Structural integrity of Notre Dame Cathedral after the fire of April 15th, 2019

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Abstract

We consider the consequences of the fire destructions, happened on April 15th, 2019, on the static regime of the Cathedral Notre Dame of Paris. In particular, we ponder the effects of the wind on the stability of the Cathedral at its post-fire state. The objective is to assess the decrease of strength with respect to the previous, original state of the Cathedral. To this purpose, we compare the results with those calculated, with the same numerical model and assumptions, before the fire. We show that a major consequence of the fire destruction is the considerable reduction of the wind strength of the Cathedral.

Key words: Notre Dame, wind strength, gothic structures.

1 Introduction

The fire of April 15th, 2019, has destroyed a large part of the structure of the Cathedral Notre Dame of Paris. The whole original roofing structure, composed by the wood of, approximately, 1300 oaks and covered with 210 tons of lead tiles, has been entirely destroyed by the fire, as well as the spire. The intensity of the fire, that has produced high temperatures, has probably damaged some parts of the stone structure: a large part of the sexpartite rib vault has collapsed, in the main aisle, in the transept and also at the center of the Cathedral, where choir, aisle and transept meet together.

As a consequence, changes have probably been engendered in the static regime of the Cathedral. This is important for different reasons: on one hand, the restoration of the

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Cathedral will last, probably, several years, so the new static regime has a rather permanent character. On the other hand, it is important, for all the duration of the restoration phase, to assess the vulnerability of the Cathedral to different actions, in order to avoid further collapse and adequately reinforce the structure.

A gothic Cathedral is a rather complicate structure, and in particular Notre Dame of Paris, a church with five aisles, two rows of lateral chapels and large tribunes, see Fig 1. The equilibrium of such a structure is determined by the mutual interactions of all of its parts: pillars, buttresses, vaults, flying buttresses, choir, towers and also the timber structure of the roofing, all together collaborating to ensure the statical equilibrium of the whole construction and to withstand the two main actions to which a Cathedral is submitted: the vertical action of its own weight and the horizontal one due to wind. In particular, the weight of the roofing, estimated at about 700 tons (including the weight of the lead tiles) served as a ballast contributing to withstand the horizontal wind forces; it also connected together the two guttering walls at the top of the main aisle, helping to transmit the action of the wind from the windward side to the leeward one. The long-span ($\sim$ 12 m) flying buttresses provided a thrust at the level of the high vault to counterbalance its outward thrust.

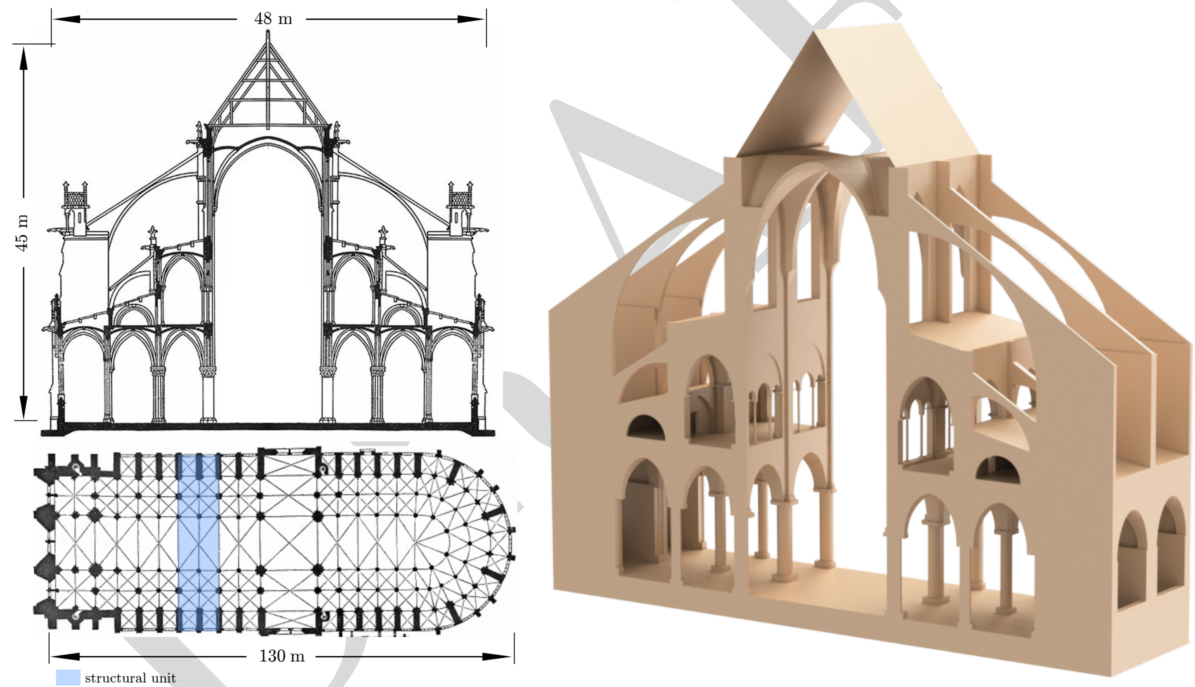

Figure 1: The Cathedral of Notre-Dame in Paris, as it was before the fire. From left to right: section (top) and plan view (bottom), numerical model (overall view). We model the blue-shadowed area, which corresponds to a part of the high vault that has collapsed after the fire.

In the present situation, all the timber structure of the roofing has burnt and large parts of the high vault have collapsed. Hence, the transmission of the forces, at the level of the clearstorey, is deeply altered. The thrust exerted by the flying buttresses is no more counterbalanced by the high vault and the equilibrating ballast of the roof has disappeared. Moreover, when the wind acts on the wall of the clearstorey, its action adds to that of the flying buttresses, producing an overturning moment at its base that can lead to its collapse. In addition, due to the lack of the wooden structure at the top of the guttering walls, the action of the wind is no more transmitted from the windward wall to the leeward one, that, together with the flying buttresses on its side, were equilibrating the wind thrust. Finally, the wall of the clearstorey directly hit by the blow can collapse for a sufficiently strong wind.

The purpose of this paper is hence *to determine how much important is the decrease of the wind strength of the Cathedral, after the fire, with respect to its undamaged state*. To give an answer to this question, we compare the results of the new structural analysis, done on the damaged Cathedral, with those published in a previous paper, Vannucci et al. [2019], and concerning the original, undamaged state of the building. In order to properly assess the wind strength reduction, a consistent comparison is set up, following... the same nonlinear approach used in the cited previous paper. Namely, we use exactly the same numerical model and mechanical assumptions of Vannucci et al. [2019]. All these points are detailed in the sections below.

2 The numerical model

As presented above, the structural analysis has been performed using the finite element model established in Vannucci et al. [2019], whose geometry can be seen in Fig. 1, properly modified to simulate the major structural destructions caused by the fire. This has been done by removing from the original model the high vault and the roof, as can be seen in Fig. 2. For what concerns the material, we still model the stone structure through a nonlinear constitutive law, and in particular:

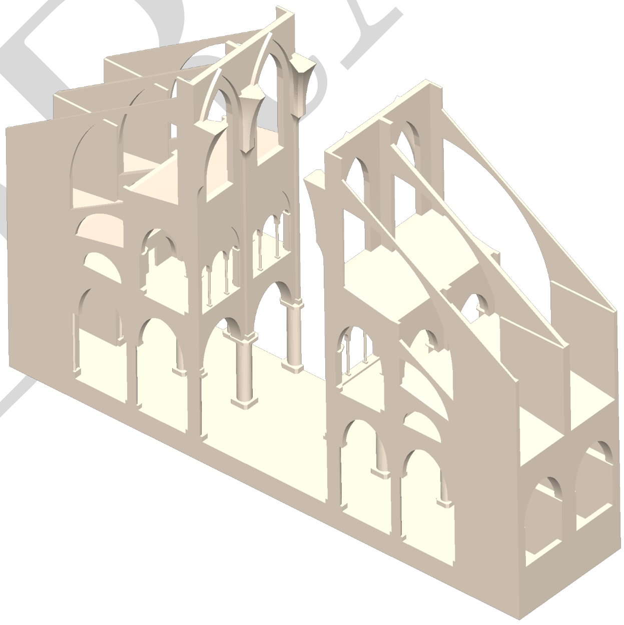

Figure 2: The numerical model of the Cathedral in the present state.

