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**Modeling Flow and Heat Transfer in a Scraped Surface Heat Exchanger during the
Production of Sorbet**

Oscar Darío Hernández-Parra (1), Artemio Plana-Fattori (2)(*), Graciela Alvarez (1),
Fatou-Toutie Ndoeye (1), Hayat Benkhelifa (1)(2), Denis Flick (1)(2)

(1) IRSTEA, UR Génie des Procédés Frigorifiques, 92761 Antony Cedex, France

(2) UMR Ingénierie Procédés Aliments, AgroParisTech, INRA, Université Paris-Saclay,
91300 Massy, France

(*) author corresponding address:
AgroParisTech, Dept. MMIP, 16 rue Claude Bernard, 75231 Paris CEDEX 5
E-mail: artemio.planafattori@agroparistech.fr

1. Introduction

Scraped surface heat exchangers (SSHEs) are commonly found in the food industry as a way of controlling the heat transfer in products of high viscosity or that could lead to fouling on the internal walls. Many designs exist; they generally consist of a double pipe heat exchanger which contains a rotating element inside the inner pipe, where a number of blades are attached with the purpose of scraping the inner cylinder at every rotation (Rao and Hartel, 2006). One application for SSHEs has been the industrial production of ice cream-like desserts (e.g. sorbets), where the purpose of the SSHE is to chill and freeze a formulated mix in order to crystallize the water present on the walls of the inner pipe (Goff and Hartel, 2013; Rao and Hartel, 2006). The blades also serve in this case as a dispersion system for the ice formed and a certain amount of air that could be introduced into the product. Air bubbles and ice crystals sizes are major quality parameters in sorbets, and their occurrence is closely related to the heat transfer mechanism and the flow characteristics in the SSHE. Actually, complicated heat transfer mechanisms and very complex flow profiles take place in the SSHE as a result of the presence of rotating elements, of the crystallization phenomena, and of the product rheology (which is highly dependent on the ice fraction). The experimental analysis of flow characteristics and heat transfer mechanisms inside the SSHE, particularly during the production of frozen desserts, is onerous and technically difficult to carry out. This is a situation in which physics-based modeling can effectively be used as a tool for providing insight about relevant phenomena.

Flow through the inner pipe of SSHEs has been extensively studied experimentally for single-phase Newtonian and non-Newtonian fluids (e.g. Wang et al., 1999, 2000; Dumont et al., 2000a, 2000b; Yataghene et al., 2011; see also the reviews by Abichandani et al., 1986; Härröd, 1986; Rao and Hartel, 2006). In summary, the flow structure can be

understood as an annular gap flow (i.e. Taylor-Couette) disturbed by the presence of rotor parts. Once a critical Taylor number value is reached, stable or wavy unstable forms of vortices begin to establish throughout the inner pipe. However, the flow patterns can be strongly affected by the rheological behavior of the fluid, the presence of blades in the system, their angle, and particularly, the gap (i.e. the clearance) between the tips and the inner wall. Large local shear rates are observed to appear in the clearance, as well as secondary vortexes in zones related to the blades. Modeling approaches have been developed to overcome the complex experimentation required, i.e. whether more precise investigation of flow phenomena or changes on the geometry of the blades make impractical the development of experimental facilities (e.g. Baccar and Abid, 1997, 1999; Stranzinger et al., 2001; Duffy et al., 2007; Yataghene et al., 2008, 2009, 2011; D'Addio et al., 2013; Yataghene and Legrand, 2013; Blasiak and Pietrowicz, 2016).

Numerous studies dealing with heat transfer phenomena in single-phase fluids in SSHEs have been reported (e.g. Härröd, 1986; Rao and Hartel, 2006). Experimental studies usually rely on obtaining the average heat transfer coefficient in the inner pipe of the exchanger; results are often expressed in terms of Nusselt number correlations in function of axial and rotational Reynolds as well as the Prandtl number, the number of blades, and geometrical ratios of diameters and lengths (e.g. Skelland, 1958; Trommelen et al., 1971; Abichandani et al., 1986; Boccardi et al., 2010). Numerical approaches have also been used to complement or develop heat transfer correlations (e.g. Baccar and Abid, 1997, 1999; Yataghene et al., 2009; Saraceno et al., 2011; D'Addio et al., 2013; Yataghene and Legrand, 2013; Rainieri et al., 2014; Dehkordi et al., 2015; Blasiak and Pietrowicz, 2016). ~~These correlations are applicable to liquid products without phase transitions. Focusing our interest on the production of non-aerated sorbet, whose thermal properties (apparent heat~~

~~capacity, thermal conductivity) change sharply at the initial freezing temperature, it seems clear that these Nusselt number correlations using the logarithmic average temperature difference are no more applicable because temperature evolution along the SSHE is expected to be very different before and after the freezing onset.~~

It is noteworthy that few experimental works deal with heat transfer in freezing solutions in SSHEs (e.g. Qin et al., 2003, 2006; Lakhdar et al., 2005; Martinez et al., 2014). In such cases, as freezing takes place, the apparent physical properties of the fluids change strongly and affect the heat transfer coefficient. For instance, Arellano et al. (2013b) have shown that the apparent viscosity of non-aerated sorbet can increase from 1 to 12 Pa·s along its pathway in the SSHE. Lakhdar et al. (2005) included the influence of initial non-freezing substance concentration in their heat transfer coefficient correlations. Simple equations for calculating the heat transfer coefficient during freezing of milk and aqueous glycerol mixtures which include a correction term for the ice heat of fusion can be found in the literature (e.g. Qin et al., 2003, 2006).

