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Supervised classification of multidimensional and irregularly sampled signals.

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Introduction

Background:
Recent space missions, such as Copernicus Sentinel-2, provide high resolution Satellite Image Time Series (SITS) to study continental surfaces, with a very short revisit period (5 days for Sentinel-2). In order to process such signals, statistical models are regularly used [1, 2], which usually require a regular temporal sampling. However, for SITS, clouds and shadows (e.g. figure from [3]), as well as the satellite orbit, an irregular temporal sampling is common.

Contribution:
A new statistical approach using Gaussian processes is proposed to classify irregularly sampled signals without temporal rescaling. Moreover, the model offers a theoretical framework to impute missing values such as cloudy pixels.

Model

Gaussian Processes (GP) model:
Let \( S = \{(y_i,z_i)\}_{i=1}^n \) be a set of multidimensional and irregularly sampled signals. A signal \( Y \) is modeled as a vector of \( p \) independent random processes \( T \rightarrow \mathbb{R}^p \), with \( T = [0,T] \). The associated label is modeled by a discrete random variable \( Z \) taking its values in \( \{1, \ldots, C\} \). The model introduced here is based on two assumptions: 1) The coordinate processes \( Y_{b,c} \), \( b \in \{1, \ldots, p\} \) of \( Y \) are independent, 2) Each process \( Y_{b,c} \) is, conditionally to \( Z = c \), a Gaussian process. Then

\[
Y_c(t)|Z = c \sim GP(m_{b,c}(t), K_{b,c}(t,s)),
\]

where \( m_{b,c} : T \rightarrow \mathbb{R}^p \) is a mean function, and \( K_{b,c} \) a covariance kernel with hyperparameters \( \theta_{b,c} \). For example \( \theta_{b,c} = \{\gamma_{b,c}, \sigma_{b,c}^2, \beta_{b,c}\} \) with

\[
K_{b,c}(t,s) = \gamma_{b,c}^2 \delta(t-s) + \sigma_{b,c}^2 \delta_{b,c}.
\]

An irregularly sampled noisy signal \( y_i \) is observed on \( T_i \) time stamps \( \{t_1, \ldots, t_r\} \) in \( T \) and its \( b \)th coordinate is represented by a vector in \( \mathbb{R}^p \). We write \( y_{i,b} = [y_i(t_1)^T, \ldots, y_i(t_r)^T]^T \), with

\[
y_{i,b}|Z_i = c \sim N_{T_i}(\mu_{i,b,c}, \Sigma_{b,c}^I).
\]

There \( \mu_{i,b,c} = B_i \alpha_{b,c} \) is the sampled mean projected on a finite-dimensional space (\( B_i \) is the fixed design matrix, \( \alpha_{b,c} \) is the unknown vector of coordinates). \( \Sigma_{b,c}^I \) is the matrix kernel \( K_{b,c} \) evaluations at \( \{t_1, \ldots, t_r\} \).

Estimation:
- \( \alpha_{b,c} \) and \( \theta_{b,c} \) are estimated by maximizing the log-likelihood,

\[
-\frac{1}{2} \sum_{i,Z_i=c} \left( \log |\Sigma_{b,c}^I| + (y_{i,b} - B_i \alpha_{b,c})^T \Sigma_{b,c}^{-1} (y_{i,b} - B_i \alpha_{b,c}) \right).
\]

- \( \alpha_{b,c} \) is given by an explicit formula, while \( \theta_{b,c} \) is computed thanks to a gradient technique.

Classification and Imputation of missing values

The assigned class is given by the MAP rule from the posterior probability

\[
P(Z|y) = \frac{\exp \sum_{i=1}^n f_{T_i}(y_{i,b}, B_i \alpha_{b,c}, \Sigma_{b,c}^I(\theta_{b,c}))}{\sum_{c=1}^C \exp \sum_{i=1}^n f_{T_i}(y_{i,b} - B_i \alpha_{b,c}, \Sigma_{b,c}^I(\theta_{b,c})))}
\]

When the class is known to be \( c \), the missing value at \( t^* \) is estimated through the computation of conditional expectation.

\[
\hat{Y}_{b,c}(t^*) = B_i(t^*) \alpha_{b,c} + K_{b,c}(t^*, t_1:T) \Sigma_{b,c}^{-1} (y_{i,b} - B_i \alpha_{b,c})
\]

\[
\text{var}(\hat{Y}_{b,c}(t^*)) = K_{b,c}(t^*, t^*) - K_{b,c}(t^*, t_1:T) \Sigma_{b,c}^{-1} K_{b,c}(t_1:T, t^*)
\]

We also generalized this imputation when the class is unknown.

Validation (Synthetic data)

Example of two signals (dots) that belongs to two different classes

<table>
<thead>
<tr>
<th>Classification rate based on average time samples</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acc_{cap} (%)</td>
<td>52.8</td>
<td>52.9</td>
<td>74.3</td>
<td>93.9</td>
<td>94.2</td>
</tr>
<tr>
<td>Acc_{cap} (%)</td>
<td>64.3</td>
<td>85.3</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Imputation on two signals belonging to the same class.

Future work

We are now implementing the model for massive real data (Sentinel-2). We are also working on a new model when the bands are correlated.

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References: