in several manners. It can take an additive form:

$$u(x,0^+) = u(x,0) - \tau(x) \tag{2}$$

[1]The discounting function of Loewenstein and Prelec (1992) is $d(t) = (1+ht)^{-r/h}$ with $h \geq 0$ and $r > 0$. The model nests two special cases: proportional discounting (Mazur, 1987) with $h = r$ and power discounting (Harvey, 1986) with $h = 1$.

with $0 < \tau(x) < u(x,0)$, $\forall x \in X$, or a multiplicative form:

$$u(x,0^+) = \beta(x)u(x,0) \tag{3}$$

with $0 < \beta(x) < 1$, $\forall x \in X$. The higher $\tau(x)$ or the lower $\beta(x)$, the stronger the bias toward present utility. If discounted utility is time-separable: $u(x,t) = d(t)u(x)$, and given the normalization $d(0^+) = 1$, the weight $d(0)$ on present utility can be interpreted as a present bias parameter. With an additive form, it is

$$d(0) = \frac{u(x)}{\tau(x)}$$

and with a multiplicative form:

$$d(0) = \frac{1}{\beta}$$

A weight above 1 reflects a bias toward the present and below 1 toward the future. The additive formulation is a key ingredient in Benhabib, Bisin, and Schotter (2010) whose discounting model is $u(x,0) = u(x)$ and $u(x,t) = \exp\{-rt\}u(x) - \tau$, $t > 0$. A bias toward the present is generated by a fixed utility cost $\tau > 0$ interpreted as the "psychological restraint from the impulse of choosing the immediate reward". The model has a jump discontinuity at date 0:

$$u(x,0) - u(x,0^+) = u(x) - \lim_{\mu \to 0^+} \exp\{-r\mu\}u(x) + \tau = \tau > 0$$

which makes preference present biased according to Condition (1).

The multiplicative formulation is exemplified by the quasi-hyperbolic model of Laibson (1997). Long-term impatience is driven by an exponential discounting function: $d(t) = (1+\rho)^{-t}$, $t > 0$. Short-term impatience is affected by an extra weight on present utility $d(0) = 1/\beta > 1$. The discounting function is also present biased according to Condition (1):

$$u(x,0) - u(x,0^+) = u(x)\Big(1/\beta - \lim_{\mu \to 0^+} (1+\mu\rho)^{-1}\Big) = (1/\beta - 1)u(x) > 0$$

with $1/\beta - 1$ a measure of the bias.

3.2 Axiomatic inclusion

Axiom 1 of present bias and Axiom 2 of future bias can mix with consistent sets of axioms, changing present neutral preferences into present biased or future biased preferences.[2] Any axiom-based preference relations become present biased once Axiom 1 is added to the set of axioms:

Proposition 1 *Let us consider a preference relation* $(\succ_A, \sim_A)$ *over* $X \times \{0, 1, ..., \bar{t}\}$, *complete, transitive, present neutral, and satisfying a set* A *of axioms.*

1. *There exists a preference relation* $(\succ_B, \sim_B)$, *complete and transitive, over* $X \times \{0, 0^+, 1, ..., \bar{t}\}$ *satisfying all axioms in* A *over* $X \times \{0^+, 1, ..., \bar{t}\}$ *and Axiom 1 over* $X \times \{0, 0^+\}$.

2. $(x, t) \sim_A (y, s)$ $\forall x, y \in X$ *and* $t, s > 0 \Rightarrow (x, t) \sim_B (y, s)$

3. $(x, 0) \sim_A (y, s)$ $\forall x, y \in X$ *and* $s > 0 \Rightarrow (x, 0) \succ_B (y, s)$

Proof 1. Since $(\succ_A, \sim_A)$ is complete and transitive over $X \times \{0, 1, ..., \bar{t}\}$, so it is over the modified time set $\{0^+, 1, ..., \bar{t}\}$, where immediate outcomes are replaced by asymptotically immediate outcomes. Present neutrality preserves all orderings: $(x, 0) \sim (y, s) \iff (x, 0^+) \sim (y, s)$. The relation $(\succ_B, \sim_B)$ is complete over $X \times \{0, 0^+, 1, ..., \bar{t}\}$ as $(\succ_A, \sim_A)$ is complete over $X \times \{0^+, 1, ..., \bar{t}\}$ and Axiom 1 implies $(x, 0) \succ_B (x, 0^+)$ and therefore $(x, 0) \succ_B (x, s)$ $\forall s > 0$. It is transitive over $X \times \{0, 0^+, 1, ..., \bar{t}\}$ since $(\succ_A, \sim_A)$ is transitive over $X \times \{0^+, 1, ..., \bar{t}\}$ and $(x, 0) \succ_B (x, 0^+)$ plus $(x, 0^+) \succ_B (x, s)$, $\forall s > 0$, implies $(x, 0) \succ_B (x, s)$.