- in compression, the material is modeled as isotropic linearly elastic with infinite strength, an assumption often used in such situations, [Heyman, 1995], [Stefanou et al., 2015];

- in tension, the material is assumed to be isotropic linearly elastic until the maximum principal stress does not exceed the tensile strength; a tensile strength $f_t = 0.5$ MPa is assumed for the material;
- when the maximum principal stress exceeds the tensile strength f_t, failure is modeled using a nonlinear constitutive law based on the softening model proposed in [Hillerborg et al., 1976];
- an equivalent homogenized Young's modulus $E_{eq} = 14000$ MPa is assumed, [Como, 2013];
- the Poisson's ratio is taken equal to $\nu_s = 0.25$;
- the density is evaluated at $\rho_s = 2000$ kg/m^3;
- the softening behavior is approximated by a bilinear law, characterized by a fracture energy $G_f = 11.3$ N/m.

Concerning the wind profile, like in other works on the wind strength of Gothic Cathedrals, [Coccia et al., 2015], we have used the power law

$$v = v_0 \left(\frac{z}{z_0} \right)^{\alpha}, \tag{1}$$

with z_0 a reference height, where the wind speed v_0 is known, and α an exponent, set equal to 0.35, value suggested for urban areas, see [Coccia et al., 2015]. Considering the skyline of Paris, we have chosen for z_0 the value of 10 m. The wind pressure p is then obtained as a drag force per unit of exposed surface using the relation

$$p = \frac{1}{2} C_D \, \rho \, v^2, \tag{2}$$

where ρ is the mass density of air, $\rho = 1.225$ kg/m^3 at an ambient temperature of 15°C, and C_D is a pressure coefficient, depending upon the shape and dimensions of the structure impinged by the wind, that we have assumed to be equal to 1.5. Considering the situation of the Cathedral, after the fire destruction, and being interested in knowing the action of the wind on the clearstorey wall directly hit by the wind, it is sufficient to apply the pressure p on the windward side of the Cathedral, see Fig. 3.

The model of the Cathedral, used in this study and in the previous one, is a 3D model of the main aisle. Of course, some structural support to the strength of the Cathedral is given by the transept and the West towers but, as often done in structural calculations in civil engineering, neglecting their presence gives a final estimation on the safety side. For the same reason, we neglect in the damaged model, Fig. 2, the parts of the high vault still in place.

We stress that the model for the calculation of the wind force is extremely similar to that used in previous papers on the matter, Mark and Jonash [1970], Mark [1982], Como [2013], Coccia et al. [2015] and, as demonstrated in Vannucci et al. [2019], Appendix B, it practically coincides with the prescriptions of Eurocode 1. In other words, we have adopted, for the wind force calculation, a model commonly accepted by the scientific community and by engineers. It is correct for structures like Notre Dame, with large dimensions, huge inertia and no slender parts submitted to possible dynamical effects produced by wind (we are not interested, here, in possible local collapses of minor parts like pinacles).

Due to the fact that the fire has destroyed the structural connection between the windward and leeward parts of the Cathedral, as apparent in Fig. 2, such two parts, on the two opposite sides of the main aisle, are structurally independent. Hence, the wind blowing on the windward part does not, practically, affect the leeward one, which is sheltered from the wind by the windward side. Because there is no structural interaction between these two parts, it is sufficient, to assess the structural wind strength of the damaged Cathedral, to consider only the part of the Cathedral directly impinged by wind, the windward part. We have applied on it the wind pressure perpendicularly to the directly exposed surfaces, Fig. 3.

