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Graph-Based Approaches to Structural Universals and Complex States of Affairs

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Abstract. Structural universals have been introduced in the philosophical literature on examples such as chemical molecules composed of bonded atoms of different kinds. They are useful to handle complex abstract entities such as designs of artefacts, shapes, strings, words and texts, relevant in many areas of applied ontology. We use graph-theory as a unifying framework to review and compare the formal accounts proposed by Armstrong, Bennett and Mormann. We then propose a more expressive account, combining features of Bennett's and Mormann's proposals, able to model both the mereology of complex states of affairs and the structure of the particulars involved in them. Structural universals are explicitly represented; their structure, where a same universal can be part of a structural universal several times over, can be read off the graphs representing complex states of affairs.

Keywords. structural universal, state of affairs, mereology, graph theory

1. Introduction

Structural universals have been discussed by philosophers at least since the debate between Armstrong, Forrest and Lewis on the Australasian Journal of Philosophy (vol. 64) in 1986. Their very existence was contested by Lewis who defended the use of complex states of affairs instead, on the basis of mereological arguments. This debate is reflected today with two main proposals: Bennett's, who develops a formal account of structural universals and their parts [1], and Mormann's, who argues with Lewis in favour of complex states of affairs [2]. Structural universals are properties whose instances are particulars presenting a complex mereological structure. A key to the debate is whether that mereological structure can be described among universals or among the corresponding states of affairs. To take the classical example of chemical molecules such as methane CH_4, can one characterize the universal *being methane* in terms of the universals *being carbon* and *being hydrogen*? If so, how to account for the difference between methane CH_4 and methylene CH_2? Can *being hydrogen* be part of *being methane* four times over and of *being methylene* twice over? How to account for the difference between butane and isobutane molecules, both having the same number of carbon and hydrogen atoms (and the same number of bonding links)?

We hold that this debate on the duality between structural universals and complex states of affairs is relevant to applied ontology. When abstract entities such as kinds,

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species, designs, shapes, strings, words, or texts, that all can have multiple realizations, are in the domain of discourse, their structure becomes at stake. How do we represent that a car model has four times a certain kind of wheel? How can a sentence contain several times the same word?

In this paper, we review part of the formal solutions offered in the recent philosophical literature, taking graphs as a unifying framework. We start by giving elements of graph theory and mereology in the next two sections. Bennett's and Mormann's proposals are then analyzed in Sect. 4 and 5. We show that these solutions have limitations and then propose in Sect. 6 a way of combining and extending them so as to overcome these issues, illustrating our proposal on a few prototypical cases.

2. Graphs

A *graph* is pair $G = \langle V, E\rangle$ where V is a non-empty set of *vertices* or *nodes*, and E is a (possibly empty) set of non-ordered pairs of vertices in V, i.e., a set of *edges* or *arcs*. In a *direct graph* (*digraph*) the edges are oriented, they are ordered pairs of vertices. We denote the set of vertices (edges) of a graph G by V_G (E_G). The subgraph and induced subgraph relations $\subseteq_{\mathrm{G}}$ and $\sqsubseteq_{\mathrm{G}}$ are defined in (d1) and (d2), while (d3) introduces a structural isomorphism between graphs that abstracts from specific vertices and edges.

d1 $G \subseteq_{\mathrm{G}} H$ iff $V_G \subseteq V_H$ and $E_G \subseteq E_H$
d2 $G \sqsubseteq_{\mathrm{G}} H$ iff $V_G \subseteq V_H$ and $E_G = E_H/V_G$ (where / is the restriction operator)
d3 $G \equiv_{\mathrm{G}} H$ iff there is a bijection $\phi : V_G \to V_H$ s.t. $(u,v) \in E_G$ iff $(\phi(u),\phi(v)) \in E_H$

A *labeled (di)graph* is 4-tuple $G = \langle V, E, L, \lambda\rangle$ where V is a non-empty set of nodes, E is a set of directed edges (ordered pairs of nodes), L is a set of labels and $\lambda : V \to L$ is a total function (different nodes can share labels). We write v:**l** to say that **l** is the label of v, i.e., $\lambda(v) = \mathbf{l}$. (d4), (d5), and (d6) extend $\subseteq_{\mathrm{G}}$, $\sqsubseteq_{\mathrm{G}}$, and $\equiv_{\mathrm{G}}$ to the case of labelled graphs.

d4 $G \subseteq_{\mathrm{L}} H$ iff $V_G \subseteq V_H$, $E_G \subseteq E_H$, and $\lambda_G = \lambda_H/V_G$, i.e., the restriction of λ_H to V_G
d5 $G \sqsubseteq_{\mathrm{L}} H$ iff $V_G \subseteq V_H$, $E_G = E_H/V_G$, and $\lambda_G = \lambda_H/V_G$.
d6 $G \equiv_{\mathrm{L}} H$ iff there is a bijection $\phi : V_G \to V_H$ such that
(i) $(u,v) \in E_G$ iff $(\phi(u),\phi(v)) \in E_H$ and *(ii)* $\lambda_G(v) = \lambda_H(\phi(v))$.

A multidigraph is a tuple $G = \langle V, E, \tau, \eta\rangle$ where V is a non-empty set of vertices, E is a set of edges, and τ and η are two functions from E to V such that τ determines the *tail* of an edge (the initial vertex) while η its *head* (the final vertex), e.g., for an edge e from v_1 to v_2, $\tau(e) = v_1$ and $\eta(e) = v_2$. It is then possible to have different edges with the same tail and head. A *path* in a multidigraph G is a sequence $\langle e_1, \ldots, e_n\rangle$ of nodes in E_G such that for $1 \leq i \leq n-1$, $\eta(e_i) = \tau(e_{i+1})$.

3. Mereology

Parthood is probably the best studied formal relation in ontology. Depending on the domain of application and on the underlying motivations, some properties of parthood are considered as too strong, and then ruled out, or as evident and then accepted without hesitation. For instance, transitivity (a3) and extensionality (a7), i.e., that two entities with the same parts must be identical, have been extensively discussed (see, e.g. [3, 4]). We here only consider the following definitions and axioms, assuming a first-order language with the primitive predicate $\mathtt{P}$, where $\mathtt{P}(x,y)$ stands for "x is *part of* y".

- **d7** $\mathtt{PP}(x,y) \triangleq \mathtt{P}(x,y) \wedge \neg x = y$ — *Proper part*
- **d8** $\mathtt{O}(x,y) \triangleq \exists z(\mathtt{P}(z,x) \wedge \mathtt{P}(z,y))$ — *Overlap*
- **d9** $\mathtt{AT}(x) \triangleq \neg\exists y(\mathtt{PP}(y,x))$ — *Atom*
- **a1** $\mathtt{P}(x,x)$ — *Reflexivity*
- **a2** $\mathtt{P}(x,y) \wedge \mathtt{P}(y,x) \rightarrow x = y$ — *Antisymmetry*
- **a3** $\mathtt{P}(x,y) \wedge \mathtt{P}(y,z) \rightarrow \mathtt{P}(x,z)$ — *Transitivity*
- **a4** $\exists y(\mathtt{P}(y,x) \wedge \mathtt{AT}(y))$ — *Atomicity*
- **a5** $\mathtt{PP}(x,y) \rightarrow \exists z(\mathtt{P}(z,y) \wedge \neg\mathtt{O}(z,x))$ — *Weak supplementation*
- **a6** $\neg\mathtt{P}(y,x) \rightarrow \exists z(\mathtt{P}(z,y) \wedge \neg\mathtt{O}(z,x))$ — *Strong supplementation*
- **a7** $(\exists z(\mathtt{PP}(z,x)) \wedge \forall z(\mathtt{PP}(z,x) \rightarrow \mathtt{PP}(z,y))) \rightarrow \mathtt{P}(x,y)$ — *Extensionality*
- **t1** $(\text{a6}) \vdash (\text{a5})$
- **t2** $\{(\text{a1}),(\text{a2}),(\text{a3}),(\text{a6})\} \vdash (\text{a7})$