2. Straightforward since $(\succ_B, \sim_B)$ satisfies all axioms in A over $\{0^+, 1, ..., \bar{t}\}$.

[2] The analysis focuses on present bias, but a symmetric reasoning would apply to the case of future bias.

3. $(x,0) \sim_A (y,s)$ over $X \times \{0,1,...,\bar{t}\}$ implies $(x,0^+) \sim_A (y,s)$ over $X \times \{0^+,1,...,\bar{t}\}$ due to present neutrality. Since $(\succ_B, \sim_B)$ ranks dated outcomes over $X \times \{0^+,1,...,\bar{t}\}$ the same way than $(\succ_A, \sim_A)$, this implies $(x,0^+) \sim_B (y,s)$ over the same domain. Axiom 1 implies in turn $(x,0) \succ_B (y,s)$. □

The relation $(\succ_B, \sim_B)$ replicates the ordering of $(\succ_A, \sim_A)$ except when one of the two compared outcomes is obtained immediately. Since a present neutral DM identically values an immediate outcome and an asymptotically immediate one, the axioms in A equivalently apply on a modified time set where "now" is replaced by "next moment". Present outcomes are then ordered relative to other dated outcomes thanks to Axiom 1.

Proposition 1 implies that any axiom-based preference relations can display present bias once Axiom 1 is included. Furthermore, if all future outcomes are identically valued under the relations $(\succ_A, \sim_A)$ and $(\succ_B, \sim_B)$, then a present outcome is more valued with Axiom 1 than without:

Proposition 2 *Let us assume that the complete, transitive and present neutral preference relation $(\succ_A, \sim_A)$ over $X \times \{0,1,...,\bar{t}\}$ is represented by a continuous real-valued function $u(x,t)$ such that $(x,t) \succ_A (y,s) \Leftrightarrow u(x,t) > u(y,s)$. There exists a continuous real-valued function $v(x,t)$ on $X \times \{0,0^+,1,...,\bar{t}\}$ representing the relation $(\succ_B, \sim_B)$ and characterized by:*

1. $v(x,t) = u(x,t)$, $\forall x \in X$ *and* $t \in \{0^+,1,...,\bar{t}\}$,
2. $v(x,0) > u(x,0)$ $\forall x \in X$.

Proof 1. Straightforward, since the preference relation $(\succ_B, \sim_B)$ ranks dated outcomes over $T - \{0\} + \{0^+\}$ the same way than $(\succ_A, \sim_A)$.

2. $u(x, 0^+) = v(x, 0^+)\ \forall x \in X$ (claim 1). Since $u(x, 0^+) = u(x, 0)$ due to present neutrality and $v(x, 0^+) < v(x, 0)$ due to present bias, claim *2.* obtains. □

Present bias can be interpreted as a psychological drive that weakly interacts with other behavioral properties like decreasing impatience. Its ability to mix with other axioms makes possible a simple and transparent axiomatization of present biased preferences. As an illustration, let us start from the set of axioms posed in Fishburn and Rubinstein (1982) which utility representation is consistent with exponential discounting. They demonstrate that a set of axioms (weak ordering, monotonicity, continuity and stationarity) guarantees the existence of a real scalar δ such that the ordering can be represented by the function $u(x, t) = \delta^t u(x)$ up to a positive and multiplicative factor. Proposition 2 can be applied on top of this result. The time set is expanded by the inclusion of the date 0^+. The axioms of Fishburn and Rubinstein and the resulting present neutral functional form u hold for the modified time set $\{0^+, 1, 2, ..., \bar{t}\}$. The addition of Axiom 1 in the set of axioms changes utility u from present neutral to present biased: $v(x, t) = u(x, t) = \delta^t u(x)\ \forall t \in \{0^+, 1, 2, ..., \bar{t}\}$ and $v(x, 0) > v(x, 0^+) = u(x)$. $v(x, 0)$ can then take the additive formulation (2) or the multiplicative formulation (3).

3.3 Behavioral implications

The definition of present bias allows a sharp distinction with several behavioral properties often interpreted as a unique phenomenon in the literature: decreasing impatience, choice reversal and short-term impatience.

We saw in Proposition 1 that any consistent set of axioms defining a weak order are compatible with Axiom 1, once defined over the expanded time set $\{0, 0^+, 1, 2, ..., \bar{t}\}$. Those preferences may be characterized either by decreasing,

constant or increasing impatience, regardless of the presence of present or future bias over the time set $\{0, 0^+\}$. Experimental studies which test whether subjects are increasingly or decreasingly impatient do not test present bias defined by Axiom 1.