For what concerns the mechanical properties of the stone, as already said we have taken the same ones already used in Vannucci et al. [2019]. In particular, we have neglected the possible, and at present still to be evaluated, decrease of the bearing capacity, namely in compression, of the stone. In fact, as the final results show clearly, the critical points of the structure, i.e. where the failure mechanism takes place, are at the springing of the clearstorey wall and in the flying buttresses. These parts of the structure cannot have been affected by the fire, because this one developed above the high vault, far from them.

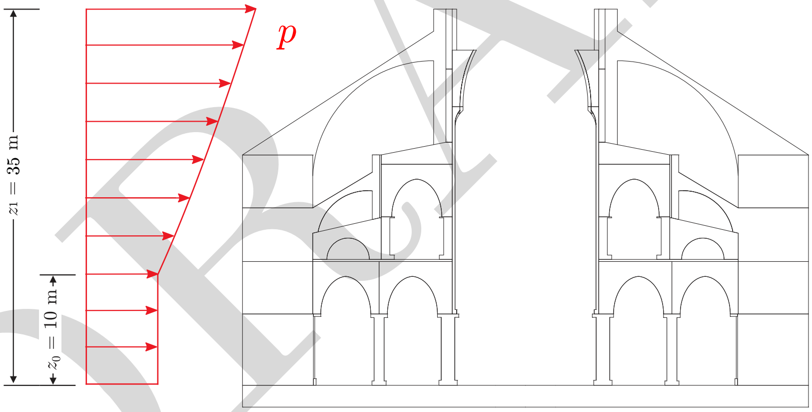

Figure 3: Wind pressure profile.

Finally, as in Vannucci et al. [2019], we identify the ultimate wind speed as the one at which the displacement of the top of the guttering wall becomes unbounded.

3 Effects of gravity and internal stress distribution change

We first consider the effects of the own weight of the structure and we compare the internal stress distribution between its current, damaged state and its intact one. This is done through an implicit, static Finite Element analysis.

As it could be expected, the absence of the high vault produces an unbalanced thrust of the flying buttresses, that makes the clearstorey wall lean inward. The horizontal displacement of the top of the guttering wall, for the simple static action of gravity, can be evaluated at 2.7 mm. At the same time, the maximum tensile stress in the flying buttresses attains the value of 0.25 MPa.

The situation is sketched in Fig. 4, where the maximum principal stresses are shown. It is apparent that, compared to the state of the Cathedral before the fire, the static regime has changed, especially in the flying buttresses, where now important tensile stresses appear.

