Dynamics under location uncertainty and other energy-related stochastic subgrid schemes
Valentin Resseguier, Long Li, Gabriel Jouan, Pierre Derian, Etienne Memin, Bertrand Chapron

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Dynamics under location uncertainty and other energy-related stochastic subgrid schemes

V. Resseguier
L. Li, G. Jouan, P. Derian,
E. Memin, B. Chapron
Motivations

• More rigorously identified sudgrid dynamics effects

• Quantification of modeling errors (UQ)

    Ensemble forecasts and data assimilation
Contents

- Models under location uncertainty (LU)
- Some parameterization of the models under location uncertainty
- A new energy-budget-based stochastic scheme: WaveHyperv
- Numerical comparisons
Part I
Models under location uncertainty (LU)
LU : Adding random velocity

\[ v = w + \sigma \dot{B} \]
LU: Adding random velocity

Resolved large scales

\[ \nu = \omega + \sigma \dot{B} \]
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

Resolved large scales

White-in-time small scales
LU: Adding random velocity

Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)

Variance tensor:
\[
a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}
\]

Resolved large scales
White-in-time small scales

\[
v = w + \sigma \dot{B}
\]
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

Resolved large scales

White-in-time small scales

References:

- Memin, 2014
- Resseguier et al. 2017 a, b, c
- Cai et al. 2017
- Chapron et al. 2018
- Yang & Memin 2019
- Resseguier et al. 2019 a,b
- Holm, 2015
- Holm and Tyranowski, 2016
- Arnaudon et al. 2017
- Cotter and al. 2017
- Crisan et al., 2017
- Gay-Balmaz & Holm 2017
- Cotter and al. 2018 a, b
- Cotter and al. 2019

Large scales: 

\( w \)

Small scales:

\( \sigma \dot{B} \)

Variance tensor:

\[ a = a(x, x) = \frac{\mathbb{E} \{ \sigma dB (\sigma dB)^T \}}{dt} \]

Memin, 2014
Resseguier et al. 2017 a, b, c
Cai et al. 2017
Chapron et al. 2018
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Holm, 2015
Holm and Tyranowski, 2016
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Crisan et al., 2017
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Cotter and al. 2018 a, b
Cotter and al. 2019
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

Large scales:
\[ w \]
Small scales:
\[ \sigma \dot{B} \]

Variance tensor:
\[ a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} dt \]

Resolved large scales

White-in-time small scales

References:
- Mikulevicius & Rozovskii, 2004
- Flandoli, 2011
- Holm, 2015
- Memin, 2014
- Resseguier et al. 2017 a, b, c
- Cai et al. 2017
- Chapron et al. 2018
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- Crisan et al., 2017
- Gay-Balmaz & Holm 2017
- Cotter and al. 2018 a, b
- Cotter and al. 2019
Advection of tracer $\Theta$

$$\frac{D\Theta}{Dt} = 0$$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:
$$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt$$
Advection of tracer \( \Theta \)

Large scales:

\( w \)

Small scales:

\( \sigma \dot{B} \)

Variance tensor:

\[
\mathbf{a} = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}
\]
Advection of tracer $\Theta$ 

Large scales: $w$  
Small scales: $\sigma \dot{B}$  
Variance tensor:  
\[
a = a(x, x) = \frac{\mathbb{E} \{ \sigma dB (\sigma dB)^T \}}{dt},
\]

\[
\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)
\]
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:

$\sigma = a(x, x) = \frac{E\{\sigma dB \, (\sigma dB)^T\}}{dt}$

\[
\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)
\]
Advection of tracer Θ

Large scales:
\( w \)
Small scales:
\( \sigma \dot{B} \)

Variance tensor:
\[
a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}
\]
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$$a = a(x,x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

**Advection**

$$\partial_t \Theta + \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

**Drift correction**

**Diffusion**
Advection of tracer $\Theta$

Large scales: $\omega$
Small scales: $\sigma \dot{B}$
Variance tensor: $\alpha = \alpha(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt$

Multiplicative random forcing

$\partial_t \Theta + \omega^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)$

Drift correction
Advection of tracer $\Theta$

Large scales:
- $w$

Small scales:
- $\sigma \dot{B}$

Variance tensor:
- $a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} dt$

Multiplicative random forcing

Drift correction

\[
\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)
\]
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \mathbf{\dot{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right) \]

**Large scales:**

- $\mathbf{w}$

**Small scales:**

- $\sigma \mathbf{\dot{B}}$

**Variance tensor:**

\[
\alpha = \alpha(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \frac{dt}{d}\]

**Multiplicative random forcing**

**Drift correction**
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \mathbf{\dot{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right) \]

