Improving Universal Multifractal parameter estimation for large data sets, a case study in the Greater Paris

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Introduction

Precipitation is a highly complex and variable phenomenon. The purpose of this work is to analyse, and inter-compare 5 different products of X-band radar with pulsed emissions for simple and dual polarization, using the Universal Multifractal analysis.

Methodology

a. ENPC polarimetric X-band radar

- Oblate drops->differential phase shift (ΦDP) between the horizontal and vertical polarized wave. The specific differential phase shift (=gradΦDP) is used for rainfall rate computation.
- Rainfall estimations
  - -Strong KDP (> 1°/km) -> directly $R = c\left(\frac{KDP}{f}\right)^d$
  - -Low KDP -> Marshall-Palmer relation $Z=aR^b$

b. Spectral analysis: the first indication of the scaling behaviour of the field

$E(k) \approx k^{-\beta}$, where β: spectral slope calculated

k: wavelength

c. Universal Multifractals parameters fully define statistics across scales[2,3,4]

- $\alpha \in [0,2]$ (multifractality index): the variability of intermittency with respect to intensity level
- C_1 (mean intermittency): mean inhomogeneity of the field (C_1: 0 for homogeneous fields)
- H: the degree of non-conservation, the scale dependency of the mean field (H=0 considered in our case)

$$K(q) = \begin{cases} \frac{C_1}{\alpha-1}(q^\alpha - q), a \neq 1 \\ C_1 q \log(q), a = 1 \end{cases}$$

C_1 and α estimated with

- the **Trace Moment** (TM)

$$C_1 = \left.\frac{dK(q)}{dq}\right|_{q=1} \qquad C_1\alpha = \left.\frac{d^2K(q)}{dq^2}\right|_{q=1}$$

- and with the (uni/mono) fractal correction[5]

$$K^*(q) = K(q) + c(q-1) \qquad C_1^* = \left.\frac{dK^*(q)}{dq}\right|_{q=1} = C_1 + c \qquad C_1^*\alpha^* = \left.\frac{d^2K^*(q)}{dq^2}\right|_{q=1} = C_1\alpha$$

- and **Double Trace Moment** (DTM) technique [1]

$$\langle \left(R_\lambda^{(\eta)}\right)^q \rangle \approx \lambda^{K(q,\eta)} \qquad K(q,\eta) = \eta^\alpha K(q)$$

Data

Table 1: The five radar products compared in this study (-SP: Simple Filter, -PS: Pseudo, TFL: Terrain Following Layer)

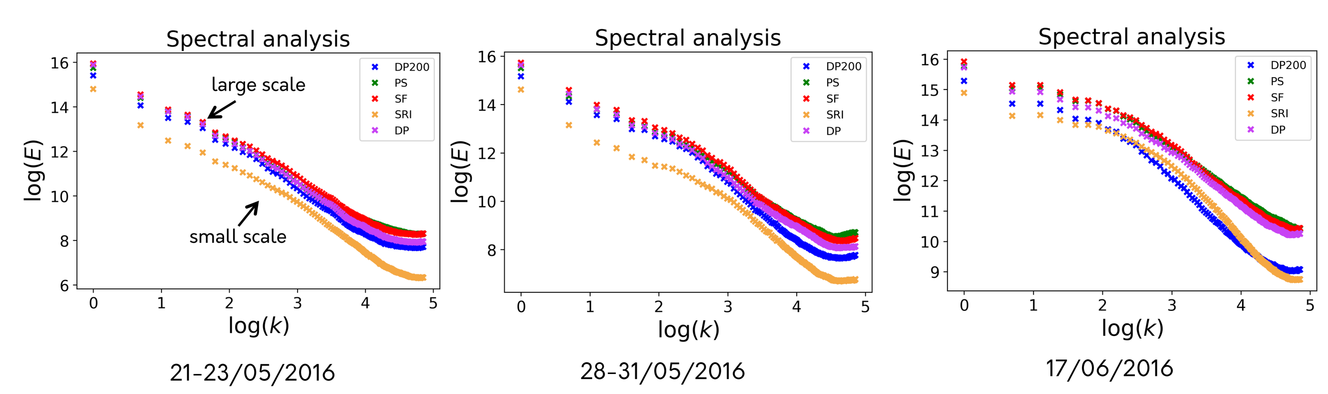
Product	Dual Polarization	Data used for products	Required parameters	Equation used	Filter	Data considered for TFL below/above lowest/highest elevation
DPSRI_150	✓	dBZ, K_{DP}	a=150 b=1.3	$Z=aR^b$	FIR	no data
DPSRI_200	✓	dBZ, K_{DP}	a =200 b =1.6	$Z=aR^b$	FIR	no data
DPSRI_SP	✓	dBZ, K_{DP}	a =150 b =1.3	$Z=aR^b$	Median	no data
DPSRI_PS	✓	dBZ, K_{DP}	a =150 b =1.3	$Z=aR^b$	FIR	Data of lowest/highest elevation used
SRI	✗	dBZ	a =200 b =1.6	$Z=aR^b$		no data