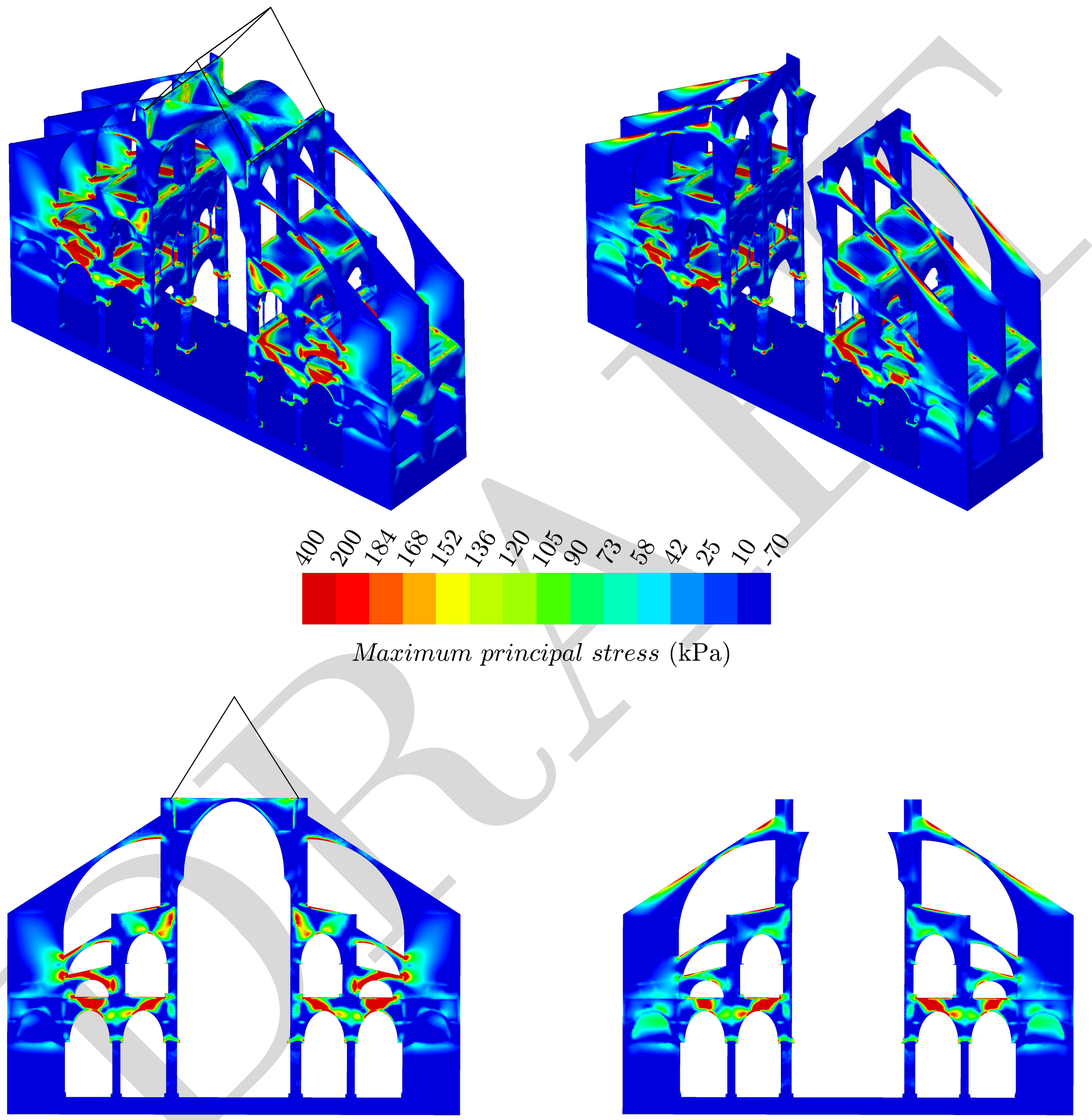

Figure 4: Maximum principal stress distribution, before (left) and after (right) the fire destructions.

These results reveal the strong impact that the destructions engendered by the fire have on the static regime of the Cathedral. More important are the effects related to the action of the wind, considered in the next Section.

4 Residual wind strength of the Cathedral

In Vannucci et al. [2019], we evaluated the ultimate wind speed of the Cathedral at $v_0 = 222$ km/h. We consider now which is the decrease of wind strength of the Cathedral after the destructions made by the fire. A certain degree of approximation is intrinsic and ineradicable in this kind of analysis, so in order to place our study in a conventional frame, we have used an approach commonly accepted by engineers, namely for the evaluation of the wind actions, as already mentioned above. Nevertheless, given that we use the same numerical model and assumptions as in our previous study, the evaluation of the ultimate wind speed at the current, damaged state of the structure and its comparison with the ultimate wind strength at its previous, intact one, is a measure of the extent of damage and loss of structural integrity of the Cathedral.

The configuration resulting from the previous implicit analysis, considering only gravity, is now used as the starting point for a nonlinear explicit analysis in which the wind loads are applied in a quasi-static manner. A fictitious mass proportional damping is assumed in order to reach equilibrium rapidly and to dissipate unwanted oscillations (quasi-static condition). We evaluate the displacement of the top of the guttering wall for different wind speeds; when such a displacement becomes unbounded, this means that the Cathedral has reached its ultimate state: cracks have formed a collapse mechanism.

The result of the numerical simulations under wind and gravity actions are represented in Fig. 5. Here, we show the two curves relating the wind speed and the maximum horizontal displacement of the vault keystone, for the original state, and of the top of the guttering wall, for the damaged structure. From this Figure, it appears clearly that the highest wind speed that the Cathedral can now withstand is evaluated at ~ 90 km/h, i.e. 40% of the ultimate wind speed found when intact. In Fig. 6 we show the collapse mechanism of the Cathedral under the action of the wind and gravity.

