Solution to the cosmological constant problem from dark energy of the quark vacuum

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Abstract

We solve the cosmological constant puzzle by attributing its origin to gravitational self-energy of quark vacuum quantum fluctuations and to Mach-Einstein's principle of the relativity of all inertia. The Dirac sea of negative energy quarks and its fluctuations, which should be subjected to the expansion of the universe, is actually frozen because of quark-antiquark pair creation from the QCD confinement energy, which is well described by a string potential. When the members of a pair separate beyond a critical distance, the string breaks and a new pair is created, so that the density remains constant under adiabatic expansion, as required for a Lorentz invariant vacuum. We argue that the energy scale at which the vacuum no longer follows the expansion is therefore given by the effective mass of quarks in the neutral pion. This yields a predicted value of the reduced cosmological constant $\Omega_\Lambda h^2(\text{pred}) = 0.31152 \pm 0.00006$, which is 100 times more precise while in excellent agreement with its observational value recently determined by the 2018 *Planck* mission reanalysis, $\Omega_\Lambda h^2(\text{obs}) = 0.3153 \pm 0.0065$.

Introduction. The cosmological constant has been introduced in gravitational field equations by Einstein in 1917 in order to satisfy to Mach's principle of the relativity of inertia [1]. Then it was demonstrated by Cartan in 1922 [2] that the Einstein field tensor including a cosmological constant Λ, $E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}$, is the most general tensor in Riemannian geometry having null divergence like the energy momentum tensor $T_{\mu\nu}$. This theorem has set the general form of Einstein's gravitational field equations as $E_{\mu\nu} = \kappa T_{\mu\nu}$ and established from first principles the existence of Λ as an unvarying true constant.

The cosmological constant problem dates back to the realisation that it is equivalent to a vacuum energy density [3, 4], while theoretical expectations of this "dark energy" from modern theories of elementary particles seem to exceed its observational value by some 120 orders of magnitude.

One of the main consequences in cosmology of a positive cosmological constant is an acceleration of the expansion of the universe. Such an acceleration has been first detected in 1981 in the Hubble diagram of infrared elliptical galaxies (which already reached at that time redshifts $z \approx 1$), yielding a positive value close to the presently measured one, but with still large uncertainties [5]. Accurate measurements of the acceleration of the expansion since 20 years [6, 7, 8, 9] have reinforced the problem. Indeed, all analyses so far support that the "dark energy" which drives this acceleration owns every properties of a cosmological constant: no variation of its value is needed to account for the precision observations, suggesting that it is a true constant, and the coefficient of its state equation is $w = -1$ within uncertainties, as expected for a Lorentz invariant vacuum that is equivalent to a cosmological constant (the 2018 reanalysis of *Planck* mission [11] obtains $w = -1.028 \pm 0.032$, and a flat universe $\Omega_k = 0.0007 \pm 0.0019$ to high accuracy).

We show in this paper that the problem is solved both in principle and quantitatively by attributing the emergence of a cosmological constant to the effect of quark confinement on the self-gravitational energy of quantum fluctuations. Through evaluating this energy from the quark effective mass in the neutral pion (which is a material implementation of this process), we predict a theoretical value $(\Omega_\Lambda h^2)_{\rm pred} = 0.31152 \pm 0.00006$. As early as 2004 the combination of WMAP and SDSS data yielded a best fit value close to this prediction, $(\Omega_\Lambda h^2)_{\rm obs} = 0.317 \pm 0.038$ [10]. The *WMAP* nine years mission has obtained $(\Omega_\Lambda h^2)_{\rm obs} = 0.341 \pm 0.012$ [8]. More recently, the *Planck* mission has yielded $(\Omega_\Lambda h^2)_{\rm obs} = 0.318 \pm 0.011$ [9], still improved by a factor of 2 in its 2018 re-analysis to $(\Omega_\Lambda h^2)_{\rm obs} = 0.315 \pm 0.006$ [11], which agrees remarkably well with the theoretical prediction.