Mathematical models have also been developed specifically for SSHE ice cream production; we can identify classes of models associated with increasing degree of complexity about coupled phenomena. Bongers (2006) proposed a model considering the freezer barrel as a series of well mixed stages, taking into account four contributions to the energy balance: refrigeration, crystallization, dissipation of mechanical energy, and scraping friction of the blades on the ice layer. In a further development of this model, Dorneanu et al. (2009) have included conservation equations for selected dispersed phases (water, sugar, fat and remaining components); they considered an immobile frozen layer at the wall, and the plug-flow assumption for the mean axial flow. Lian et al. (2006) have

combined a 2D computational fluid dynamics model and a population balance approach with the aim to predict the ice crystal size distribution in the final product.

Non-aerated lemon sorbet production has been studied in a series of articles by Arellano et al. (2013a, 2013b, 2013c), Gonzalez-Ramirez et al. (2013a, 2013b) and Casenave et al. (2013, 2014). Modeling efforts were devoted by those authors in predicting ice crystal size distributions. These models used residence time distributions and reduced heat transfer equations under transient operation with population balance approaches in order to predict crystal chord length distributions during steady-state operations. Particularly, the model developed by Arellano et al. (2013c) reached good agreement with experimental crystals mean chord lengths and temperature profiles along the exchanger. In such a goal, key parameters as global heat transfer coefficients, and crystal growth and nucleation rate constants, were adjusted in the model. Although the results show a good agreement, the reduced models cannot offer a full insight into the heat transfer mechanisms taking place inside the exchanger.

In the logical continuation of experimental and modeling efforts devoted by Arellano et al. (2013a, 2013b, 2013c), this study presents a numerical model for predicting the non-aerated lemon sorbet production in a pilot-scale SSHE under different operating conditions. The main objectives were: firstly, to implement a 3D computational fluid dynamics (CFD) model for solving the coupled problem of fluid flow and heat transfer within the SSHE, while respecting the strong influence of temperature on key properties of the liquid food product; secondly, to analyze the 3D distributions of key variables throughout the SSHE, at scales beyond usual experiments; and lastly, to assess the performance of this model under different operating conditions, using as reference measurements conducted in those early contributions.

2. Development of the model

2.1. Physical problem description

Before developing the model equations, a short description of the physical problem is required. The modeling approach could be applied to the production of non-aerated sorbet in any SSHE freezer; here it was implemented for the pilot unit used by Arellano et al. (2013a, 2013b, 2013c). Those authors carried out the freezing of a commercial UHT-pasteurized lemon sorbet mix (14.6 % w/w sucrose, 8 % w/w fructose, 0.09 % w/w dextrose, 3 % w/w lemon juice concentrate 60 Brix, 0.5 % w/w locust bean gum/guar gum/hypromellose stabilizer blend) in a laboratory scale continuous pilot freezer (WCB Model MF 50). Within the freezer, the SSHE consists of a double pipe heat exchanger whose inner pipe (0.05 m inner diameter, 0.40 m long) contains a solid flattened cylindrical rotor equipped with two scraper blades placed at opposite positions. A scheme of the SSHE is presented in Figure 1.

Four wireless data loggers with built-in temperature sensors (Thermochron iButton DS1922L – Maxime; 16.3 mm diameter and 6.4 mm thick, calibrated to an accuracy of ± 0.2 °C within a range from -20 to 20 °C) were placed at regular distances on the flat dasher surface as viewed in Figure 1B. Also, a calibrated type T thermocouple (accuracy of ± 0.2 °C) was placed at the entrance of the heat exchanger, and a calibrated Pt100 probe (Baumer, accuracy of ± 0.1 °C) was placed at the outlet pipe of the exchanger. These probes allowed the measurement of temperature during steady-state operation of the freezer.

The processing unit has a nominal capacity of 25 to 75 kg/h (about 0.007 to 0.021 kg/s), with accuracy of $\pm 9 \times 10^{-5}$ kg/s. Refrigerant fluid was evaporating dichlorodifluoromethane (R22), which circulated through the cooling outer pipe of the SSHE at controlled temperature. This temperature was measured through a calibrated type-T thermocouple, with an accuracy of ± 0.2 °C, fixed with conductive aluminum tape on the external surface wall of the outer pipe. The rotational speed of the dasher and blades varied from 545 to 1000 rpm (about 9 to 17 rev/s), with accuracy of 1 rpm (about 0.017 rev/s). Using this device, Arellano et al. (2013a) measured the fluid temperature at selected positions inside the SSHE under seven combinations of operating conditions, using three refrigerant temperatures ($T_R = -10.6^\circ\text{C}$, -15.3°C and -20.1°C), three mass flow rates ($\dot{m}_{mix} = 25, 50$ and 75 kg/h) and three dasher rotation frequencies ($f_{rot} = 545, 750$ and 1000 rpm). In all cases the sorbet mix inlet temperature was held constant at 5°C .

2.2. Governing equations

Although non-aerated sorbets are two-phase systems containing ice and concentrated liquid mix, in this work it is assumed that sorbet mix and freezing sorbet can be treated as a single-phase fluid with apparent physical properties varying with temperature; hence mass, momentum and energy conservation equations for single-phase fluids were used, following the approach proposed by Lakhdar et al. (2005). The inner cylinder and the rotating elements are the most relevant parts in the SSHE, and therefore, it is convenient to establish the governing equations using a reference frame rotating with them since that allows deal with a non-moving geometry. From this, a relative velocity in the moving frame, $\vec{v}$, can be defined from the velocity in the Galilean frame, $\vec{u}$, such that:

$$\vec{u} = \vec{v} + \vec{\Omega}_{rotor} \times \vec{r} \quad (1)$$

and the conservation equations for mass, momentum and energy can be expressed as:

$$\vec{\nabla} \bullet \vec{v} = 0 \quad (2)$$

$$\rho \left(\vec{v} \bullet \vec{\nabla} \right) \vec{v} = -\vec{\nabla} \bullet \vec{\bar{\tau}} - \vec{\nabla} p - 2 \rho \left(\vec{\Omega}_{rotor} \times \vec{v} \right) - 2 \rho \left(\vec{\Omega}_{rotor} \times \vec{\Omega}_{rotor} \times \vec{r} \right) \quad (3)$$

$$\rho C_p \vec{v} \bullet \vec{\nabla} T = \vec{\nabla} \bullet \left(k \vec{\nabla} T \right) - \left(\vec{\bar{\tau}} : \vec{\nabla} \vec{v} \right) \quad (4)$$

where $\vec{v}$ is the velocity (magnitude in m/s), $\vec{\bar{\tau}}$ the viscous stress tensor obtained from a power-law model, p the pressure (Pa), and T the temperature (K); ρ is the product density (kg.m^{-3}), C_p its specific heat capacity ($\text{J.kg}^{-1}.\text{K}^{-1}$), and k its thermal conductivity ($\text{W.m}^{-1}.\text{K}^{-1}$).

Since the internal dasher and the blades occupy a non-negligible portion of the volume inside the inner pipe in the heat exchanger, it is possible that heat conduction in the solids affects the temperature profiles in the fluid. In other works this has been sometimes neglected (Lian et al., 2006; Yataghene et al., 2009) or included (Yataghene and Legrand, 2013). In the present work, heat transfer across the dasher and blades were included while dasher and blade supports were neglected; the influence of heat conduction in the dasher and the blades is assessed through numerical simulation while that associated with their supports remains a matter of discussion (see subsection 5.2). Under the rotating reference frame, both elements (dasher and blades) appear static, and therefore, the following equation holds for the heat transfer:

$$\vec{\nabla} \bullet \left(k \vec{\nabla} T \right) = 0 \quad (5)$$

The following boundary conditions were considered for solving fluid flow equations:

a) Under the rotating reference frame: the inner wall of the internal cylinder, the inlet and outlet bowls and pipes were perceived to be rotating at the speed of the rotor on the opposite sense of rotation; hence the non-slip condition in these surfaces becomes

$$\vec{v} = -\vec{\Omega}_{rotor} \times \vec{r}.$$

b) On the dasher and blades surfaces: $\vec{v} = 0$ (no-slip condition).

c) At the inlet boundary: laminar flow profile was imposed from the specified mass flow rate and the rheology of the sorbet mix:

$$v_x = \Omega_{rotor} y; \quad v_y = -\Omega_{rotor} x; \quad v_z = \frac{3n+1}{n+1} \frac{\dot{m}_{mix}}{\rho \pi r_{pipe}^2} \left(1 - \left(\frac{r}{r_{pipe}} \right)^{\frac{n+1}{n}} \right) \quad (6)$$

where n is the flow behavior exponent.

d) At the outlet boundary, the same flow profile was re-imposed to insure mass conservation.

e) The relative pressure was set to be zero at the center point of the exit boundary.

The following boundary conditions were considered for solving the equation of conservation of energy:

a) The product was assumed to enter in the inlet pipe of the SSHE at a uniform temperature of 5 °C.

b) At exit the outlet pipe of the SSHE, a null gradient was assumed for the product temperature.

c) The product was assumed to flow under adiabatic conditions in the inlet and outlet pipes as well as in the inlet and outlet bowls of the SSHE: these computational sub-domains were assumed to be delimited by thermally-insulated walls.

d) Inside the SSHE cylinder, the product flows confined between the wall, the dasher and the blades. Conduction is solved throughout these solid elements; at the interface between product and dasher or blades, continuity of temperature and heat flux was assumed.

e) At the internal wall of the SSHE inner cylinder, the heat flux density was expressed as the product of a global heat transfer coefficient ($\underline{U}$) and the difference between the refrigerant evaporating temperature (T_R) and the liquid product temperature in contact with the wall ($T = T_{inside}$) (see Figure 2):

$$\underline{U} (T_R - T) = k \vec{\nabla} T \cdot \vec{n} . \quad (7)$$

We applied the value $\underline{U} = 2500 \text{ W.m}^{-2}.\text{K}^{-1}$ reported by Arellano et al. (2013c). This heat transfer coefficient is associated with a) convection between the refrigerant fluid and the external side of the SSHE inner cylinder, which is characterized by the outside convective heat transfer coefficient $h_{outside}$, and b) conduction through the stainless steel wall which separates the refrigerant fluid and the product inside the SSHE inner cylinder; the wall is characterized by thickness $e_w = 0.002 \text{ m}$ and conductivity $k_w = 15 \text{ W.m}^{-1}.\text{K}^{-1}$. This way, $\underline{U}$ can be expressed as:

$$\underline{U} = \frac{1}{\frac{e_w}{k_w} + \frac{1}{h_{outside}}} . \quad (8)$$

The SSHE inner cylinder was not included in the computational domain because, in the reference frame of the blades, it is in rotation; hence the problem would become unsteady. The heat transfer resistance of the wall (e_w / k_w) represents one third of the total resistance $1/U$. An upper limit for the global heat transfer coefficient in our problem would be $U = k_w / e_w \approx 7500 \text{ W.m}^{-2}.\text{K}^{-1}$, which can be estimated by neglecting the convective heat transfer resistance between the external side of the SSHE inner cylinder and the refrigerant fluid ($1/h_{outside}$) in equation (8).

3. Product physical properties

Physical properties of sorbets were estimated from experimental measurements or from available information in the literature. Some approximations were performed in order to reduce the mathematical difficulty of solving the governing equations. Table 1 summarizes values and correlations assumed.

3.1. Freezing point curve of concentrated lemon sorbet mix and sorbet ice fraction

Although non-aerated sorbets are two-phase systems containing ice and concentrated liquid mix, in this work it is assumed that the apparent physical properties can be approximated assuming that a local thermodynamical equilibrium is established at any point of the fluid inside the SSHE. This approach has been also used before in order to simulate similar systems e.g. ice crystallization from sugar solutions (Lakhdar et al., 2005) and ice cream properties estimation (Cogné et al., 2003). At equilibrium, the solute fraction (in the liquid surrounding the ice crystals) becomes that of the freezing point curve of concentrated sorbet mix at the local temperature.

Data of freezing point temperatures for the commercial lemon sorbet mix modeled in this work were obtained experimentally by Gonzalez-Ramirez (2012), who proposed a polynomial correlation restricted to the range from $-13\text{ }^{\circ}\text{C}$ to $0\text{ }^{\circ}\text{C}$ whereas the sorbet temperature can be as low as $-20\text{ }^{\circ}\text{C}$. To improve the predictions, a correlation based on a thermodynamical approach was developed. Freezing point depression curves of thermodynamically ideal binary solutions of a freezing solvent compound (i.e. water) and a solute (i.e. soluble solids) can be expressed; their molar fraction being a function of temperature, solvent pure compound heat of fusion at its melting point temperature, solvent liquid and solid heat capacities as follows:

$$\ln x_w(T) = \frac{\Delta h_{f,mp}}{R} \left(\frac{1}{T_{mp}} - \frac{1}{T} \right) - \frac{1}{R} \left(\int_{T_{mp}}^T \frac{c_{p,w} - c_{p,i}}{T} dT' - \int_{T_{mp}}^T \frac{c_{p,w} - c_{p,i}}{T'} dT' \right) \quad (9)$$

The solute mass fraction can be expressed in function of the water molar fraction:

$$w_s^*(T) = \frac{1 - x_w(T)}{1 - C x_w(T)} \quad (10)$$

where $C = 1 - M_w / M_s$ could be regarded as a fitting parameter depending on the molar mass of water and of solute. In order to improve the quality of the fitting curve, a second constant was introduced, and the equation fitted was:

$$w_s^*(T) = C_1 \frac{1 - x_w(T)}{1 - C_2 x_w(T)} \quad (11)$$

In equation (9) the heat capacities of liquid water and ice were taken as functions of temperature according to Choi and Okos (1986). After fitting the experimental data to equation (11) the values obtained were $C_1 = 0.7316$ and $C_2 = 0.9476$. Figure 3 displays experimental data and corresponding results from equation (11) for two products: concentrated lemon sorbet mix and sucrose solution.

Once the equilibrium solute concentration is determined, a mass balance yields the ice mass fraction using the initial solute fraction in the mix, as indicated in equation (12). The estimated ice fractions at different temperatures are shown in Figure 4A.

$$w_i = 1 - \frac{w_{s0}}{w_s^*} \quad (12)$$

3.2. Density

The sorbet density was estimated from the ice mass fraction and the specific densities of ice and mix at temperatures ranging from -20 to 10 °C using the approach recommended by Choi and Okos (1986). Estimated values of density vary slightly (less than 6 %) and the constant value 1050 kg.m^{-3} was assumed for modeling purposes.

3.3. Rheological behavior of non-aerated lemon sorbet

Arellano et al. (2013b) have thoroughly investigated the influence of the temperature and thus of the ice volume fraction on the apparent viscosity of sorbet by using pipe rheometry measurements. Their main results can be summarized by an empirical rheological power-law model which predicts the consistency index and the flow behavior index as function of the ice volume fraction and the temperature, these two parameters being linked by the hypothesis of local equilibrium.

The difficulty to study the sorbet rheological behavior is put in evidence in Figure 2 of Arellano et al. (2013b): the range of shear rates permitting reliable observations decreases as the sorbet becomes colder and associated with higher ice volume fraction. Indeed, at -2.89 °C (ice volume fraction of 5.8 %), the observations were reliable for shear rates up to about 450 s^{-1} ; at -5.68 °C (ice volume fraction of 39 %), the observations were reliable for

shear rates up to 40 s^{-1} only. The rheological model proposed by those authors represents a synthesis over the whole dataset of operating conditions studied (mass sorbet flow rate and sorbet temperature).

The following approximation was fitted to describe the consistency index (in units of Pa.s^n) of both sorbet mix and sorbet:

$$K = 0.5838 \quad \text{for } T \geq T_{if} \quad (13a)$$

$$K = 0.5838 + 10.16 (T_{if} - T) \quad \text{for } T < T_{if} \quad (13b)$$

where $T_{if} = -2.72 \text{ }^\circ\text{C}$ is the initial freezing temperature. Figure 4B compares equation (13) with the consistency index estimated accordingly to Arellano et al. (2013b) at selected temperatures. Applying the local thermodynamical equilibrium assumption as discussed above between -20 and $10 \text{ }^\circ\text{C}$, the sorbet flow index (n) reported by Arellano et al. (2013b) ranges between 0.38 and 0.55. In this work a constant value of 0.5 was used for modeling purposes.