(a1)-(a3) are usually considered as the minimal basis, the *ground mereology* [3], although too weak to capture the 'essence' of parthood, since $\mathtt{P}$ simply is a classical *partial order* relation. Simons [4] assumes that, in addition to (a1)-(a3), $\mathtt{P}$ must satisfy weak supplementation (a5). A stronger version of this principle is often considered, see (a6) and (t1). In the following we use these principles to understand which type of mereological systems we are representing by means of certain relations between structured entities.[2]

4. Bennett and having a part twice over

To represent structural universals—e.g., *being methane* is a structural composition of *being carbon*, *being hydrogen*, and *being bonded*, or, in case of strings, *being baa* is a composition of *being b*, *being a*, and *preceding*—Bennett [1] develops a theory that allows an entity to be part of another entity several times over. She claims that the *being three feet from* binary relation may hold multiple times between the same entities, e.g., "consider two antipodal points on a sphere, such that the shortest distance between them along the surface is three feet" Bennett [1, p.83]. One can then assume that the same applies to parthood, e.g., the parthood relation between *being hydrogen* and *being methane* holds four times, while it holds two times between *being a* and *being baa*. Bennett's theory is based on the distinction between *slots* (roles) vs. *fillers* (occupiers) of slots and considers two primitives: $\mathtt{F}(x,s)$ stands for "x fills s" while $\mathtt{P_s}(s,x)$ stands for "s is a parthood slot of x". These two primitives are governed by the following axioms:

- **a8** $\mathtt{F}(x,s) \rightarrow \exists y(\mathtt{P_s}(s,y))$ — *only slots are filled*
- **a9** $\mathtt{F}(x,s) \rightarrow \neg\exists y(\mathtt{P_s}(x,y))$ — *slots cannot fill*
- **a10** $\mathtt{P_s}(s,x) \rightarrow \neg\exists r(\mathtt{P_s}(r,s))$ — *slots don't have slots*
- **a11** $\exists s(\mathtt{P_s}(s,x)) \rightarrow \exists r(\mathtt{P_s}(r,x) \wedge \mathtt{F}(x,r))$ — *improper parthood slots*
- **a12** $\mathtt{P_s}(r,y) \wedge \mathtt{F}(y,s) \wedge \mathtt{P_s}(s,x) \rightarrow \mathtt{P_s}(r,x)$ — *slot inheritance*

[2] We do not consider here the closure principles, i.e., rules that establish how the mereological domain can be closed under sum, product, difference, etc.

a13 $\mathtt{F}(x,s) \wedge \mathrm{P_S}(s,y) \wedge \mathtt{F}(y,r) \wedge \mathrm{P_S}(r,x) \rightarrow x = y$ *mutual occupancy is identity*
a14 $\mathrm{P_S}(s,x) \rightarrow \exists! y(\mathtt{F}(y,s))$ *single occupancy*
a15 $\exists s(\mathrm{P_S}(s,x)) \wedge \exists s(\mathrm{P_S}(s,y)) \rightarrow (\neg\exists s(\mathrm{P_S}(s,x) \wedge \mathtt{F}(y,s)) \rightarrow \exists r(\mathrm{P_S}(r,y) \wedge \neg\mathrm{P_S}(r,x)))$ *slot strong supplementation*

(d10) defines parthood, i.e., x is part of y when x fills one slot of y while the other mereological relations are defined as done in Sect. 3. An entity x is part of y twice over when x fills two different slots of y. Bennett shows that parthood is transitive, antisymmetric, and (conditionally) reflexive, but that it does not satisfy neither weak supplementation nor extensionality. On the other hand, Bennett shows that both weak supplementation and extensionality, reformulated as in (t3)-(t4), hold for parthood slots.

d10 $\mathtt{P}(x,y) \triangleq \exists z(\mathrm{P_S}(z,y) \wedge \mathtt{F}(x,z))$ *parthood*
t3 $\mathtt{PP}(x,y) \rightarrow \exists z(\mathrm{P_S}(z,y) \wedge \neg\mathrm{P_S}(z,x))$ *slot weak supplementation*
t4 $\exists z(\mathtt{PP}(z,x) \vee \mathtt{PP}(z,y)) \rightarrow (x = y \leftrightarrow \forall z(\mathrm{P_S}(z,x) \leftrightarrow \mathrm{P_S}(z,y)))$ *slot extensionality*

To understand how Bennett's theory can be integrated into a graph-based framework we show how it can be interpreted in terms of multidigraphs $G = \langle V, E, \tau, \eta \rangle$ where (i) for all $v \in V$ there exists a $e \in E$ such that $\tau(e) = \eta(e) = v$ (there are loops for every node); and (ii) $\langle V, E \backslash \{e \mid \tau(e) = \eta(e)\}, \tau', \eta' \rangle$ is acyclic (τ' and η' are the restrictions of τ and η to the new set of edges). Assume to have a graph G that satisfies such constraints, and an interpretation function I from the language of Bennett's theory into G. Our idea is that slots are interpreted into edges, fillers into vertices, and

- $\mathtt{F}(x,s)$ iff $\tau(s^I) = x^I$
- $\mathrm{P_S}(s,x)$ iff there exists a path $\langle s^I, e_1, \ldots, e_n \rangle$ s.t. $\eta(e_n) = x^I$.

We can prove that (a8)-(a15) are satisfied:

(a8) By hypothesis $\tau(s^I) = x^I$. It is enough to take $y^I = \eta(s^I)$.
(a9) By hypothesis $\tau(s^I) = x^I$ thus x^I cannot be an edge as required by $\mathrm{P_S}(x,y)$.
(a10) By hypothesis there is a path $\langle s^I, e_1, \ldots, e_n \rangle$ s.t. $\eta(e_n) = x^I$, so s^I cannot be a vertex as required by $\mathrm{P_S}(r,s)$.
(a11) By hypothesis there is a path $\langle s^I, e_1, \ldots, e_n \rangle$ s.t. $\eta(e_n) = x^I$. By the hypothesis (i) on the admitted multidigraphs there is an $e \in E$ s.t. $\tau(e) = \eta(e) = x^I$. Consider $r^I = e$.
(a12) By hypothesis there exist two paths: $\langle r^I, e_1, \ldots, e_m \rangle$ such that $\eta(e_m) = y^I$ and $\langle s^I, e'_1, \ldots, e'_n \rangle$ such that $\eta(e'_n) = x^I$. The condition $\mathtt{F}(y,s)$ implies that $\tau(s^I) = y^I$, thus $\eta(e_m) = \tau(s^I)$ and consequently $\langle r^I, e_1, e_m, s^I, e'_1, \ldots, e'_n \rangle$ is a path where $\eta(e'_n) = x^I$.
(a13) By hypothesis there are two paths $\langle s^I, e_1, \ldots, e_n \rangle$ and $\langle r^I, e'_1, \ldots, e'_m \rangle$ s.t. $\tau(s^I) = \eta(e'_m) = x^I$ and $\tau(r^I) = \eta(e_n) = y^I$, thus $\langle s^I, e_1, \ldots, e_n, r^I, e'_1, \ldots, e'_m \rangle$ is a cycle. By the hypothesis (ii) on the acyclicity of admitted multidigraphs, the only cycles are paths $\langle e''_1, \ldots, e''_i \rangle$ s.t. $\eta(e''_1) = \tau(e''_1) = \cdots = \eta(e''_i) = \tau(e''_i)$. So $x^I = \tau(s^I) = \eta(e_n) = y^I$.
(a14) It is enough to observe that τ is a function so that the tail of an edge is unique.
(a15) From the hypotheses, $x \neq y$, otherwise, by (a11), there is a slot s s.t. $\mathrm{P_S}(s,x) \wedge \mathtt{F}(x,s)$. Take two paths $\langle e_1, \ldots, e_n \rangle$, $\langle e'_1, \ldots, e'_m \rangle$ s.t. $\eta(e_n) = x^I$ and $\eta(e'_m) = y^I$. Since η is a function from edges to nodes and $x^I \neq y^I$, $e_n \neq e'_m$. It is enough to take $r^I = e'_m$.[3]

[3] According to this interpretation, different objects cannot share *direct* slots (slots represented by paths of length 1), i.e., direct slots depend on the object they are slots of. Objects can share a slot only when they have a common part filling their slots that, in its turn, has a slot. It is not clear to us if Bennett had this in mind.

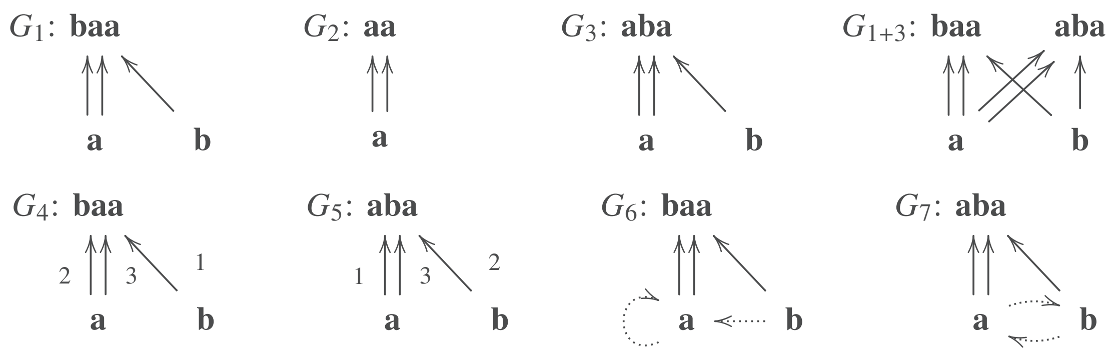

Figure 1. Alternative representations of structural universals.

An object x is part of an object y twice (or several times) over when two (or several) edges have (the vertex representing) x as tail and (the vertex representing) y as head. For instance, the universal *being baa* can be represented as G_1 in Fig. 1 where there are two different slots linking **a** with **baa**, and *being aba* as in G_3.[4] G_2 and G_{1+3} (**baa** and **aba** in a single graph) in Fig. 1 show, respectively, that parthood (see (d10)) does not satisfy weak supplementation (a5) and extensionality (a6). Furthermore, (a15) (and (t3)-(t4)) is quite weak. For instance, the graph $H : \mathbf{a} \leftarrow \mathbf{b}$ satisfies Bennett's axioms. In H not only **a** has as unique proper part **b** but it also has only one proper (excluding the loop) slot. As we will see, this type of graphs can be useful to represent the *constitution* relation.

Discussing the butane vs. isobutane example, very similar (but not identical, see below) to the **baa** vs. **aba** example, Fisher [5] claims that in Bennett's approach they become identical because of the slot extensionality (t4). We do not agree on that analysis. G_1 and G_3 have the same number of slots filled by the same universals, but the slots —in this graph interpretation, the edges—are private to them, **baa** and **aba** do not share any slot (or edge), only all their parts, as can be clearly seen on the whole graph G_{1+3}. However, Fisher has a point about the structural difference between the representation of **baa** vs. **aba** in Bennett's approach. In fact, G_1 and G_3 are isomorphic ($G_1 \equiv_G G_3$), they differ only because of their top nodes. To obtain a representation clarifying the structural difference between **baa** and **aba**, one possibility is to label slots (as suggested by Fisher), where the labels represent kinds of slots, that is, different ways an entity can be involved in the whole, as in G_4 and G_5 in Fig. 1.[5] This solution however leaves all the structural information necessary to distinguish the two wholes encapsulated into edge labels. A more interesting possibility is the preceding relation between the letters composing a string. Edges are labelled here too, but in this case we only distinguish parthood (solid arrows) from preceding relations (dotted arrows) rather than different kinds of parthood, as in G_6 and G_7 in Fig. 1. The reason why two edges link **a** to **baa** is explicit in G_6: one edge corresponds to the **a** preceded by **b**, the other to the **a** preceded by **a**; while in G_7 one edge corresponds to the **a** preceding **b**, the other to the **a** preceded by **b**.

This last solution has nevertheless some limitations. Consider, for instance, the universals $\mathbf{u_1}$ and $\mathbf{u_2}$ that have the complex structure depicted in Fig. 2. These diagrams do not show graphs interpreting Bennett's representations as discussed above, since the universal **a** appears twice and the structural universals $\mathbf{u_1}$ and $\mathbf{u_2}$ are missing, but a visual

[4] We omit to represent the loops and the *preceding* universal (that however is considered later).

[5] Fisher talks about ordering or arranging slots, here we are considering different kinds of slots.

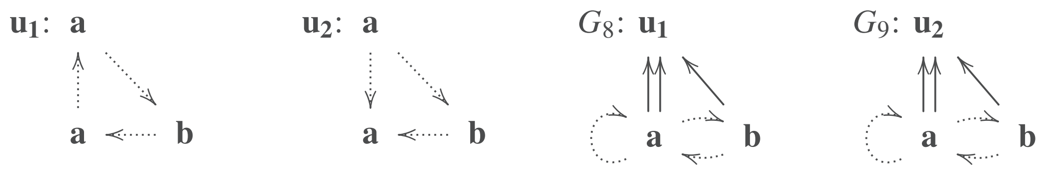

Figure 2. Problem with some structured universals.

representation akin to standard molecule diagrams in which multiple occurrences of, say, H appear. Clearly, $\mathbf{u_1}$ and $\mathbf{u_2}$ have non-isomorphic structures. However, when we turn them into the graphs G_8 and G_9 in Fig. 2 they become isomorphic because we cannot distinguish the cases where the top **a** is linked to the bottom **a** from the case where the bottom **a** is linked to the top **a**. What the example shows is that, starting from the universals **a** and **b**, different wholes can satisfy the conditions that **a** is part of the whole twice over while **b** only once, and that $\mathbf{a} \dashrightarrow \mathbf{a}$, $\mathbf{b} \dashrightarrow \mathbf{a}$ and $\mathbf{a} \dashrightarrow \mathbf{b}$. The difference between G_8 and G_9 is then still encapsulated into their top nodes.[6] This is also what happens in the case of butane vs. isobutane. Not only these molecules are composed by the same number of hydrogen and carbon atoms, but the number of carbon-carbon and hydrogen-carbon bonding links are the same, so Bennett's proposal, even extended with bonding universals as relations among universals, cannot account for their difference.

For a more explicit representation of the structure, while retaining Bennett's idea of multiple parthood relationships defined on the same universals, we build on Mormann's work [2] aiming at representing complex *states of affairs* rather than structural universals.

5. Mormann and complex states of affairs

Mormann [2] studies the mereological structure of complex states of affairs (states hereafter) by using the 'tool' of (labelled) graphs. In his account, possibly complex states are represented by graphs and parthood between states by the subgraph relation $\subseteq_G$ (or $\subseteq_L$). In this way he intends to specify the mereology of states, that actually are structured entities, without committing to structural universals and structured particulars. More specifically, Mormann assumes that (labelled) nodes represent *thick* particulars, as opposed to *thin* particulars, a distinction introduced by Armstrong [7]. For Armstrong, a *thin* particular is a particular "in abstraction from its properties" [7, p.123] while a *thick* particular is a particular "taken along with all and only the particular's non-relational properties" [7, p.125], i.e., the thick particular is identical with the state *a's being N*, where N is the nature of a, i.e., the conjunction of all its non-relational properties. In Mormann's graphs, edges represent instantiations of binary universals. Mormann too focuses on the field of chemistry. This means that the bonding universal in molecules holds between *thick* rather than *thin* particulars as assumed by Armstrong in [7]. Similarly, the labels of the nodes identify kinds of thick particulars that can be taken to correspond to the universal that is the nature of the thin particulars (we write in the same way the nature of the thin particular and the kind of the thick particular). For instance, the labelled graph M_1 in Fig. 3 represents the complex state where a, b, c, d are thick particulars of kind **h**, i.e,

[6] As observed by Cotnoir [6] defining the mereological sum operator in the framework proposed by Bennett also poses a general problem, which we ignore here.

$$
\begin{tikzcd}
& a{:}\mathbf{h} \arrow[d, dash, dotted] & \\
M_1{:}\ b{:}\mathbf{h} \arrow[r, dash, dotted] & e{:}\mathbf{c} \arrow[r, dash, dotted] \arrow[d, dash, dotted] & c{:}\mathbf{h} \\
& d{:}\mathbf{h} &
\end{tikzcd}
\qquad
S_1{:}\ a{:}\mathbf{b} \dashrightarrow b{:}\mathbf{a} \dashrightarrow c{:}\mathbf{a}
\qquad
S_2{:}\ d{:}\mathbf{a} \dashrightarrow e{:}\mathbf{b} \dashrightarrow f{:}\mathbf{a}
$$

Figure 3. A molecule of methane and two strings.

instantiations of the universal *being hydrogen* by different particulars, *e* is a thick particular of kind **c** that corresponds to an instantiation of *being carbon*, and the edges are instantiations of the bonding universal holding between specific thick particulars. M_1 then represents the instantiation of *being methane* by the thin particulars involved in the thick particulars represented by the nodes. Similarly, the graphs S_1 and S_2 in Fig. 3 represent, respectively, the instantiation of *being baa* and *being aba* by the thin particulars involved in the thick particulars of their nodes. S_1 and S_2 are not L-isomorphic (see (d6)), and, importantly, the same would hold for instantiations of the complex universals $\mathbf{u_1}$ and $\mathbf{u_2}$ of Fig. 2. As said, Mormann refuses both structural universals and structured particulars. For instance, M_1 represents a complex state without referring to *being methane* or to a thick methane-particular. The instantiation of *being methane* is a *façon de parler* to express the complex pattern of instantiations (by several thin particulars) in M_1.

To represent the mereological structure of complex states, Mormann establishes a parallel with the mereological structure of sets. Starting from a set X, one can build the powerset of X (the set of all the subsets of X) denoted by $\mathcal{P}(X)$ and $\mathcal{P}^{-\emptyset}(X) = \mathcal{P}(X)\backslash\emptyset$. The relational structure $\langle \mathcal{P}^{-\emptyset}(X), \subseteq\rangle$ is a classical mereological system which satisfies (a1)-(a7), i.e., a Boolean algebra without the bottom element. In the case of states, one can start from a graph G and consider the relational structure $\langle \mathcal{P}_{\subseteq}(G), \subseteq_{\mathrm{L}}\rangle$ where $\mathcal{P}_{\subseteq}(G)$ denotes the set of all the $\subseteq_{\mathrm{L}}$-subgraphs of G (note that $\langle\emptyset,\emptyset\rangle$ is not a graph because the set of nodes must be non-empty). Mormann shows that $\langle \mathcal{P}_{\subseteq}(G), \subseteq_{\mathrm{L}}\rangle$ (and $\langle \mathcal{P}_{\subseteq}(G), \subseteq_{\mathrm{G}}\rangle$ in case of non-labelled graphs) is not a classical mereological system, actually he proves (cf. [2] theorem 2.9) that, by adding $\langle\emptyset,\emptyset\rangle$ into $\mathcal{P}_{\subseteq}(G)$, $\langle \mathcal{P}_{\subseteq}(G), \subseteq_{\mathrm{L}}\rangle$ is a Heyting algebra that in general is not a Boolean algebra. In particular $\langle \mathcal{P}_{\subseteq}(G), \subseteq_{\mathrm{L}}\rangle$ satisfies reflexivity, antisymmetry and transitivity but not weak supplementation (a5) and extensionality (a7). For counterexamples, take $G = \langle\{u{:}\mathbf{a}, v{:}\mathbf{b}, z{:}\mathbf{c}\}, \{(u,v),(v,z)\}\rangle$, $X = \langle\{u{:}\mathbf{a}, v{:}\mathbf{b}\}, \{(u,v)\}\rangle$, $Y = \langle\{u{:}\mathbf{a}, v{:}\mathbf{b}\}, \emptyset\rangle$ and $Z = \langle\{u{:}\mathbf{a}, v{:}\mathbf{b}, z{:}\mathbf{c}\}, \emptyset\}\rangle$. For (a5): both X and Y are in $\mathcal{P}_{\subseteq}(Y)$, $Y \subseteq_{\mathrm{L}} X$, and $X \neq Y$; however, all graphs $\subseteq_{\mathrm{L}}$-included in X, i.e., Y, $\langle\{u{:}\mathbf{a}\},\emptyset\rangle$, and $\langle\{v{:}\mathbf{b}\},\emptyset\rangle$, are also $\subseteq_{\mathrm{L}}$-included in Y. For (a7): both X and Z are in $\mathcal{P}_{\subseteq}(G)$; X has three proper subgraphs, $\langle\{u{:}\mathbf{a}, v{:}\mathbf{b}\},\emptyset\rangle$, $\langle\{u{:}\mathbf{a}\},\emptyset\rangle$, and $\langle\{v{:}\mathbf{b}\},\emptyset\rangle$, that are also proper subgraphs of Z but $X \not\subseteq_L Z$.

As noted, the graphs S_1 and S_2 in Fig. 3 are not L-isomorphic. In addition, the mereological systems $\langle \mathcal{P}_{\subseteq}(S_1), \subseteq_{\mathrm{L}}\rangle$ and $\langle \mathcal{P}_{\subseteq}(S_2), \subseteq_{\mathrm{L}}\rangle$ contain other non-L-isomorphic graphs, e.g., $X = \langle\{b{:}\mathbf{a}, c{:}\mathbf{a}\}, \{\langle b,c\rangle\}\rangle \subseteq_{\mathrm{L}} S_1$ is not L-isomorphic to any subgraph of S_2 and $Y = \langle\{d{:}\mathbf{a}, e{:}\mathbf{b}\}, \{\langle d,e\rangle\}\rangle \subseteq_{\mathrm{L}} S_2$ is not L-isomorphic to any subgraph of S_1.