In other words, the destructions due to the fire have reduced the wind strength of the Cathedral of about 60%. More important, is the fact that such a wind speed can be rather frequent in the Paris region, especially given the current climate deregulation. Consequently, it appears urgent the need for providing the Cathedral with temporary support structures, until the end of the restoration phase.

5 Conclusion

Our analysis shows that the fire occurred on April 15th, 2019, in the roofing structure of Notre Dame of Paris has lead to an important stress redistribution under gravity loads. Moreover, the fire caused a reduction of about 60% of the strength of the Cathedral to wind actions. According to our calculations, the ultimate wind speed leading to collapse is now ~ 90 km/h, instead of ~ 222 km/h that was before the fire.

It is worth mentioning that this is a first study based on our previous analysis of the intact structure after removing the roof and the collapsed parts of the monument due to the fire. We hope that this work can provide insight and a rational basis for decisions to be taken regarding reinforcement and structural support. Moreover, it can help the

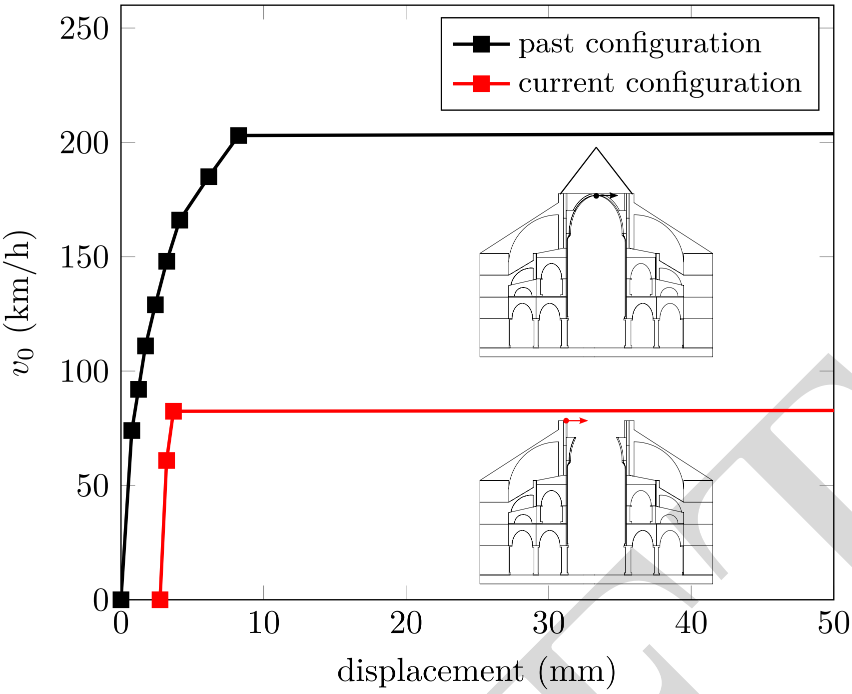

Figure 5: Wind speed v_0 versus horizontal displacement.

authorities in the design of future interventions.

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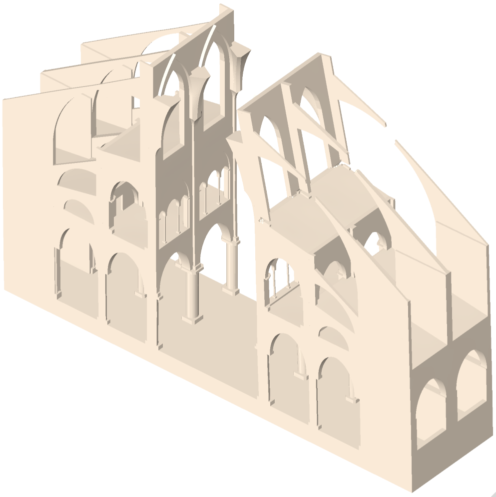

Figure 6: Collapse mechanism.

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