3.4. Apparent heat capacity

In order to include the effect of water freezing into the enthalpy of the sorbet, the apparent heat capacities of the sorbet mix and the frozen non-aerated sorbet were estimated using the approach proposed by Cogné et al. (2003) for ice cream and by Lakhdar et al. (2005) for crystallizing sugar solutions. It consists of a weighted average between the values for mix and ice, including a correction term for the change in the amount of ice according to the local thermodynamical equilibrium approximation:

$$C_p = C_{pi} w_i + C_{ps} w_s + C_{pw} (1 - w_i - w_s) - \Delta H_f \frac{dw_i}{dT} \quad (14)$$

In order to simplify the heat capacity calculation, the following equation was fitted from estimation:

$$C_p = 3500 \quad \text{for } T \geq T_{if} \quad (15a)$$

$$C_p = 3266 + 7.95 \times 10^4 \exp\left(0.543(T - T_{if})\right) \quad \text{for } T < T_{if} \quad (15b)$$

Figure 4C compares estimated values provided by equations (14) and (15).

3.5. Thermal conductivity

Non-aerated sorbet was represented as by Cogné et al. (2003) as a two-phase material, which consist of a solid phase (ice crystals) dispersed in a continuous fluid phase made of dry matter and unfrozen water (Maxwell-Eucken model). The thermal conductivity of the continuous phase was estimated by using the intrinsic thermal conductivity values of each main components and their volume fractions estimated from their weight fraction and their density (Choi and Okos, 1986). In the present study, the solute mass fractions were obtained from equation (10). As before, a simpler equation was fitted:

$$k = 0.432 \quad \text{for } T \geq T_{if} \quad (16a)$$

$$k = 0.432 + 0.303 \left(1 - \exp\left(-0.367(T_{if} - T)\right)\right) \quad \text{for } T < T_{if} \quad (16b)$$

A comparison between estimated values and equation (16) is shown in Figure 4D.

4. Numerical model

4.1. Geometry

In order to avoid a computationally-intensive transient 3D simulation with moving boundaries, we adopt here the geometrical representation of the SSHE which is displayed in

Figure 1A. While being quite simple, this representation allows us to conduct a steady-state simulation of all the relevant phenomena occurring in the cylindrical section of the SSHE: scraped surfaces, dasher, and blades.

The inner cylinder of the SSHE under consideration contains several elements: i) the sorbet that is subjected to heat transfer and flow, ii) the internal solid elements that constitute the rotor (i.e. the dasher, the blades and their supports), and iii) the temperature sensors added to the dasher. Dasher and blades were considered for simulations given their major influence on the global performance of SSHEs; nevertheless, the blade supports and temperature sensors were not included since they have a relative small size and hence a expected negligible influence on flow profiles as well as on heat transfer mechanisms. Both headers of the exchanger contain reservoirs for the fluid entering or leaving the inner cylinder, as well as the dasher supports and the mechanical joins required. They were simplified, as shown in Figure 5, replacing them by co-axial cones of 0.01 m length connecting the inner cylinder of the SSHE to the inlet and outlet pipes. Also, in the exchanger the mix enters the bowl laterally, while the sorbet exits downwards. This was simplified by using co-axial horizontal tubes of 0.0125 m diameter, and 0.1 m long for ensuring a complete development of flow profiles.

Although the gaps between the blades and the wall of the internal cylinder have been shown to have an effect on the flow patterns inside SSHEs with liquid flow (Dumont et al., 2000a, 2000b), they were neglected in this work by extending the blades up to the inner wall of the internal cylinder. This was made considering that ice formation takes place mostly on the inner wall of the internal cylinder during the production of ice creams and sorbets and the action of the scraping blades on this ice could block the gap from the fluid (Cook and Hartel, 2010).

352

353 **4.2. Mesh construction**

354 Looking for the resolution of equations which govern the coupled phenomena, every
355 computational domain has to be subdivided into a number of small, non-overlapping cells.

356 The mesh was constructed using the COMSOL Multiphysics integrated builder, version
357 5.2. In order to account correctly for the momentum and thermal boundary layers, the mesh
358 around the surfaces was built using four-element layer of prismatic elements, while the
359 domain interiors were filled out with tetrahedral elements of varying minimum sizes.

360 Results achieved with the mesh constituted by 1.5×10^6 elements provide a satisfactory level
361 of reliability in terms of the predicted profile of temperature along the SSHE axis, as
362 indicated by a comparison with two other meshes (see subsection 5.3). Figure 6 displays
363 this mesh around the dasher and the blades, as well as a cut view at $z = 0.2$ m.

364

365 **4.3. Calculation strategy**

366 The solution of the linear system resulting from the linearization of the governing equations
367 is reached through the Parallel Sparse Direct Linear Solver (PARDISO) (Schenk and
368 Gartner, 2004), as implemented with the finite-elements method in the simulation package
369 COMSOL Multiphysics, version 5.2 (Zimmermann, 2006). Satisfactory convergence of
370 numerical solutions was reached after hundreds of iterations, looking for a relative
371 tolerance better than 10^{-5} . Simulations were implemented in a PC station with 6 CPU at
372 3.50-GHz and 128-Gb RAM.

373 The coupled problem to be solved is challenging from different perspectives: on one hand,
374 the product is characterized by physical properties which vary significantly with
375 temperature; on the other hand, the product experiences a complex flow pattern, including

obstructions and relatively fast rotation. The mathematical solution associated with every operating condition of interest was obtained as the result of a particular step-by-step calculation strategy. Firstly, preliminary fields for velocity and pressure were estimated at the temperature 5 °C, by solving equations (2-3); secondly, a preliminary temperature field was estimated for that preliminary fluid flow field by solving only equations (4-5); finally, assuming these first-guesses as starting condition, the ensemble of coupled equations (2-5) was solved simultaneously by a direct method, iteratively, while increasing the maximal value for the sorbet consistency index, until no further increase influenced the mathematical solution.

5. Results and discussion

5.1. Analysis of flow and temperature profiles

Main features of fluid flow in the SSHE as predicted by the numerical model are displayed in Figure 7. These results were obtained under the reference operating conditions $\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3^\circ\text{C}$ and $f_{rot} = 750$ rpm. The rotation direction of the inner wall of the heat exchanger is clockwise as seen from the exit of the outlet pipe.

Firstly, the velocity field is largely dominated by rotation: the highest velocities occur in the vicinity of the exchanger wall, and the maximal tangential velocity (1.97 m/s) is about 200 times higher than the mean axial velocity (about 0.01 m/s).

Secondly, the flow is confined between the exchanger wall, the dasher and the blades; as a consequence, changes of velocity direction are significant. The region indicated by the red ellipse in Figure 7 puts in evidence the occurrence of recirculation in the vicinity of the

blades endings. Indeed, the occurrence of recirculation near the blades endings has been predicted by mathematical modeling of fluid flow in scraped surface heat exchangers (Duffy et al., 2007). Fluid parcels travelling between the dasher and the blades near the exchanger outlet are reoriented back towards the exchanger inlet, following pathways between the blades and the wall. The reciprocal feature takes place near the exchanger inlet, where parcels coming between the dasher and the blades are reoriented back towards the exchanger outlet, between the blades and the wall. Such features are particularly important for the temperature axial distribution, because they foster mixing between fluid parcels experiencing different thermal histories. Indeed, near the exchanger inlet, colder parcels travelling between the dasher and the blades do encounter relatively warmer parcels just coming from the entrance of the inlet pipe (at 5 °C), allowing a faster decrease of temperature than we might expect without such a recirculation.

Figures 8A to 8C display the velocity, pressure and temperature profiles in the SSHE as predicted by the numerical model under the operating conditions $\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3$ °C and $f_{rot} = 750$ rpm. In the left-hand side of Figures 8A, 8B, and 8C, as well as in Figures 8D and 8E, the product flows from the left to the right, as indicated by the vector velocity in the inlet and the outlet pipes in Figure 8B.

From the rotating reference frame, the inner cylinder wall is perceived turning clockwise (right-hand side of Figures 8A, 8B, and 8C); the liquid flows from the annular space left by the dasher (point 1) towards the zone near the lower face of the blade (point 2). In this zone, the high-speed flow is blocked suddenly by the lower surface of the blade and immediately forced to change direction. This momentum accumulation yields a pressure increase in this region, whose maximum occurs where the blade encounters the wall. The region on the

opposite side of the blade (point 6) experiences the opposite phenomena, with the pressure reaching its lowest value at the contact between the blade and the wall. These trends can be appreciated in the cross-section view of the pressure field in Figure 8B.

Therefore, from point 2, the fluid is forced to follow the lower surface of the blade (point 3), get around it (point 4) and follow the upper side of the blade (point 5) towards point 6. From there, the rotating wall forces the flow to change direction drastically (point 7) leading the fluid back to the opposite annular section (point 1') where the sequence described before is repeated following the symmetry in the SSHE. It is noteworthy that the entire trajectory is carried out 12.5 times per second (for $f_{rot} = 750$ rpm).

The longitudinal view in the pressure profile shown in Figure 8B also indicates the axial flow direction in the SSHE. It can be observed that, inside the SSHE between the dasher and the blades (high-pressure zone, point 9), pressure increases from the inlet pipe towards the outlet pipe, reaching a maximum before the blades ending ($z \sim 0.36$ m). This forces the fluid in this zone to go back to the exchanger inlet promoting back-mixing. Between the blades and the wall (low-pressure zone, point 10) the opposite effect (decreasing pressure) directs the flow towards the outlet pipe. Near the blades ending ($z > 0.36$ m), where the pressure gradients are inversed, a recirculation zone is observed (point 11), as displayed in Figure 7.

The range of shear rate conditions in the SSHE is wide, actually from zero up to more than 2000 s^{-1} . On one hand, as suggested by the blue regions in the shear rate display of Figure 8A, the velocity field can exhibit quite "gentle" features (e.g. between the dasher and one blade), being associated with relatively low shear. On the other hand, shear rates are very high near the region where the blades touch the inner wall of the SSHE.

The volume-averaged value of the shear rate in the SSHE can indicate the typical shear conditions experienced by the sorbet while it undergoes progressive freezing. Considering the heat exchanger itself only, without the inlet and the outlet cylindrical pipes and bowls (see Figure 1A), the volume-averaged shear rate was about 420 s^{-1} . We are aware that such a value is near the higher limit of shear rates observed in the experiments conducted by Arellano et al. (2003b) (see section 3). The application of the rheological model developed by those authors is assumed to be satisfactory in most of the volume of the SSHE. Nevertheless the lack of rheological data at high shear rate could impact the accuracy of velocity prediction in the region where the blades touch the inner wall of the SSHE.