According to Mormann, by assuming binary universals to hold between thick particulars, his approach solves some problems the framework of Armstrong suffers of. Adopting Armstrong's view, nodes would represent thin (not thick) particulars, edges binary (bonding) relationships between thin particulars, and labels universals (and the labelling function a sort of instantiation). Mormann claims that "for many essentially different molecules it [Armstrong's approach] gives the same mereological structure" [2, p.413]. In particular he refers to the case of butane and isobutane molecules but a similar problem applies to S_1 and S_2 in Fig. 3. A sketch of Mormann's argument follows.

Suppose to have states in the domain and to denote by $\{\mathbf{p}|a\}$ and $\{\mathbf{r}|ac\}$ the states that correspond to 'a's being **p**' and 'a and c being in the relation **r**'. Thus, the graphs S_1 and S_2 in Fig. 3 correspond, respectively, to the complex states $SOA_1 = \{\mathbf{b}|a, \mathbf{a}|b, \mathbf{a}|c, \mathbf{prec}|ab, \mathbf{prec}|bc\}$ and $SOA_2 = \{\mathbf{a}|d, \mathbf{b}|e, \mathbf{a}|f, \mathbf{prec}|de, \mathbf{prec}|ef\}$. The mereological structures $\langle \mathcal{P}^{-\emptyset}(SOA_1), \subseteq\rangle$ and $\langle \mathcal{P}^{-\emptyset}(SOA_2), \subseteq\rangle$ are different because $SOA_1 \neq SOA_2$. Mormann argues that by assuming an equivalence relation between complex states that just counts the number of atomic states (singletons) of the same kind (e.g., in SOA_1 and SOA_2 there are three kinds of states, namely, **a**-, **b**- and **prec**-states) then SOA_1 and SOA_2 are equivalent (both have 2 **a**-states, 1 **b**-state, and 2 **prec**-states).

However, this is a very rough way of defining equivalence between complex states. Armstrong's notion of state is richer, every atomic state involves not only a universal (the kind of the state) but also the thin particulars that instantiate such universal. In this view it is possible to introduce a stronger notion of equivalence: states S and R are equivalent if there is a 1-1 correspondence between the particulars involved in S and R such that for every atomic state part of S there is an atomic state *of the same kind* that is included in R and that *involves the corresponding particulars* and vice versa. According to this stronger definition, SOA_1 and SOA_2, as well as some of their parts, are not equivalent. The criticism of Mormann to Armstrong seems then to hold only in the case one embraces a very rough definition of equivalence between states. According to our view, the structures considered by Mormann are just more restrictive than Armstrong's. For instance $\{\mathbf{b}|a, \mathbf{a}|b, \mathbf{bond}|bc\} \in \mathcal{P}^{-\emptyset}(SOA_1)$ but there are no subgraphs of S_1 that correspond to this state. Vice versa, for every subgraph of S_1 it is easy to find a corresponding state in $\mathcal{P}^{-\emptyset}(SOA_1)$. For instance $a{:}\mathbf{b} \dashrightarrow b{:}\mathbf{a}$ corresponds to $\{\mathbf{b}|a, \mathbf{a}|b, \mathbf{prec}|ab\} \in \mathcal{P}^{-\emptyset}(SOA_1)$. It is because some of the states considered by Armstrong are ruled out by Mormann that the extensionality of $\subseteq_{\mathrm{L}}$ (and $\subseteq_{\mathrm{G}}$) is lost. Mormann's proposal provides then a stricter characterization of the notion of state of affairs. For instance, the fact that $\{\mathbf{b}|a, \mathbf{a}|b, \mathbf{bond}|bc\}$ does not correspond to any graph in $\langle \mathcal{P}_{\subseteq}(S_1), \subseteq_{\mathrm{L}}\rangle$ means that, according to Mormann, such configuration of the world is not a state of affairs. Interestingly, this way of ruling out some of Armstrong's states is based on the notion of graph itself, i.e., it is the assumption of representing states by graphs that rules out some states accepted by Armstrong. This assumption has nothing to do with the mereological structure taken into account, it concerns the specific commitment about the nature of states. It seems to us that it is also orthogonal with the choice of assuming the bonding relation as defined on thick (rather than thin) particulars. However, we do agree with Mormann that forcing states to include all the thick particulars whose thin particulars are involved in instances of binary relations makes sense. This is why we adopt the approach based on labelled graphs.

Starting from Mormann's position, one could be still more restrictive. Assume that states are represented by graphs but consider $\langle \mathcal{P}_{\sqsubseteq}(G), \sqsubseteq_{\mathrm{L}}\rangle$ where $\mathcal{P}_{\sqsubseteq}(G)$ denotes the set of all the $\sqsubseteq_{\mathrm{L}}$-, instead of $\subseteq_{\mathrm{L}}$-, subgraphs of G, i.e., *in the context of the graph G*, some $\subseteq_{\mathrm{L}}$-subgraphs are ruled out to keep only induced G-subgraphs. For instance, $\langle\{a{:}\mathbf{b}, b{:}\mathbf{a}\}, \emptyset\}\rangle$ belongs to $\mathcal{P}_{\subseteq}(S_1)$ but not to $\mathcal{P}_{\sqsubseteq}(S_1)$, while $\langle\{a{:}\mathbf{b}, b{:}\mathbf{a}\}, \{\langle a, b\rangle\}\rangle$ belongs to both. The idea is that when in the graph G (representing a complex state) there is an edge between two nodes (a binary relation between two thin/thick particulars), one cannot exclude this edge in subgraphs including the two nodes. In other words, admissible substates are

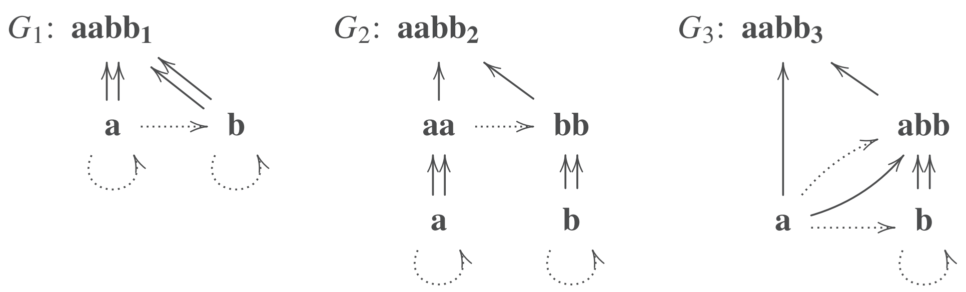

Figure 4. Three different structural universals specializing *being aabb*.

those portions of a state obtained by focusing on a subset of particulars, together with all the facts internal to this subset—not only the properties of these particulars as in Mormann's subgraphs, but all binary relations holding among them as well. We prove that $\langle \mathcal{P}_{\sqsubseteq}(G), \sqsubseteq_{\mathrm{L}} \rangle$ satisfies (a1)-(a7), so by ruling out some graphs, supplementation and extensionality are recovered. For reasons of space we provide only the proof of strong supplementation that implies weak supplementation and extensionality, see (t1) and (t2).