The signal of KDP is noisy so the use of a smoothed signal is used to compute KDP. Two different filters are used: the Finite Impulse Response (FIR) and the median one. The data were processed with standard Rainbow software.

Table 2: Rainfall events and their characteristics

Event	Date	Start time (UTC)	End time (UTC)	Duration (in h)	Time steps
E1	21-23/05/2016	12:25:00	23:50:00	59.42	1264
E2	28-31/05/2016	08:25:00	00:00:00	87.58	1686
E3	17/06/2016	00:00:00	21:15:00	21.25	422

Table 3: Figures of spectral analysis of the selected events



Analysis

The values of UM parameters of event 28-31/05/2016 for TM and TM corrected are presented:

Table 4: Values of UM parameters for TM and TM corrected

Radar Products	α *	α	C^*_1	C_1	c
DPSRI_150_s	1.443	*4.016*	0.047	0.017	0.03
DPSRI_150_l	0.861	*2.146*	0.278	0.112	0.17
DPSRI_200_s	1.854	*4.85*	0.049	0.019	0.03
DPSRI_200_l	1.137	*2.735*	0.278	0.116	0.16
DPSRI_SF_s	1.743	*4.20*	0.051	0.021	0.03
DPSRI_SF_l	1.743	*2.244*	0.288	0.121	0.17
DPSRI_PS_s	1.545	*4.20*	0.047	0.017	0.03
DPSRI_PS_l	0.952	*2.053*	0.277	0.129	0.15
SRI_s	1.296	*5.715*	0.039	0.009	0.03
SRI_l	0.812	*2.178*	0.226	0.084	0.14

For every simulation the following figures are obtained (in red: small and in blue: large scales):

Table 6: TM analysis and K(q) for small (in red) and large (in blue) scales

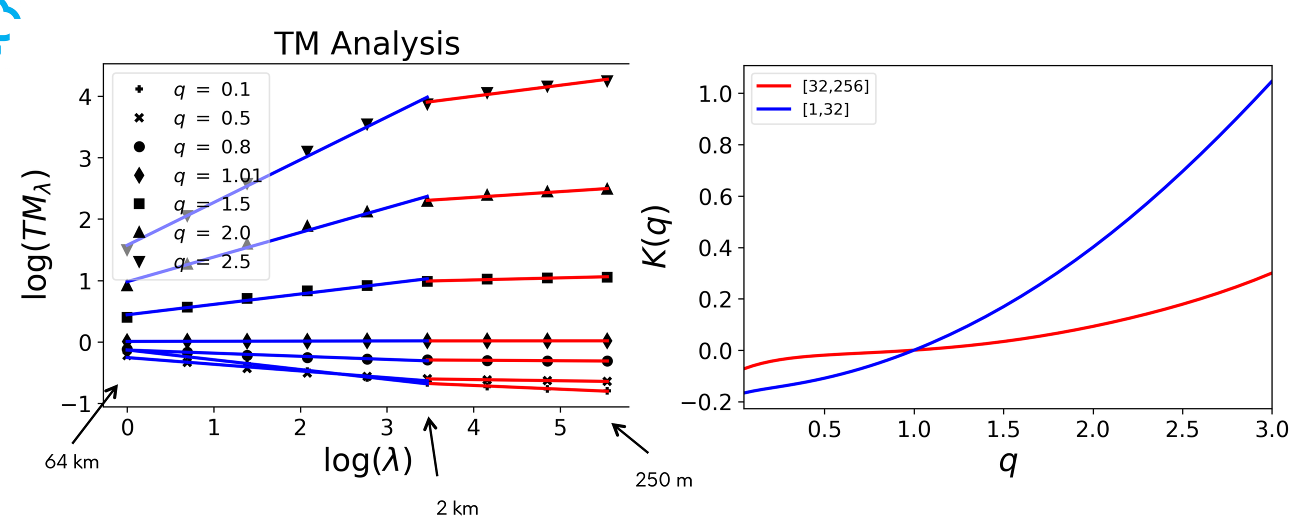
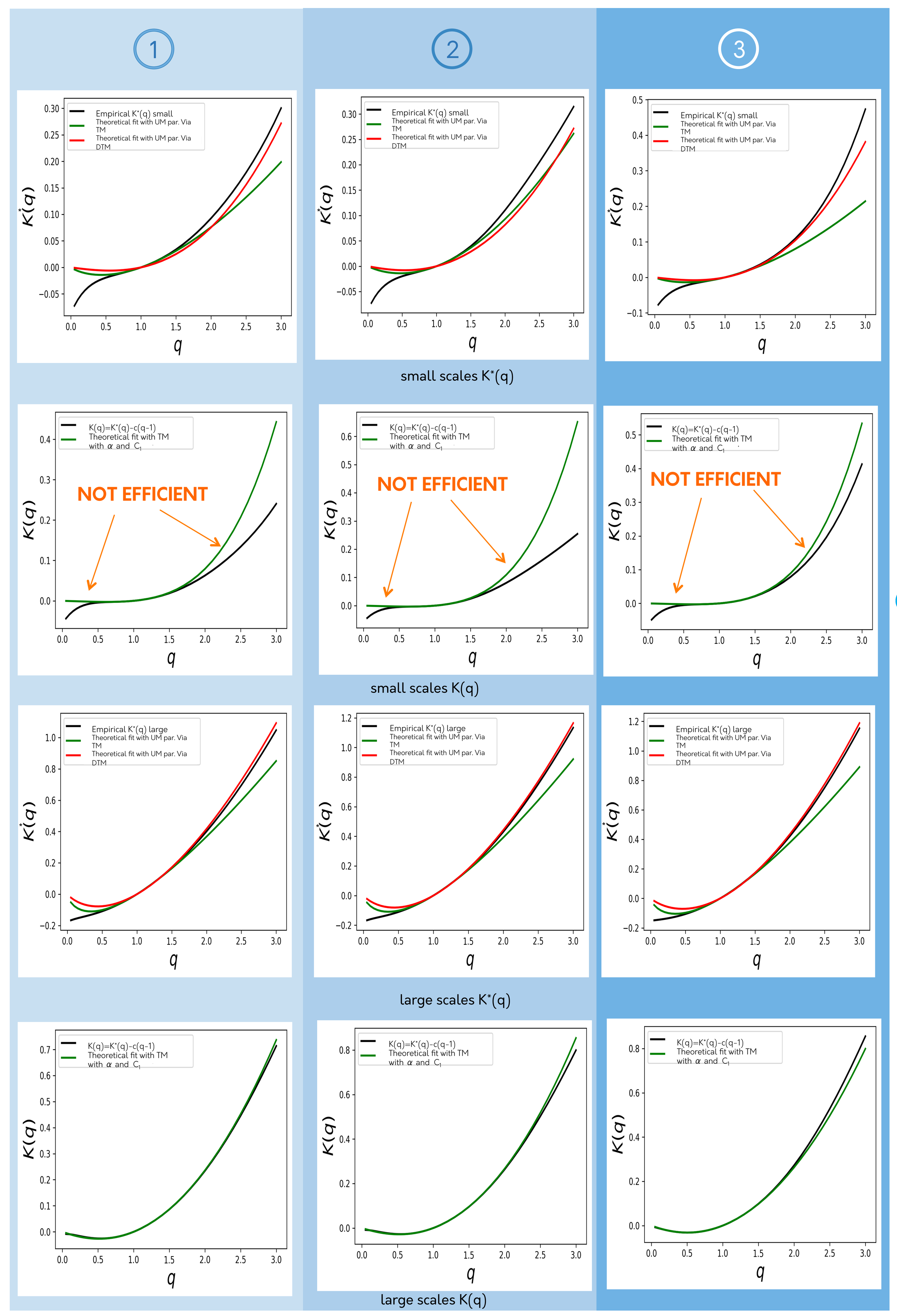


Table 5: Figures of spectral analysis and DTM method



① **DPSRI a=150 b=1.3** *28-31/05/2016*

② **DPSRI a=150 b=1.3 simple filter** *28-31/05/2016*

③ **PsDPSRI a=150 b=1.3** *28-31/05/2016*

Conclusions

- The correction does not work for small scales (difficulty in estimating c)
- Efficient for large scales (but strong values for α)

Future work

- Test it for events of higher intensity
- Comparing with other types of radar data and with rain gauge measurements
- Use them in urban hydrological models (Multi Hydro) to find out which filter is better to use

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