Present bias is related to the definition of decreasing impatience in the following way. Let us pose a modified definition of decreasing impatience:

Definition 3 *(decreasing impatience)* $\forall\, x, y \in X$, $\forall\, t \in T$, *such that* $(x, 0) \sim (y, t)$, *impatience is decreasing if* $(x, \Delta) \prec (y, t + \Delta)$, $\forall \Delta > 0$.

The definition is the same as Definition 1 except that the early date is the present and the delay Δ may take any real positive values instead of only discrete time intervals.

Proposition 3 *Definition 3 of decreasing impatience implies Axiom 1 of present bias.*

Proof Definition 3 holds for Δ as small as desired: $(x, 0^+) \prec (y, t^+)$. If $(y, t^+) \sim (y, t)$ or $(y, t^+) \prec (y, t)$, then $(x, 0^+) \prec (y, t) \sim (x, 0)$. □

A preference relation is present biased according to Axiom 1 if it is decreasingly impatient according to Definition 3. The proof relies on the exclusion of future bias for delayed trade-offs. Preferences can be either "future present neutral" ($(y, t) \sim (y, t^+)$, $\forall\, t > 0$) or "future present biased" ($(y, t) \succ (y, t^+)$, $\forall\, t > 0$), but cannot be "future future biased" ($(y, t) \prec (y, t^+)$, $\forall\, t > 0$). Conversely, present bias does not imply decreasing impatience since the delay Δ may span larger intervals of time. In this sense, the definition of present bias is weaker than the definition of decreasing impatience.

Likewise, present bias does not necessarily imply choice reversal toward the immediate outcome, even if preferences are stationary. Following Definition 2

of choice reversal, suppose the outcomes (x,t) and (y,s), $t<s$, are equivalent from date 0 perspective, or $u(x,t)=u(y,s)$. t periods later, when the early outcome is available now, the DM choice is tilted toward the immediate option if $u(x,0)-u(y,s-t)>0$ or if:

$$\Big(u(x,0)-u(x,0^+)\Big)+\Big(u(x,0^+)-u(y,s-t)\Big)>0$$

The choice is reversed if the DM preferences satisfy both present bias (the first difference is positive) and decreasing impatience (the second difference is positive). Yet, if time preferences are characterized by increasing impatience (the second difference is negative), present bias could well be associated with preference reversal toward the late outcome. A corollary is that the definition of present bias is insensitive to the delicate issue of whether individuals choose naively (they do not foresee their self control problems) or in a more sophisticated way (they foresee them, at least partially).

O'Donoghue and Rabin (2006, 2015) define present bias as short-term impatience where "short" means any durations which, which once exponentially compounded over longer time intervals imply implausible impatience. Present bias defined here as impatience over vanishingly small time intervals, is close in spirit to theirs with however notable differences: it can be formally stated, defines future bias symmetrically, does not rely on model-dependent consequences for long-run impatience, and allows to sort out truly present biased discount functions from others ones.

4 Conclusion

How concepts are defined influences the way researchers frame their reasoning and build new knowledge. Sound and operational definitions facilitate scientific progress and open new directions for research. This article argues that a focused

definition of present bias is missing in the literature. The concept is either confounded with its behavioral implications, like decreasing impatience or choice reversal, or identified to functional forms, principally the $(\beta-\delta)$ model of Laibson (1997).

The definition of present bias, a strict preference for immediate outcomes over ones postponed in the arbitrarily near future, has affinities with, yet is distinct from behaviors previously associated with the concept. Individual may be present biased and not decreasingly impatient with regards to immediate or future trade-offs. As time elapses, they may not necessarily reverse their choice toward the immediate outcome, even if their preferences are stationary. The definition takes a simple axiomatic form and discriminates between models commonly associated with present bias. The $(\beta - \delta)$ model of Laibson (1997) and the fixed cost model of Benhabib, Bisin, and Schotter (2010) satisfy the definition, while the generalized model of hyperbolic discounting of Loewenstein and Prelec (1992) or the discount functions of Bleichrodt, Rohde and Wakker (2009) and Ebert and Prelec (2007) do not.

The paper proposes an intuitive axiomatic framework in which the elicitation of present bias is agnostic about preferences over delayed trade-offs, and as a result not tied to a specific functional form. The definition suggests new ways of measuring present bias. Most experiments estimate a $(\beta-\delta)$ model by collecting data on multiple types of choices, some involving trade-offs between immediate utility and future utility, and others involving trade-offs between future utilities at different dates. A simpler measure consistent with the present definition would elicit the discount parameter over short periods of time, an empirical strategy recently followed by Augenblick (2018) for unpleasant tasks.

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