Supervised classification of multidimensional and irregularly sampled signals.

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Introduction

Background:

Recent space missions, such as Copernicus Sentinel-2a, provide high resolution Satellite Image Time Series (SITS) to study continental surfaces, with a very short revisit period (5 days for sentinel-2). In order to process such statistical models are regularly used [1, 2], which usually require a regular temporal sampling. However, for SITS, clouds and shadows (e.g. figure from [3]), as well as the satellite orbit, an irregular temporal sampling is common.

Contribution:

A new statistical approach using Gaussian processes is proposed to classify irregularly sampled signals without temporal rescaling. Moreover, the model offers a theoretical framework to impute missing values such as cloudy pixels.

Model

Gaussian Processes (GP) model:

Let \( S = \{ (y_i, z_i) \}^C_{i=1} \) be a set of multidimensional and irregularly sampled signals. A signal \( y \) is modeled as a vector of \( p \) independent random processes \( T \rightarrow \mathbb{R}^p \), with \( T = [0, T_i] \). The associated label is modeled by a discrete random variable \( Z \) taking its values in \( \{1, \ldots, C\} \). The model introduced here is based on two assumptions: 1) The coordinate processes \( Y_{b,c} \), \( b \in \{1, \ldots, p\} \) of \( Y \) are independent, 2) Each process \( Y_b \) is, conditionally to \( Z = c \), a Gaussian process. Then

\[
Y_b(t) | Z = c \sim \mathcal{GP}(m_{b,c}(t), K_{b,c}(t, s)),
\]

where \( m_{b,c} : T \rightarrow \mathbb{R}^p \) is a mean function, and \( K_{b,c} \) a covariance kernel with hyperparameters \( \theta_{b,c} \). For example \( \theta_{b,c} = \{ \gamma_{b,c}^2, \sigma_{b,c}^2 \} \) with

\[
K_{b,c}(t, s) = \gamma_{b,c}^2 \kappa(t, s|h_{b,c}) + \sigma_{b,c}^2 \delta_{t,s}
\]

An irregularly sampled noisy signal \( y_i \) is observed on \( T_i \) time stamps \( \{ t_{i,1}, \ldots, t_{i,C} \} \) in \( T \) and its \( b \)th coordinate is represented by a vector in \( \mathbb{R}^2 \). We write \( y_{i,b} = [y_{i,b}(t_{i,1}), \ldots, y_{i,b}(t_{i,C})]^T \), with

\[
y_{i,b} Z_i = c \sim \mathcal{N}(\mu_{i,b,c}, \Sigma_{b,c}).
\]

There \( \mu_{i,b,c} = B_i \alpha_{b,c} \) is the sampled mean projected on a finite-dimensional space (\( B_i \) is the fixed design matrix, \( \alpha_{b,c} \) is the unknown vector of coordinates). \( \Sigma_{b,c} \) is the matrix kernel \( K_{b,c} \), evaluations at \( \{ t_{i,1}, \ldots, t_{i,C} \} \).

Estimation:

- \( \alpha_{b,c} \) and \( \theta_{b,c} \) are estimated by maximizing the log-likelihood,

\[
\frac{1}{2} \sum_{i|Z_i=c} \log \left( \Sigma_{b,c}(\theta_{b,c}) \right) + (y_{i,b} - B_i \alpha_{b,c})^T \Sigma_{b,c}(\theta_{b,c})^{-1} (y_{i,b} - B_i \alpha_{b,c}).
\]

- \( \alpha_{b,c} \) is given by an explicit formula, while \( \theta_{b,c} \) is computed thanks to a gradient technique.

Classification and Imputation of missing values

The assigned class is given by the MAP rule from the posterior probability

\[
P(Z | y) \propto \prod_{t_i \in \mathcal{T}} f_{t_i}(y_{i,b} | \hat{\theta}_{i,b,c}, \Sigma_{b,c}(\hat{\theta}_{b,c})).
\]

Validation (Synthetic data)

Example of two signals (dots) that belongs to two different classes

<table>
<thead>
<tr>
<th>Classification rate based on average time samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_t )</td>
</tr>
<tr>
<td>( \text{Acc}_{\text{map}} ) (%)</td>
</tr>
<tr>
<td>( \text{Acc}_{\text{map}} ) (%)</td>
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Future work

We are now implementing the model for massive real data (Sentinel-2). We are also working on a new model when the classes are correlated.

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