Elements of the problem. As remarked by Weinberg [12], anything that contributes to the energy density of the vacuum acts just like a cosmological constant. Therefore the problem amounts not only to find which contribution yields the observed value and why it does, but also to understand why and how the other particles / fields do not contribute.

The vacuum energy varies with the length scale r as r^{-1}, and therefore the vacuum energy density as r^{-4}. Relating it to the Planck scale, one obtains $\rho_V(r) = \rho_{\mathbb{P}} \times (r_{\mathbb{P}}/r)^4$. With today's precisely measured value of the cosmological constant [11], one obtains a factor 3.4×10^{121} from the Planck scale contribution, but even electron-positron virtual pairs provide a far too high contribution, by a factor 1.07×10^{32}.

The cosmological constant Λ, as it appears in Einstein's equations, is a curvature (it is expressed in units of m^{-2}). As such, besides being an energy density, it is also the inverse of the square of an invariant cosmic length $\mathbb{L}$ [14],

$$\Lambda = \frac{8\pi G \rho_V}{c^2} = \frac{1}{\mathbb{L}^2}. \tag{1}$$

The recently measured value of Λ, derived from the best *Planck* solution (reduced cosmological constant $\Omega_\Lambda = 0.6889 \pm 0.0056$ and reduced Hubble constant $h = 0.6766 \pm 0.0042$), yields a cosmic length-scale $\mathbb{L} = (3.08 \pm 0.03)$ Gpc. Its ratio with the Planck scale is an invariant pure number, $\mathbb{K} = \mathbb{L}/r_{\mathbb{P}} = (5.878 \pm 0.060) \times 10^{60}$ (whose square just yields the 10^{121} discrepancy). Note that, while the status of the cosmological constant as an energy density has been often emphasized, its complementary (and original) status as an invariant cosmic length has been underestimated. The solution proposed here is just based on this double meaning [14, 15].

A vacuum is a perfect fluid that must be Lorentz invariant, and therefore it has the equation of state $\rho_V + p_V/c^2 = 0$, where ρ_V is its density and p_V its pressure. In Einstein's field equations, such a fluid contributes to the stress-energy tensor $T_{\mu\nu} = (\rho + p/c^2)u_\mu u_\nu + p g_{\mu\nu}$ by a mere negative pressure term $p_V\, g_{\mu\nu} = -\rho_V c^2\, g_{\mu\nu}$ which accelerates the expansion.

Another essential property of such a vacuum, as emphasized by Carroll et al. [16], is that its density must remain constant if a volume of this fluid is adiabatically compressed or expanded: a work $p_V dV$ provides exactly the amount of energy to fill the new volume dV to the same density ρ_V. Therefore the energy density remains a true constant, as required for its identification with a general relativistic cosmological constant. As we shall see, this property plays a central role in the solution proposed here.

Gravitational self-energy density of quantum fluctuations. A first step toward a solution to this problem, which has been pointed out by many authors [4, 12, 16], is that energy is defined only up to a constant and plays a role in physics only by its differences, i.e., it can always be renormalized as $<E>= 0$. Therefore the vacuum energy density by itself has no reason to contribute as a source for the gravitational field. Arrived at that point, the cosmological problem is reversed, since the theory now predicts $\rho_V = 0$, as was supported by Hawking's quantum gravity argument [13], while observations instead tell us that it is the dominant energy contribution in the universe.

A second step has been made by Zeldovich [4], who remarked that, although the energy density can be taken identically equal to zero, the energy of the gravitational interaction of the virtual particle pairs contained in the vacuum cannot vanish because of Heisenberg's inequality. A quantum fluctuation $\pm\Delta E$ of the vacuum at length scale r allows the appearance of a pair of virtual particles of mass $m = \Delta E/c^2 \propto 1/r$. The gravitational potential self-energy of this pair is $\phi_G = -Gm^2/r = -G <\Delta E^2> /c^4 r$, so that the corresponding energy density ϕ_G/r^3 is proportional to r^{-6}. Relating it once again to the Planck scale, one obtains

$$\rho_G = -\rho_{\mathbb{P}} \times \left(\frac{r_{\mathbb{P}}}{r}\right)^6 . \tag{2}$$

The main point here is that, while the mean energy of a fluctuation can be renormalized ($< E >= 0$) as already pointed out, its square $<E^2>$ cannot vanish according to Heisenberg's inequality.