t5 $\langle \mathcal{P}_{\sqsubseteq}(G), \sqsubseteq_{\mathrm{L}} \rangle$ satisfies (a6) (strong supplementation).
Proof. First we prove that if $X, Y \in \mathcal{P}_{\sqsubseteq}(G)$ then $V_X \subseteq V_Y$ implies $X \sqsubseteq_{\mathrm{L}} Y$. $X, Y \in \mathcal{P}_{\sqsubseteq}(G)$ implies that $E_X = E_G/V_X = \{(u,v) \in E_G \mid u,v \in V_X\}$ and $E_Y = E_G/V_Y = \{(u,v) \in E_G \mid u,v \in V_Y\}$. Then $E_Y/V_X = \{(u,v) \in E_G \mid u,v \in V_X \cap V_Y\}$. If $V_X \subseteq V_Y$, then $E_Y/V_X = \{(u,v) \in E_G \mid u,v \in V_X\} = E_X$. Similarly for the restrictions of the labelling functions. Thus $E_X = E_Y/V_X$, $\lambda_X = \lambda_Y/V_X$, and $V_X \subseteq V_Y$, i.e., $X \sqsubseteq_{\mathrm{L}} Y$. Now, from $X, Y \in \mathcal{P}_{\sqsubseteq}(G)$ but $Y \not\sqsubseteq_L X$ we have $V_Y \not\subseteq V_X$, so there exists $v \in V_Y \backslash V_X$ such that $\lambda(v) = \mathbf{a}$. $Z = \langle \{v{:}\mathbf{a}\}, E_Y/\{v\} \rangle$ is such that $Z \sqsubseteq_{\mathrm{L}} Y$ and it belongs to $\mathcal{P}_{\sqsubseteq}(G)$ ($\sqsubseteq_{\mathrm{L}}$ is transitive). But because $V_Z \cap V_X = \emptyset$, X and Z cannot have a common subgraph. □

Before moving to expounding our proposal, we want to highlight an important difference between the approaches of Bennett and Mormann (and Armstrong). Mormann's approach is combinatorial: starting from a graph, all its parts are generated. For instance, starting from $G = \mathbf{a} \dashrightarrow \mathbf{a} \dashrightarrow \mathbf{b} \dashrightarrow \mathbf{b}$, $\mathcal{P}_{\subseteq}(G)$ (and $\mathcal{P}_{\sqsubseteq}(G)$) necessarily contains all the 'intermediate wholes' $\mathbf{a} \dashrightarrow \mathbf{a}$, $\mathbf{a} \dashrightarrow \mathbf{b}$, and $\mathbf{b} \dashrightarrow \mathbf{b}$ that are $\subseteq_{\mathrm{L}}$- (and $\sqsubseteq_{\mathrm{L}}$-) included in G. On the other hand, Bennett's approach allows to explicitly select what are the relevant parts for a given complex entity (universal), i.e., there is a non-systematic filtering process that depends on the specific entity considered. For instance, the three graphs in Fig. 4 represent different ways of viewing a string universal **aabb**. All these three alternatives satisfy the order constraints $\dashrightarrow$ specified in G but they differ in *mereological* terms. In G_1 the string is directly composed by two letters of kind **a** and two letters of kind **b**, in G_2 the string is composed by two shorter strings each one composed by two letters of the same kind, and in G_3 the string is composed by one letter and one shorter string composed by three letters of two kinds. It easy to check that G_1-G_3 are not isomorphic, it is enough to count the number of edges (representing the slots). In Mormann's approach this difference cannot be represented because, once the notion of subgraph is fixed, a graph representing a complex state has a unique mereological structure. This is also linked to the refusal of structured particulars and structural universals, i.e., in Mormann's approach all the particulars and universals are atomic, there are only relational (bonding) patterns of thick particulars (of given kinds). Vice versa, the idea of having different (mereological) levels of entities with 'intermediate structured components' is important to represent, for instance, the intended assembly of mechanical artefacts. In this view, in G_1 the artefact is obtained directly assembling four constructional (atomic) parts while in G_2 it has two different components both composed of two constructional parts. To deal with these situations, we propose a framework that integrates Mormann's and Bennett's approaches.

6. Proposal

We are now in the position to propose a graph account able to represent not only complex states of affairs but also structured particulars and structural universals.

We follow Mormann in grounding the representation on particulars and states, to avoid the limitations in expressivity of Bennett's approach. But we depart from Mormann's proposal by explicitly representing both structured particulars and structural universals[7] and by distinguishing parthood among states from parthood among thin particulars called here individuals[8] (parthood among universals being derived from parthood among individuals). In our graphs, a vertex represents a structured or unstructured individual while its label denotes the structural or non structural universal that, in Armstrong's terms, is its nature.[9] We add proper parthood relationships between individuals as edges in the graph, in addition to edges in Mormann's graphs representing binary (e.g., bonding or preceding) relationships. The parthood relation between states (represented by graphs) is modeled through the induced subgraph relation $\sqsubseteq_{\mathrm{L}}$. As seen above with (t5), $\sqsubseteq_{\mathrm{L}}$ filters out some of Mormann's substates and yields an extensional mereology.

Introducing explicitly in the graph structured individuals with their structural universals and their parthood relationships is what we adopt from Bennett's account. We welcome the possibility to represent only relevant structural universals and individuals, without imposing an extensional mereology, as seen above discussing Fig. 4. In addition, a novel possibility arises in our combined approach: binary relations may hold on any individual, atomic or structured. This allows for, for instance, the representation of relationships between words without necessarily reducing them to the relationships between their composing letters. More importantly, it allows relationships on structured individuals that cannot be reduced to relationships between atomic individuals. For instance, the relationship of *Lea loving her cat* can hardly be reduced to some relations between certain subparts of Lea's body with certain subparts of her cat. We then have the possibility to represent emerging relationships that are typical of a given ontological or granularity level. In this case, the person level significantly differs from the body part level.

Formally, we consider a graph structure $\langle \mathcal{P}_{\sqsubseteq}(G), \sqsubseteq_{\mathrm{L}} \rangle$, where $G = \langle V, E^p, E^r, L, \tau, \eta, \lambda \rangle$ is a labelled multidigraph such that:

– V is a non-empty set of vertices representing individuals, atomic or structured;
– E^p is a set of edges representing proper parthood relationships among individuals;
– E^r is a set of edges representing other binary relationships among individuals;

[7] When Mormann claims that the thin particulars involved in the thick particulars in the graph M_1 in Fig. 3 'instantiate' *being methane* he just suggests that methane molecules are composed by thin particulars, but neither these molecules nor the universal *being methane* exist in his account.

[8] Individuals are assumed as disjoint from states, i.e., ontologically, they differ from states even though both individual and states are particulars.

[9] This means that we cannot represent the structure of many properties, for instance distributional properties like *being polka-dotted*. This would require to significantly extend or alter the approach.