The temperature profile in the exchanger is presented in Figure 8C. At the entrance of the inlet pipe, the temperature was assigned to $5 \text{ }^{\circ}\text{C}$, following the experimental conditions in Arellano et al. (2013a). It can be observed a sharp temperature decrease in the entrance region of exchanger, down to the initial freezing temperature of the sorbet mix ($-2.72 \text{ }^{\circ}\text{C}$, $w_{ice} = 0$). Such a decrease is most likely due to the inlet recirculation and the back-mixing observed between dasher and blade, which puts into contact frozen sorbet from the mid-section with the warmer fluid coming from the inlet pipe.

Once the initial freezing temperature is reached, the temperature decreases more slowly with the axial coordinate. This could be explained by the higher energy required to freeze the sorbet mix once passed the initial freezing temperature, which can be better appreciated following the specific enthalpy of the sorbet presented in Figure 8D. It can be seen how inside the SSHE freezing an additional 5 % of the sorbet mass required about the same amount of energy regardless of the location inside the exchanger.

Also, as a consequence of the predominant tangential component of the velocity and the fluid complex trajectory, the temperature variations in cross sections across the exchanger were found to be very small (less than 0.2 °C) suggesting a strong radial mixing. The fluid cooled at the wall is carried out around the blades and mixed with the slightly warmer fluid in the zone between the blades and the exchanger wall.

Globally, the cooling effect of the scraped wall is thwarted by viscous dissipation effects. The later are the highest in the exit area, given that the lower the temperature the higher the consistency index, and so the apparent viscosity of the sorbet. Viscous dissipation profiles are shown in Figure 8E. As discussed above, the increased presence of ice because of lower temperatures increases the viscous dissipation near the end of the SSHE. Also, viscous dissipation is caused by the friction of fluid changing direction radically (i.e. near the point 4) or friction with the solid elements in the exchanger, therefore on the dasher surface (i.e. point 1) or around the blades (i.e. points 3-6).

5.2. Comparison between experimental and simulated values

The experimental verification of numerical model predictions represents a complicated task in the case of coupled problems of rotating flow, heat transfer and product transformation. Model predictions of isothermal fluid flow without product transformation in SSHE have been compared with observations gathered through noninvasive techniques, like Magnetic Resonance Imaging with nonmagnetic and nonmetal exchangers (Wang et al., 1999), and Particle Image Velocimetry with exchangers with transparent walls (Stranzinger et al., 2001; Crespi-Llorenz et al., 2016). These techniques cannot be applied to the SSHE here considered, whose walls are made of steel. The reliability of our modeling approach is hereafter assessed by comparing temperature predictions with in situ observations which

were performed while running the SSHE. The interest on the temperature field can be justified by its determinant role on product transformation, and therefore on product rheology. Arellano et al. (2013a) have successfully employed wireless temperature sensors in order to monitor the temperature profile inside the SSHE under a variety of operating conditions; forthcoming paragraphs consider these measurements in assessing the bulk reliability of model predictions. Pressure drop measurements are unfortunately not available.

Figure 9 compares Arellano et al. (2013a)'s measurements (indicated by circles) with the temperature profile in the SSHE as predicted by the numerical model (black lines) under the operating conditions $\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3^\circ\text{C}$ and $f_{rot} = 750$ rpm. The left-most circle indicates the temperature 5°C prescribed at the entrance of the inlet pipe, upstream to the heat exchanger; the right-most circle indicates the temperature value observed at the exit of the outlet pipe, downstream to the heat exchanger. All the other four measurements correspond to positions inside the heat exchanger (see Figure 2 of Arellano et al., 2013a). The black line in displays 8A and 8B show temperature predictions obtained after setting to $15\text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ the thermal conductivity of solids, and setting to $2500\text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ the heat transfer coefficient at the SSHE inner wall. Such a thermal conductivity is typical of stainless steel employed in food processing units, while this heat transfer coefficient value was estimated by Arellano et al. (2013c) after energy balance considerations and closure experiments conducted with the SSHE. Four results emerge from this comparison.

Firstly, the experimental temperature at the first measuring position inside the exchanger inlet ($z = 0.045$ m) is under-estimated by the model; actually, this was the case for all the operating conditions considered in our study (see below). In the real SSHE pilot unit, as

displayed in Figures 5A and 5B, the dasher is in contact with its support at the inlet bowl (the header), which in turn is in contact with fresh mix at higher temperature. This could lead to a non-negligible conductive heat transfer to the dasher, eventually reaching the temperature sensor. The predicted temperature profile cannot incorporate this phenomenon because the numerical model does not include the dasher supports.

Secondly, model predictions of the temperature profile follow the experimental temperature at the second, third and fourth measuring positions inside the SSHE: differences reach ± 0.2 °C, with no systematic sign. The temperature sensors are in contact with the dasher while being submerged in a liquid product which becomes colder towards the exchanger outlet.

Thirdly, the averaged value of the temperature at the exit of the outlet pipe (-5.1 °C) agrees relatively well with the available measurement (-4.6 °C). Such a level of agreement can be considered as satisfactory, given the sensibility of temperature predictions with respect to model parameters (see below) and the influence of phenomena potentially affecting the product temperature at the measuring position which were not included in the numerical model (like conduction through dasher supports).