Arrived at that point, two problems arise : (1) the sign of this expression is the opposite of the observed energy density; (2) there is no reason for a *gravitational* potential energy to become a source for gravitation. We suggest hereafter a solution to both problems.

Consequences of Mach-Einstein's principle. The 'postulate of relativity of all inertia', which has been named by Einstein 'Mach principle' [1] allows one to solve this puzzle. Concerning matter, it has been expressed by Sciama [17] in a very simple way. According to this principle, the total energy of any body of mass m should be zero (i.e., ultimately, any body should be free). In the rest frame of the body, it is the sum of its mass energy and of the potential energy of its gravitational coupling with all the other bodies of the universe, i.e., $m_I c^2 + (-G m_p \sum_i m_i/r_i) = 0$. In agreement with the weak equivalence principle, the inertial mass and the passive gravitational mass of the body are equal, $m_I = m_p$, and we are therefore left with a black hole-like relation for the whole universe,

$$\frac{G}{c^2}\,\frac{M_U}{R_U} = 1. \tag{3}$$

Recall that Einstein's goal when introducing the cosmological constant was precisely to implement such a relation $M/R = \text{cst}$ for the Universe, in order to render general relativity 'Machian' [1]. Since non static models of the universe have $M_U \propto \rho\, a^3 = \text{cst}$ but $R \approx c/H$ variable with time (where H is the Hubble constant), Einstein suggested that the universe should be static with $R = 1/\sqrt{\Lambda} = c/H$, a solution which has been subsequently invalidated by the discovery of the expansion of the universe.

However, one can make nowadays an important remark. The very introduction of a cosmological true constant $\Lambda = 1/\mathbb{L}^2$, which is now supported by astronomical observations, introduces an *invariant* length-scale $\mathbb{L}$ at the cosmological scales (which has no reason to be identified with the time-varying $c/H(t)$) even in non static model (it applies to the whole general relativity theory). Therefore the problem of having a 'Machian' universe is actually solved in principle for *any* cosmological model, since there exists a prime integral $M_U = \text{cst}$ for each model, so that a relation $M_U/R_U = \text{cst}$ is implemented in general with $R_U = \mathbb{L}$.

The new element in the present work consists of now applying this Mach-Einstein-Sciama principle, not only to real matter, but also to virtual particles, i.e. to the vacuum itself. In the same way as the total energy of matter should vanish, the total energy of the vacuum is also expected to be zero. Therefore, since we have found that a negative gravitational self-energy density $\rho_G = -\rho_{\mathbb{P}} \times (r_{\mathbb{P}}/r)^6$ must be created by quantum fluctuations of the vacuum, we expect that it must be cancelled by a *positive* energy density ρ_V such that $\rho_V + \rho_G = 0$. And in the same way as the rest energies mc^2 of material bodies are sources of gravitation, we also expect this positive energy density,

$$\rho_V = \rho_{\mathbb{P}} \times \left(\frac{r_{\mathbb{P}}}{r}\right)^6, \tag{4}$$

to be a source for gravitation and to enter the stress-energy tensor in Einstein's field equations.

Cosmological constant as a transition scale. In order for the above formula to yield the correct numerical value of the cosmological constant $\Lambda = r_{\mathbb{P}}^{-2}(\rho_V/\rho_{\mathbb{P}})$, quantum vacuum fluctuations must have been frozen at a transition length scale r_0 given by $\rho_V = \rho_{\mathbb{P}} \times (r_{\mathbb{P}}/r_0)^6$. Introducing again the cosmic length $\mathbb{L} = 1/\sqrt{\Lambda}$, it yields the simple 'large number' relation $r_0^3 = r_{\mathbb{P}}^2\,\mathbb{L}$ [14, p. 303].