– L is a set of labels for vertices, representing nature-universals of individuals;[10]
– τ and η are total functions $E^r \cup E^p \to V$ for edges' tail and head;
– λ is a total labelling function $V \to L$.

and G satisfies the following two structural constraints:

(*i*) $\langle V, E^p, \tau/E^p, \eta/E^p\rangle$ is acyclic (and thus without loops);[11]
(*ii*) any two $G_1, G_2 \in \mathcal{P}_{\sqsubseteq}(G)$ such that $v_1 \in V_{G_1}$, $v_2 \in V_{G_2}$ and:

- $\lambda(v_1) = \lambda(v_2)$;
  i.e., v_1 and v_2 represent individuals with the same nature;
- there are $v'_1, v'_2 \in V_G$, $e_1, e_2 \in E^p_G$ s.t. $\tau(e_1) = v'_1$, $\tau(e_2) = v'_2$, $\eta(e_1) = v_1$, $\eta(e_2) = v_2$
  i.e., v_1 and v_2 represent structured individuals;
- for any $v'_1 \in V_{G_1}$, either $v'_1 = v_1$ or there is $e_1 \in E^p_{G_1}$ s.t. $\tau(e_1) = v'_1$ and $\eta(e_1) = v_1$ and similarly for G_2;
  i.e., all vertices in G_1 (G_2) are direct parts of v_1 (v_2);
- for any $e \in E^p_G$ if $\eta(e) = v_1$ then $\tau(e) \in V_{G_1}$ and if $\eta(e) = v_2$ then $\tau(e) \in V_{G_2}$
  i.e., all direct parts of v_1 and v_2 in G are also in G_1 and G_2;

are L-isomorphic (d6).

Constraint (*ii*) forces all instances of a structural universal to have the same internal structure, that is, to be isomorphic in terms of parts and relations between these parts.

Given such a graph G as a model of a whole state of affairs, the structure $\langle \mathcal{P}_{\sqsubseteq}(G), \sqsubseteq_{\mathrm{L}}\rangle$ is taken as a model of all its substates and parthood relations among states, forming an extensional mereology as shown in the previous section. Notice that states may now explicitly contain parthood relationships between individuals (edges from E^p).

Induced $\sqsubseteq_{\mathrm{L}}$-subgraphs of G will keep constraint (*i*) imposed on G but not constraint (*ii*). The verification of (*i*) is trivial. For (*ii*) consider $G : u{:}\mathbf{a} \leftarrow x{:}\mathbf{b}, v{:}\mathbf{a} \leftarrow y{:}\mathbf{b}$ and $G' : u{:}\mathbf{a} \leftarrow x{:}\mathbf{b}, v{:}\mathbf{a}$, then $G' \sqsubseteq_{\mathrm{L}} G$ but $u{:}\mathbf{a} \leftarrow x{:}\mathbf{b}$ and $v{:}\mathbf{a}$ are not L-isomorphic. An interesting extension might thus be obtained by restricting $\sqsubseteq_{\mathrm{L}}$ even more so as to keep as subgraphs of G only those G' such that for all individuals of a given kind (with the same label) in G' that are structured individuals in G, either none or all of their direct parts in G are also in G'. This would force G' to satisfy the structural homogeneity constraint on universals (*ii*) too. It would at the same time avoid considering as substates those that keep only a partial picture of the (direct) part structure of the structured individuals. This would further strengthen the idea that focusing on a (set of) individual(s), one embraces all what holds internally to them, not only their properties and, in our approach, their internal relations, but also all their parts up to a given granularity level. We leave this extension and the investigation of the mereology obtained among states for further work.

Turning to structural universals, on each graph G, one can read off (and could extract from it) a purely universal graph where vertices represent now universals—they correspond to the labels of G—i.e., all instances of a same universal are grouped into a single

[10] Here we do not use labels for edges, which could represent various relational universals when labelling edges in E^r or even ways of being part when labelling edges in E^p. This is left as a an extension. Note that G is a multigraph, i.e., it is possible to have several edges with the same tail-head pair of vertices, e.g., in the case different kinds of relationships hold between the same individuals.

[11] Parthood transitivity is supposed to be accounted for through paths in the interpretation function, similarly to the interpretation of Bennett's parthood in terms of graphs seen in Sect. 4 so there is no need here for any other constraint on τ and η on E^p. Here loops are not necessary because we are considering *proper* parthood.

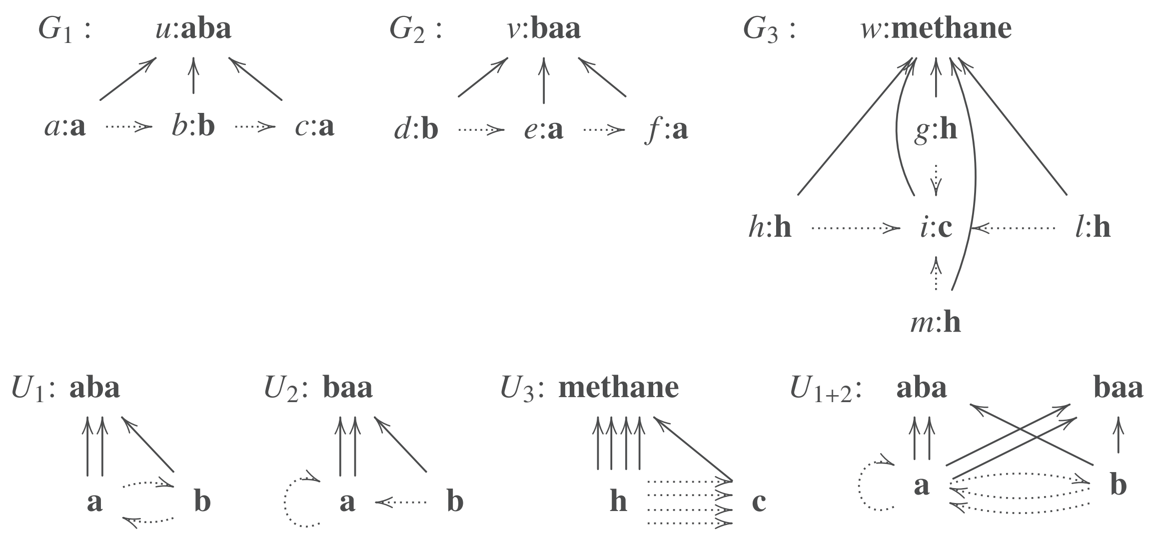

Figure 5. Our account of 3 states of affairs from Fig.1 and Fig. 3 and their corresponding structural universals.

node of the generated graph. Multiple occurrences of edges between two nodes are kept, except when grouping parthood edge heads (structural universal wholes), to retain only one occurrence of their part structure. In this way we obtain a structural universal graph similar to the graphs G_6 and G_7 in Fig. 1 or G_1, G_2 and G_3 in Fig. 4. Since constraint (*ii*) on our graphs obliges all instances of a structural universal to have the same internal structure, a unique structure is read off for each structural universal. This is adequate for universals such as molecules, strings, words or sentences, whose structure is fixed, but less for other universals like biological species and artefact kinds, whose instances are not all isomorphic as there are optional parts (e.g., hair for humans) and parts that can have variable properties (e.g. black or brown hair). Wittgenstein's example of *being a game* shows that instances of universals (concepts) might not even share any property. Of course this criticism applies to Mormann's and Bennett's proposals as well, and in fact to any classical logic approach defining a type of individuals in terms of their essential mereological structure. To handle structural universals whose instances are not all isomorphic, various refinements may be considered. For instance, one could allow for disjunctive labels, exploit some graph distance measure, or consider that a shared internal structure characterized in terms of isomorphism of (maximal) subgraphs could represent the structural universal. But we leave these possible extensions for further work.