Drift correction

Multiplicative random forcing

Balanced energy exchanges

Large scales:
- $\mathbf{w}$

Small scales:
- $\sigma \mathbf{\dot{B}}$

Variance tensor:
- $\alpha = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} / dt$
Part II
Some parameterization of LU models
Part II
Some parameterization of LU models

\[ \sigma = ? \]
Large scales: 
\( w \)
Small scales: 
\( \sigma \dot{B} \)
Variance tensor:
\[
\alpha = \alpha(x, x) = \frac{\mathbb{E}\{\sigma dB \, (\sigma dB)^T\}}{dt}
\]
Spectral model
(homogeneous and stationary $\sigma \dot{B}$)

Large scales:
$w$
Small scales:
$\sigma \dot{B}$

Variance tensor:
$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB \langle \sigma dB \rangle^T\}}{dt}$

Fixed spectrum at small scales

Reference:
Resseguier, Memin & Chapron 2017b

Code online
Spectral model
(homogeneous and stationary $\sigma \dot{B}$)

$\omega$

Small scales:
$\sigma \dot{B}$

Variance tensor:
$a = a(x,x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} / dt$

Large scales:

Code online

Reference:
Resseguier, Memin & Chapron 2017b
Absolute Diffusivity
Spectral Density
(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Large scales:
$\omega$
Small scales:
$\sigma \dot{B}$

Variance tensor:
$a = a(x, x) = \mathbb{E} \{ \sigma dB (\sigma dB)^T \} / dt$

Reference:
Resseguier, Pan & Fox-Kemper 2019a
Absolute Diffusivity
Spectral Density
(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Large scales: $\mathbf{w}$
Small scales: $\sigma \dot{B}$

Variance tensor:
$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} / dt$

Absolute Diffusivity Spectral Density

$A(\kappa) = E(\kappa) \tau(\kappa)$

Reference:
Resseguier, Pan & Fox-Kemper 2019a
Absolute Diffusivity Spectral Density
(homogeneous but non-stationary and tuning-free $\sigma B$)

**Large scales:**
$\omega$

**Small scales:**
$\sigma \dot{B}$

Variance tensor:

$\mathbf{a} = a(x, x) = \mathbb{E}\left\{\sigma dB (\sigma dB)^T\right\}$

$$\dot{B}$$

Absolute Diffusivity Spectral Density

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Absolute Diffusivity
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Absolute Diffusivity
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Absolute Diffusivity Spectral Density
$A(\kappa) = E(\kappa) \tau(\kappa)$

Residual non-stationary ADSD

$\sigma \dot{B} = \text{(filter)} \ast \text{(white noise)}$

Reference:
Resseguier, Pan & Fox-Kemper 2019a
Random switching of points
(heterogeneous and non-stationary $\sigma \dot{B}$)

Large scales:
$w$
Small scales:
$\sigma \dot{B}$

Variance tensor:

$$a(x, x) = \mathbb{E}\left\{ \sigma dB(\sigma dB)^T \right\} dt$$

Reference:
Resseguijer, Li, Jouan, Derian, Memin & Chapron 2019b
Random switching of points
(heterogeneous and non-stationary $\sigma \dot{B}$)

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

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Random switching of points
(heterogeneous and non-stationary $\sigma \dot{B}$)

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \mathbb{E}\left\{\sigma dB (\sigma dB)^T\right\}$$

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b
Random switching of points
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Large scales: $w$
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Resseguijer, Li, Jouan, Derian, Memin & Chapron 2019b
Random switching of points
(heterogeneous and non-stationary $\sigma \dot{B}$)

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}$$

Neighbor
Centered neighbor
Random selection

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b
Random switching
of points
(heterogeneous and non-stationary $\sigma \dot{B}$)

Large scales: $\mathbf{w}$
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Random switching of points
(heterogeneous and non-stationary $\sigma \dot{B}$)

Large scales:
$w$

Small scales:
$\sigma \dot{B}$

Variance tensor:
$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt$

Global ensemble

$\sigma \dot{B}$

Local ensemble:

SVD

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b
Part III
A new energy-budget-based stochastic scheme:
WaveHyperv
WaveHyperv

Transport equation

\[
\frac{Dq}{Dt} = \mathcal{L}[q] + \eta
\]

Reference:

Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b
WaveHyperv

Transport equation

\[
\frac{Dq}{Dt} = \mathcal{L}[q] + \eta
\]

Usual deterministic subgrid tensor

e.g. Hyper-viscosity

\[
\mathcal{L}[q] = -\nu \Delta^4 q
\]

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b
WaveHyperv

Transport equation

\[ \frac{Dq}{Dt} = \mathcal{L}[q] + \eta \]

Usual deterministic subgrid tensor

e.g. Hyper-viscosity

\[ \mathcal{L}[q] = -\nu \Delta^4 q \]

Random forcing built to meet the energy budget:

(Random energy intake) = \zeta \times \text{Dissipation}

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b
## Summary of UQ methods

<table>
<thead>
<tr>
<th>Name</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>MU Spec</td>
<td>LU with homogeneous and stationary small-scale velocity</td>
</tr>
<tr>
<td>MU ADSD</td>
<td>LU with homogeneous, non-stationary and tuning-free small-scale velocity</td>
</tr>
<tr>
<td>MU SVD</td>
<td>LU with inhomogeneous and non-stationary small-scale velocity</td>
</tr>
<tr>
<td>WaveHyperv</td>
<td>Energy-budget-based stochastic scheme</td>
</tr>
<tr>
<td>PIC Spec</td>
<td>Perturbed initial conditions with homogeneous noise</td>
</tr>
<tr>
<td>PIC SVD</td>
<td>Perturbed initial conditions with inhomogeneous noise</td>
</tr>
</tbody>
</table>
Part IV
Numerical comparisons
Test case 1:

**SQG**

\[
\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}
\]

\[
u = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b
\]

Reference flow:
- deterministic
- SQG
- 512 x 512

\[t = 17 \text{ days}\]
Test case 1:

SQG

\[ \frac{D b}{D t} = -\alpha_{HV} \Delta^4 b \text{ Hyper-viscosity} \]

\[ u = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b \]

Reference flow:

deterministic

SQG

512 x 512

\( t = 17 \text{ days} \)
Test case 2:

**Reference flow:**
- deterministic
- SQG
- $512 \times 512$

**SQG**

\[
\frac{D b}{D t} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}
\]

\[
u = \left( \text{cst.} \, \nabla^\perp \, \Delta^{-\frac{1}{2}} \right) b
\]
Test case 2:

SQG

\[
\frac{D b}{D t} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}
\]

\[
u = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b
\]

Reference flow:

deterministic

SQG

512 x 512

\[ t = 10 \text{ days} \]
<table>
<thead>
<tr>
<th>Metric</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>Error of ensemble members</td>
</tr>
<tr>
<td>Talagrand histogram (TH)</td>
<td>Capacity of the ensemble to <strong>explore</strong> all reference possible values</td>
</tr>
<tr>
<td>Bias^2-spread</td>
<td>Capacity of the ensemble to <strong>explore</strong> all reference possible values</td>
</tr>
<tr>
<td>CRPS</td>
<td>Point-wise distance between the ensemble CFD and the indicator function of the event</td>
</tr>
<tr>
<td>Energy Score (ES)</td>
<td>Generalized CRPS for multivariate ensemble</td>
</tr>
</tbody>
</table>
Spreading VS Errors

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

\[ \frac{(\text{MSB} - \text{MSE})}{B_0} \times 10^{-3} \]

\[ \frac{\text{MSE}}{B_0^2} \times 10^{-3} \]

\[ t = 0 \text{ days} \]

\[ x(m) \text{ vs. } t = 0 \times 10^3 \]

\[ y(m) \text{ vs. } t = 0 \times 10^3 \]

Legend:
- MU Spectral
- MU ADSD
- WavHypervis 50%
- MU SVDpseudo
- PIC Spectral
- PIC SVDpseudo

Spread

Error

Forecast Days

Forecast Days

Forecast Days

Forecast Days

Forecast Days
Spreading VS Errors

(a) and (b) show the spread of errors over forecast days for different methods:
- MU Spectral
- MU ADSD
- WavHypervis 50%
- MU SVDpseud
- PIC Spectral
- PIC SVDpseud

(c) and (d) focus on the error component of the spread, measured as MSE/B^2.

(e) and (f) compare the spread and error components using the same methods as in (a) and (b).
Spreading VS Errors

Opposite conclusions

Spread

Error

(a)

(b)

(c)

(d)

(e)

(f)
Ensemble point-wise skills: CRPS

MU Spec

MU ADSD

MU SVD

WaveHyperv

PIC Spec

PIC SVD
Ensemble point-wise skills: CRPS
Ensemble point-wise skills: CRPS

CRPS spatial of each model at day 19

CRPS spatial of each model at day 10
Ensemble point-wise skills: CRPS

CRPS spatial of each model at day 19

CRPS spatial of each model at day 10
Conclusion
## Conclusion

<table>
<thead>
<tr>
<th></th>
<th>RMSE (errors)</th>
<th>B^2-Var (spread)</th>
<th>CRPS (point-wise)</th>
<th>ES (global)</th>
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## Conclusion

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