This result is remarkable, since the question of calculating the vacuum energy density $\rho_V = \Lambda c^2/8\pi G$ has been replaced by the problem of identifying the transition scale r_0 (or equivalently the corresponding mass scale $m_0 = \hbar/r_0 c$). It is no longer necessary to calculate explicitely the vacuum energy density in order to obtain the cosmological constant, since it is now given directly by a ratio of length scales:

$$\Lambda = \frac{1}{\mathbb{L}^2} = \frac{r_{\mathbb{P}}^4}{r_0^6}, \tag{5}$$

from which one can easily derive the observed parameter $\Omega_\Lambda = \Lambda c^2/3H_0^2$.

This is an important point for an explicit numerical calculation of the cosmological constant since the value of the energy density depends on numerical constants which are difficult to estimate and may change in dependence of the models.

Quark vacuum Dirac sea. We have suggested more than 25 years ago that the cosmological constant could be a relic of the quark-hadron transition and derived at that time a value $\Omega_\Lambda h^2 = 0.36$ [21], [14, p. 305], in satisfying agreement with the observational values measured after 1998 (see also Beck [22]). However, this initial result was based on the observation of the convergence of fundamental elementary particle scales around ≈ 70 MeV, the classical radius of the electron, the effective mass of quarks in the neutral and charged pion and the QCD scale for 6 quark flavors (which is today estimated to a larger value, 89 ± 6 MeV [31]). This leads to a reduced cosmological constant in the observed range $[0.31 - 0.39]$, but an explicit mechanism of transition was still lacking. It was suggested in [15, p. 547] that such a mechanism is just quark confinement, which imposes a largest possible value for the interdistance between two quarks (or a quark and an antiquark) given by the effective Compton length of quarks in the neutral pion. It is this proposed solution that we elaborate here.

Under such a view, the solution to the cosmological constant problem lies in the thermal history of the primordial Universe. It is now well known that, when the temperature of the Universe was larger than about 150 MeV, it was dominated by a plasma of quarks and gluons [23]. Because of asymptotic freedom, the quarks and gluons are almost free and deconfined at high temperature. However, when the Universe cools because of its expansion, a transition occurs at an epoch of about 10 μs, since quarks and gluons are forced to be confined as colorless hadrons at low temperature. Simulations of three-flavor QCD on a lattice have given a fairly well-defined temperature of $T_c = (154 \pm 8)$ MeV [18, 19, 20]. Just after the transition, the Universe is dominated by pions, which rapidly decay, leaving only the residual nucleons (plus electrons, positrons, neutrinos and photons).

However, besides this history of matter, we are more closely concerned here with the parallel history of the quantum vacuum and of its fluctuations. Our suggestion is that the clue to the solution of the cosmological constant problem lies in the understanding of the behavior of the vacuum under the action of the expansion of the Universe.

In order to be more specific, let us consider a Dirac sea representation of the quark vacuum. In the Dirac sea model, the vacuum is filled by all the possible quark states. A quantum fluctuation extracts a quark from the vacuum, leaving a hole which is seen as an antiquark, thus yielding a virtual quark-antiquark pair.

The structure of the QED Dirac sea has been elaborated by e.g. [24, 25]. The quark Dirac sea has been specifically addressed by Molodtsov and Zinovyev [28, 29, 30], who find that its ground state is identical to a BCS state, therefore involving a condensate of quark-antiquark pairs into bosons. They find that the quasiparticles obtained are identical to the pion and that their derived dynamical mass depends only on the coupling constant.

The question addressed now is what happens to the Dirac sea of quarks and to its fluctuations when the universe expands. The Dirac vacuum is not empty, but filled by an infinity of virtual particles which endow it with a rich structure. As for the quark Dirac sea, it is therefore a quark-gluon plasma of negative energy states.

Pair creation is usually described in terms of a momentum-energy representation, but a more complete description is possible in terms of position and momentum [24]. One can therefore consider the effect on the vacuum and on its fluctuations of the increase of the interdistance between the negative energy quarks due to expansion of the universe. If, because of quantum fluctuations, a negative energy quark from the Dirac sea jumps to positive energy, leaving a hole viewed as an antiparticle, the quark-antiquark interdistance is expected to follow the same evolution.

Concerning real particles, it is known that if one pulls on the two quarks forming a pion, the energy needed to break it is larger than the energy of creation of light $q\bar{q}$ pairs. In other words, this operation results in two pions (four confined quarks) instead of two free quarks. This confinement of quarks is well described by a linear + Coulomb-like string potential, $V(r) = \sigma r + \mu + \beta/r + ..$, which agrees with lattice results up to large r (see e.g. [26]). The dominant term $V = \sigma r$ is directly proportional to distance, and it therefore yields a constant force between the two quarks and a potentially infinite binding energy.

The string breaking has been observed in lattice simulations of QCD [27], yielding a typical string breaking distance $r_c = (1.25 \pm 0.06)$ fm for two sea quark flavors, slightly increasing to $r_c = (1.27 \pm 0.08)$ fm if a third (strange) quark is added. This corresponds to an energy of 155 MeV, i.e. precisely the QH transition temperature. Extrapolated to real QCD, this value is reduced to (1.13 ± 0.12) fm, including the uncertainty on the extrapolation and on the physical scale of reference.

Let us consider the structure of the quark vacuum and of its fluctuations around this Fermi scale. At length scales smaller than $\Delta r \approx 1.25$ fm, energy fluctuations $\Delta E > \hbar/\Delta r \approx 150$ MeV allow the creation of quark-antiquark pairs of mass $m_q \approx 70$ MeV, which is the effective mass of quarks in the pion. The new question here concerns the behavior of these virtual particle pairs under the effect of the expansion of the universe. The interdistance between the quark and the antiquark is expected to increase due to this expansion. But they should be submitted to exactly the same string potential as the real pairs in the pion. As a direct consequence, when the interdistance reaches the string-breaking value, a new virtual quark-antiquark pair is expected to appear and to contribute to the vacuum fluctuations. Contrarily to the case of real matter, where this pair manifests itself as a pion, the virtual particles remain as a quark and an antiquark, as required by the fact that they are part of the Dirac quark vacuum sea and its fluctuations. Therefore the volume density of the pairs become frozen at this scale, since the density decrease expected from the expansion of the universe is continuously compensated by particle creation from the 'confinement' energy field (which is probably just a manifestation of the 'large' scale non-perturbative QCD field, as now supported by lattice QCD simulations). The freezing of the pair density (i.e. of the scale of the vacuum fluctuations of the Dirac quark sea) finally yields a freezing of the gravitational self-energy density ρ_G, then of the dark energy $\rho_V = -\rho_G$. This is exactly the kind of process expected for implementing a vacuum energy that remains invariant under adiabatic expansion (a property, as already mentioned, explicitly required for a dark energy to be identical to a cosmological constant [16]).

Two complementary problems are solved by this proposal: (i) we understand the order of size of the cosmological constant, linked to the typical confinement scale ≈ 70 MeV; (ii) we understand why other fields do not contribute to dark energy, since the quark confinement field is the only one which owns this property of being able to cancel the expansion of the universe by continuous pair creation involving such a freezing process. The other fields are diluted as r^{-6} in the course of the universe expansion and have finally a vanishing contribution.

Predicted accurate numerical value of the cosmological constant. The last step amounts to obtain a precise theoretical determination of the numerical value of the cosmological constant from this process. A difficulty could be the uncertainty on the precise numerical constants which intervene in it. But this difficulty is actually shunted by the use of the Planck scale as reference scale. Only scale ratios intervene in this calculation, not the dimensioned scales themselves, and we just have to check that the same definition is used for the reference (Planck) scale and for the transition scale. Defining the Planck length scale as $r_{\mathbb{P}} = \sqrt{\hbar G/c^3}$ means that it is defined as the 'Compton length' of a Planck mass (instead of its *wave* length based on h), and therefore that it should be related to the transition scale r_0 also defined as a Compton length, $r_0 = \hbar/m_0 c$.

