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Supervised classification of multidimensional and irregularly sampled signals.

Alexandre Constantin¹, Mathieu Fauvel², Stéphane Girard¹ and Serge Iovleff³

¹Université Grenoble Alpes, Inria, CNRS, Grenoble INP, LJK, Grenoble, France
²CESBIO, Université de Toulouse, CNES/CNRS/IRD/UPS/INRA, Toulouse, France
³Laboratoire Paul Painlevé - Université Lille 1, CNRS, Inria, France

Introduction

Background:
Recent space missions, such as Copernicus Sentinel-2a, provide high resolution Satellite Image Time Series (SITS) to study continental surfaces, with a very short revisit period (5 days for Sentinel-2). In order to process such statistical models are regularly used [1, 2], which usually require a regular temporal sampling. However, for SITS, clouds and shadows (eg. figure from [3]), as well as the satellite orbit, an irregular temporal sampling is common.

Contribution:
A new statistical approach using Gaussian processes is proposed to classify irregularly sampled signals without temporal rescaling. Moreover, the model offers a theoretical framework to impute missing values such as cloudy pixels.

Gaussian Processes (GP) model:
Let $S = \{(y_i, z_i)\}_{i=1}^p$ be a set of multidimensional and irregularly sampled signals. A signal $Y$ is modeled as a vector of $p$ independent random processes $T \rightarrow \mathbb{R}^p$ with $T = [0,T]$. The associated label is modeled by a discrete random variable $Z$ taking its values in $\{1, \ldots, C\}$. The model introduced here is based on two assumptions: 1) The coordinate processes $Y_{i,b}, b \in \{1, \ldots, p\}$ of $Y$ are independent, 2) Each process $Y_i$ is, conditionally to $Z = c$, a Gaussian process. Then

$$Y_i(t)|Z = c \sim \mathcal{GP}(m_{b,c}(t), K_{b,c}(t,s)),$$

where $m_{b,c} : T \rightarrow \mathbb{R}^p$ is a mean function, and $K_{b,c}$ a covariance kernel with hyperparameters $\theta_{b,c}$. For example $\theta_{b,c} = \{\gamma_{b,c}, h_{b,c}, \sigma_{b,c}^2\}$ with

$$K_{b,c}(t,s) = \gamma_{b,c}^2 h_{b,c} + \sigma_{b,c}^2 \delta_{t,s}.$$ 

An irregularly sampled noisy signal $y_i$ is observed on $T_i$ time stamps $\{t_i^1, \ldots, t_i^n\} \subset T$ and its $b$th coordinate is represented by a vector in $\mathbb{R}^2$. We write $Y_{i,b} = [Y_{i,b}(t_1^1), \ldots, Y_{i,b}(t_1^n)]^T$, with

$$y_{i,b}|Z_i = c \sim \mathcal{N}(\mu_{i,b,c}, \Sigma_{i,b,c}^b).$$

There $\mu_{i,b,c} = B_i \alpha_{b,c} \Sigma_i$ is the sampled mean projected on a finite-dimensional space ($B_i$ is the fixed design matrix, $\alpha_{b,c}$ is the unknown vector of coordinates). $\Sigma_{b,c}$ is the kernel $K_{b,c}$ evaluations at $\{t_1^1, \ldots, t_1^n\}$.

Estimation:
• $\alpha_{b,c}$ and $\theta_{b,c}$ are estimated by maximizing the log-likelihood,

$$-\frac{1}{2} \sum_{i \mid Z_i = c} \log |\Sigma_{b,c}^i| + (y_{i,b} - B_i \alpha_{b,c})^T \Sigma^{-1}_{b,c}(y_{i,b} - B_i \alpha_{b,c}).$$

• $\alpha_{b,c}$ is given by an explicit formula, while $\theta_{b,c}$ is computed thanks to a gradient technique.

Classification and Imputation of missing values

The assigned class is given by the MAP rule from the posterior probability

$$P(Z = c | y_i) \propto \frac{\pi_c \prod_{l=1}^n f_{T_l}(y_j, B_l \alpha_{b,c}, \Sigma_{b,c}^l(\theta_{b,c})))}{\sum_{c} \pi_c \prod_{l=1}^n f_{T_l}(y_j, B_l \alpha_{b,c}, \Sigma_{b,c}^l(\theta_{b,c})))}.$$

When the class is known to be $c$, the missing value at $t^*$ is estimated through the computation of conditional expectation.

$$\hat{Y}_{b,c}(t^*) = B(t^*) \alpha_{b,c} + K_{b,c}(t^*, t_1: t_n) \Sigma^{-1}_{b,c}(y_{i,b} - B(t^*) \alpha_{b,c})$$

$$\text{var}(\hat{Y}_{b,c}(t^*)) = K_{b,c}(t^*, t^*) - K_{b,c}(t^*, t_1: t_n) \Sigma_{b,c}^{-1}(t_1: t_n, t^*) K_{b,c}(t_1: t_n, t^*)$$

We also generalized this imputation when the class is unknown.

Validation (Synthetic data)

Example of two signals (dots) that belongs to two different classes

Classification rate based on average time samples

<table>
<thead>
<tr>
<th>$n_t$</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
</tr>
</thead>
<tbody>
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<td>Acc$^\text{exp}$ (%)</td>
<td>52.8</td>
<td>52.9</td>
<td>74.3</td>
<td>93.9</td>
<td>94.2</td>
</tr>
<tr>
<td>Acc$^\text{imp}$ (%)</td>
<td>64.3</td>
<td>85.3</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Imputation on two signals belonging to the same class.

Future work

We are now implementing the model for massive real data (Sentinel-2). We are also working on a new model when the bands are correlated.

References