Running example. Fig. 5 depicts our standard **aba** and **baa** string examples and the methane example as states of affairs involving individuals in our graph representation system.[12] As previously, parthood edges of E^p are drawn with solid arrows and binary relation edges of E^r with dotted arrows. U_1, U_2, and U_3 are the structural universal graphs read off or extracted from G_1, G_2, and G_3 (U_1 and U_2 are G_7 and G_6 in Fig. 1). Note that if we consider G_1 and G_2 to represent substates of a larger state, then the universal that can be read off is much more confusing, as shown by U_{1+2}. This is because the relational universals (*preceding* links between letters in the strings) loose the context of the structural universal in which they appear. As was seen in Sect. 4 discussing the graphs

[12]Even though *being bonded* is usually considered as a symmetric relation, here we assume a stronger oriented version (because our graphs are directed).

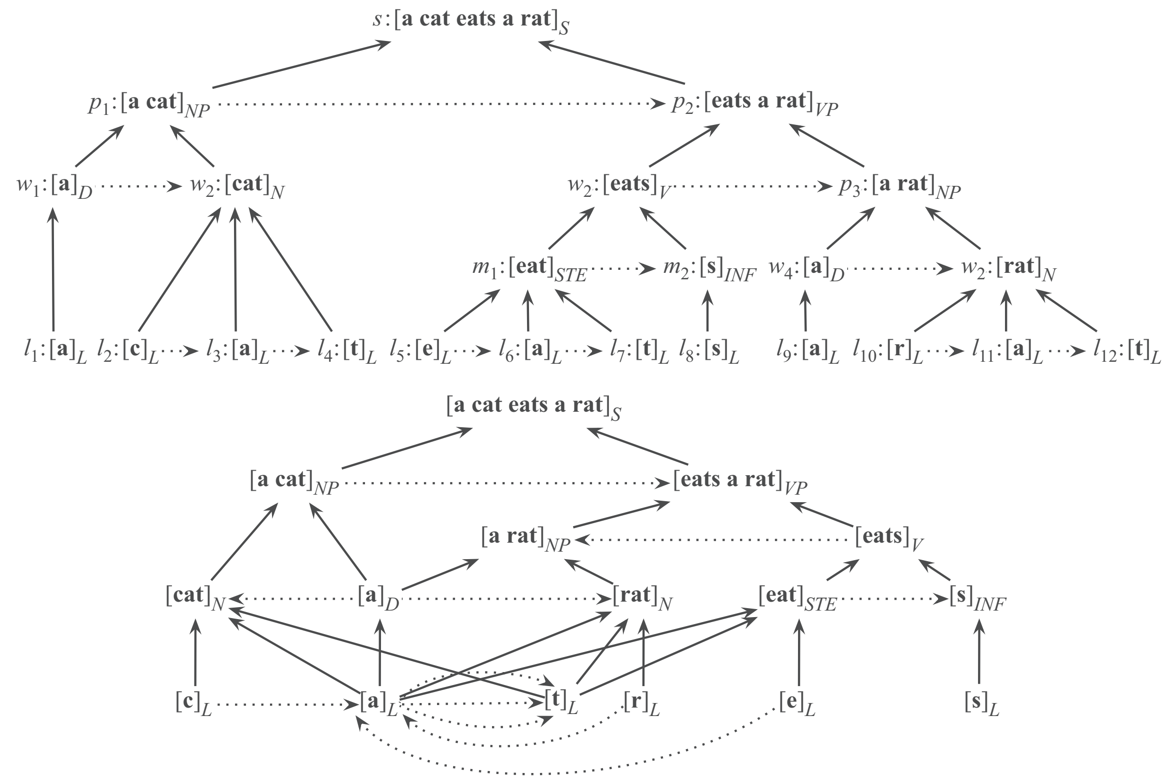

Figure 6. The structure of an occurrence s of the sentence **a cat eats a rat**$_S$ and the corresponding universal.

on Fig. 2, it is in general not possible to recover the internal structure of an instance of a structural universal from such graphs describing structural universals, even when their mereological structure is extended with binary relations. There is an irremediable information loss during the extraction process from states. This is why our approach to structural universals is founded on the representation of states of affairs involving individuals.

More complex example. Linguistic structural universals, such as texts, phrases, words, morphemes and letters (or phonemes), illustrate the usefulness of the multiple levels of decomposition. The sentence [**a cat eats a rat**]$_S$, an abstract entity that can have many occurrences and thus can be considered as a case of structural universal, is composed of two phrases, the noun phrase [**a cat**]$_{NP}$ and the verb phrase [**eats a rat**]$_{VP}$, in turn composed of a verb and another noun phrase. This abstract sentence is also composed of four words, [**a**]$_D$, [**cat**]$_N$, [**eats**]$_V$, [**rat**]$_N$, one of which, the determiner [**a**]$_D$, twice over. The verb [**eats**]$_V$ is composed of two morphemes, the stem [**eat**]$_{STE}$ and the inflexion [**s**]$_{INF}$. Finally, the sentence is composed of six letters, [**a**]$_L$, [**c**]$_L$, [**e**]$_L$, [**r**]$_L$, [**s**]$_L$, and [**t**]$_L$, [**a**]$_L$ being part 4 times over and [**t**]$_L$ 3 times over. Note that the letter [**a**]$_L$ and the determiner [**a**]$_D$ are different universals, and that one could also distinguish simple words such as [**cat**]$_N$ from the unique morpheme that make them up (for the sake of conciseness, we do not pursue this here). Preceding relationships hold among letters as well as among morphemes, words and phrases. The top graph on Fig. 6 shows the state of affairs describing the structure of an occurrence s and its parts, from which the structure of the universals they instantiate can be read off as the bottom graph. Note that the fact that the mereology defined on individuals does not satisfy weak supplementation allows to represent a sort of *constitution* relation, e.g., the relation between a statue and its material substratum. This is the case, for instance, of the parthood relation between the [**a**]$_L$ letter l_1 and the [**a**]$_D$ determiner w_1 on the left of the top graph on Fig. 6.

7. Conclusion

We have critically reviewed three state-of-the-art approaches to structural universals and complex states, including one that actually claims that structural universals are a mere *façon de parler* and cannot be accounted for explicitly. This analysis has been done in a graph-theoretical framework shedding light on formalisms at first sight very different. It leads us to propose a new account, more expressive than the three approaches considered.

Our representational system gives an explicit account of a universal being part of a structural universal several times over, while distinguishing structural universals sharing the same parts on the basis of the internal structure of their instances. In addition, explicitly representing structured individuals, labeling them with structural universals and using parthood relations among individuals allow for distinguishing mereology among states from mereology among individuals and maintaining mereological extensionality on states while leaving open the nature of mereology among individuals and universals. By not assuming that any substate corresponds to a (possibly spurious) structural universal, the system can accommodate several levels of universal decomposition. Finally, relational universals can hold among atomic and structured individuals alike.

We have seen on examples that our system allows for a detailed account of strings, molecules and linguistic entities. We conjecture that it is also adequate to represent other structural universals important in applied ontology. For an actual implementation, one could at first envisage to translate our graphs in formulas of (a fragment of) FOL, as done in [8] on molecule examples, using graphs as compact specifications. This would represent the individuals and their properties involved in the state of affairs denoted by the formula. But it would leave the very notion of structural universal and the mereological structure of these universals not explicitly accounted for; moreover, such a translation could not avoid spurious subformulas corresponding to spurious states of affairs and to spurious structural universals. Enriching the representation with the reification of structural universals and complex states of affairs could be part of a solution. We instead anticipate more powerful applications within a mixed logic-and-graph framework taking advantage of graph-based reasoning. To fully exploit such reasoning mechanisms, future work will aim at extending our approach to handle universals not limited to the whole nature of their instances (a limitation inherited from Armstrong and Mormann) and subsumption relationships between them.

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