Arrived at this point, we postulate a full similarity between the real (matter) particle properties and the virtual (vacuum fluctuation) particles, as supported by experimental data and by QED and QCD theoretical calculations. We assume that the lightest meson, the neutral pion of mass m_{π^0}, is a matter manifestation of the largest possible volume density of quark-antiquark pairs (the charged pions being excluded because they contain an electromagnetic contribution to their mass-energy), and that it is reflected in the vacuum and its fluctuations with the same property. The Compton length corresponding to the individual quarks in the pion is therefore $r_0 = 2\hbar/m_{\pi^0}c$.

With this value, the transition scale is $r_0 = (2.92386 \pm 0.00001)$ fm from $m_q = m_{\pi^0}/2 = 67.48850 \pm 0.00025$ MeV [31], and one obtains (assuming no other contribution)

$$\Omega_\Lambda h^2(\text{pred}) = 0.311524 \pm 0.000064. \tag{6}$$

This expectation is in excellent agreement with the observational value determined by the *Planck* mission, $\Omega_\Lambda h^2(\text{obs}) = 0.318 \pm 0.011$ [9], improved by a recent re-analysis [11] to

$$\Omega_\Lambda h^2(\text{obs}) = 0.3154 \pm 0.0065, \tag{7}$$

from their best fits $\Omega_\Lambda = 0.6889(56)$ and $h = 0.6766(42)$. The theoretical prediction being still ≈ 100 times more precise than the observational measurement, its validity will be testable in possible future observational improvements.

The value of the corresponding theoretically predicted cosmic length-scale $\mathbb{L} = \Lambda^{-1/2}$ is

$$\mathbb{L} = (3.10109 \pm 0.00032) \text{ Gpc}, \tag{8}$$

to be compared to the observational *Planck* 2018 value $\mathbb{L} = (3.08 \pm 0.03)$ Gpc. The vacuum dark energy density of the cosmological constant is

$$\rho_v = \frac{\Lambda c^2}{8\pi G} \approx 5.91 \times 10^{-27} \text{ kg.m}^{-3}, \tag{9}$$

i.e. ≈ 49 quarks by m^3 (taking an effective mass $m_q = m_{\pi 0}/2$ for quarks). The density compensation of the expansion by quark energy creation from QCD field is given by $\delta\rho/\rho_v = 3H(t)\delta t$, corresponding to one quark by m^3 every 100 million years at the present epoch.

Toward a full first principle theoretical solution. The above predicted value of the cosmological constant is not fully theoretical since it remains derived from the experimentally measured mass of the neutral pion and on the hypothesis that the pion, which is the lightest two quark meson, is a strict equivalent at the level of matter of the quark vacuum behavior. This assumption is supported by the fact that only a small fraction of its mass (5%) comes from the constituent quarks. Most of its mass finds its origin in spontaneous chiral symmetry breaking of the QCD Lagrangian by the quark condensate induced by non perturbative strong interactions, i.e., therefore, ultimately from QCD vacuum energy density. The Goldstone bosons resulting from this process are just the three neutral and charged pions (in terms of the two lightest quarks, u and d) [31].

The resulting neutral pion mass is given to lowest order by the relation $m_{\pi^0}^2 = B(m_u + m_d)$, where B is an unknown constant, which applies also in a consistent way to similar relations for the other Goldstone bosons (K and η) derived from three light quarks u, d, and s [31].

This has opened the possibility of predicting the pion mass from fundamental principles. Recent lattice non perturbative QCD calculations have brought this goal closer: the mass of the proton and of other light hadrons has been derived from that of the pion [32]. The mass of the pion has been extracted to 10% accuracy by Molodtsov et aL. [28] and more recently a pion mass differing only by less than 3% of its physical mass has been reversely derived in a self-consistent way with the proton mass as input [33]. A new step has been taken by the recent decomposition of the proton mass [34], showing that $\approx 70\%$ of its mass comes from quark energy and glue field energy, and by the now most precise derivation of quark masses from lattice QCD [35]. Molodtsov and Zinovjev [29] underlign the remarkable fact that the quasiparticle size they obtain in their BCS-like model of the quark Dirac sea is close to the pion size and does not depend on the scale but on the mere coupling constant.

The proton mass result is naturally extended to the pion mass, so that we can conclude that most of the effective mass of quarks in the neutral pion comes from the quark and gluon field *energy*, not from their rest mass, and that it fairly represents the limit vacuum energy which gives rise to the cosmological constant. More profoundly, one can expect that the quark masses themselves find their origin in gauge fields energy. One can hope that, in a near future, a full theoretical solution of the cosmological problem be obtained from non perturbative lattice QCD, although the uncertainty on such a future possible achievement should reach 0.5 MeV on the theoretical pion mass in order to attain the present observational error on the measurement of the cosmological constant.

Conclusion and prospect. Let us sum up the various results obtained in the present work and some of their implications.

(1) With the proposed solution of the cosmological constant problem from gravitational self-energy of the quark vacuum, there is no longer any contradiction between an explanation of the expansion acceleration in terms of cosmological constant or of dark energy: they become the same.

(2) The $\sim 10^{120}$ discrepancy is solved by accounting for the explicit scale dependence of the vacuum gravitational self-energy, which varies as r^{-6}: there is no reason to attribute at cosmological scales the value obtained at the Planck scale. This well-known factor 10^{120} is just the result of the cosmological constant being the inverse of the square of a length, while the ratio of this cosmic length to the Planck scale is $\sim 10^{60}$.

(3) While the energies of the various quantum fields remain too large to make the

observed dark energy since they vary as r^{-4} (or can be naturally normalized to $< E >= 0$), the gravitational self-energy of their fluctuations both has the correct order of size thanks to its r^{-6} variation and cannot be made vanishing because of Heisenberg's relation.

(3) The Eddington-Dirac large number coincidence is no longer an approximate assumption but becomes a precise exact and well understood relation between the neutral pion scale and the cosmological constant length-scale. The $\sim 10^{120}$ factor is just the ratio of Planck mass over pion mass, $\sim 10^{20}$, to the above power six. This is achieved without variation of constant, which is no longer needed.

(4) Mach-Einstein's principle (i.e., the principle of the relativity of all inertia) is implemented, not only for a specific cosmological model, but at the fundamental level of the general relativity theory (as was initially hoped by Einstein in his foundation of the theory), both for matter and for the quantum vacuum.

(5) The process of continuous extraction of energy from the confinement (i.e. probably QCD) field compensates in real time the cooling of the universe due to expansion. It ressembles, but now applied to the vacuum, Hoyle's model of continuous creation leading to a stationary universe (but here, only the dark energy remains stationary, while the universe is expanding).

(6) This process is still at work now, although it has been established first in the primordial universe, at the end of the quark-hadron transition. The cosmological constant therefore appeared at this epoch as a relic of the previous hot phase, in a way similar to the transitions giving rise to the emergence of nucleons, then nuclei, then light atoms and the CMB radiation at the (re)-combination epoch.

(7) Finally, the fact that an apparently purely general relativistic and then physical "object" of gravitational nature be made from the QCD field points toward an underlying profound unity of field theories in physics that remains to be unveiled, not only at the Planck energy scale where the quantum, gravitational and gauge field effects are expected to become of the same order, but here, now and in a fundamental way.

It is fascinating that the length-scale of the cosmological constant, which is at some level a largest distance and horizon for our universe, be connected to (and derived from) the largest possible distance between quarks in the QCD inner "universe" of hadrons. These two interconnected distances play a similar role, one for gravitation and the other for QCD, while the largest one finds its origin in a continuous extraction of the quasi-infinite QCD confinement energy.

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