Lastly, the initial freezing temperature constitutes a key value in the coupled phenomena represented by the numerical model. When the temperatures decrease below -2.72 °C, the sorbet thermal properties as well as its consistency coefficient experience a sharp increase (see Figure 4); as a consequence, both the fluid flow and the heat transfer are significantly affected, promoting a major decrease in the gradient of axial temperature near the dasher (see Figure 9).

Two sensitivity tests were conducted in order to evaluate the influence of key parameters on model predictions.

In building the numerical model, we have assumed a typical value for the thermal conductivity of the solid elements which were taken into account (dasher and blades). One may ask the importance of heat conduction in these solid elements on the predicted temperature. Figure 9A presents the results of a first sensitivity test: black and grey lines correspond to the temperature profiles after setting the thermal conductivity of solids to $15 \text{ W.m}^{-1}.\text{K}^{-1}$ and to zero, respectively. The thermal conductivity of solids is many times greater than that for non-aerated sorbet (see Figure 4D); solid elements contribute to reduce temperature gradients within the SSHE. With no conduction in solids, the predicted temperature profile along the SSHE axis becomes slightly colder, from 0.1°C at the exchanger inlet ($z = 0$) up to about 0.35°C at its outlet ($z = 0.4 \text{ m}$); the latter difference can be found also between the corresponding surface-averaged temperatures at the exit of the outlet pipe (squares in Figure 9A). We can conclude that the influence of conduction in solids on the predicted temperature along the SSHE axis is relatively weak, because it exhibits the same order of magnitude than the difference between temperature measurements and predictions.

Figure 9B summarizes the results of a second sensitivity test with the numerical model, in order to assess the influence of the heat transfer coefficient assumed at the internal side of the SSHE inner cylinder wall. The black line displays the temperature profile obtained after setting this coefficient to $2500 \text{ W.m}^{-2}.\text{K}^{-1}$; dark-grey and light-grey lines correspond to results obtained after increasing and decreasing that value by 20 %, respectively. After increasing the heat transfer coefficient, the liquid is exposed to more effective cooling while travelling in the SSHE. As a consequence, the predicted temperature profile along the

SSHE axis becomes relatively colder, from $-1.3\text{ }^{\circ}\text{C}$ at the exchanger inlet to $-1.8\text{ }^{\circ}\text{C}$ at its outlet; hence, the initial freezing temperature is reached early (at about $z = 0.025\text{ m}$, see dark grey line in Figure 9B). Effects of opposite sign and smaller magnitude take place after decreasing the heat transfer coefficient. Therefore, the value $2500\text{ W.m}^{-2}.\text{K}^{-1}$ proposed by Arellano et al. (2013c) allows a reliable prediction of the product temperature.

The convective heat transfer resistance between the external side of the SSHE inner cylinder and the refrigerant fluid ($1/h_{outside}$) cannot be neglected. Indeed, such an assumption is equivalent to estimate $U = k_w / e_w \approx 7500\text{ W.m}^{-2}.\text{K}^{-1}$ in equation (8). As demonstrated above, even for $U = 3000\text{ W.m}^{-2}.\text{K}^{-1}$ the product temperature is largely underestimated.

In this CFD approach the heat transfer between the internal side of the wall and the product is directly calculated by the convection-conduction equation. Therefore no convective heat transfer coefficient is required on this side of the SSHE inner cylinder. Nevertheless, the order of magnitude of an hypothetical coefficient can be estimated from model predictions. The averaged heat transfer coefficient between the liquid product and the internal side of the SSHE inner cylinder could be defined by

$$h_{inside} = \frac{\dot{m} (\hat{H}_{outlet} - \hat{H}_{inlet})}{A_{wall} (\bar{T}_{wall} - \bar{T})} \quad (16)$$

where $\dot{m}$ is the product mass flow rate; $\hat{H}_{inlet}$ and $\hat{H}_{outlet}$ are specific enthalpy (J/kg) values at the SSHE inlet and outlet respectively; A_{wall} is heat exchange area, $\bar{T}_{wall}$ the averaged wall temperature, and $\bar{T}$ an averaged product temperature. The difficulty to

define and to calculate such a coefficient comes from that, towards the SSHE outlet, the phase change of the product occurs under recirculation (see for instance Figure 7); the flow pattern is very far from plug flow. Therefore the logarithmic mean temperature difference between SSHE inlet and outlet cannot be applied. We choose to use the surface averaged temperature at the internal side of the wall and the volume averaged temperature of the product in the cylindrical part of the SSHE (say, excluding the inlet and outlet bowls). In the case of reference operating conditions ($\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3$ °C and $f_{rot} = 750$ rpm), we obtain $\bar{T}_{wall} = -3.49$ °C and $\bar{T} = -3.43$ °C; the difference is very small, about 0.06 °C. Similar small differences in temperature were observed at a typical cross-section of the SSHE (Figure 8C, right display): the range of temperature is about 0.1 °C. These findings denote that either the radial mixing or the heat transfer between wall and product are very efficient. Equation (16) provides an estimate for the "internal" heat transfer coefficient (above 10^5 W.m⁻².K⁻¹) which appears too large compared to the "external" one

$$\text{obtained from equation (8): } h_{outside} = \frac{I}{\frac{I}{U} - \frac{e_w}{k_w}} \approx 3750 \text{ W.m}^{-2}.\text{K}^{-1}.$$

At this stage it is difficult to propose a general correlation for the inside heat transfer coefficient, notably because only one product was considered. At least, products with different initial freezing point and consistency should be tested and / or simulated.

5.3. Sensibility to mesh refinement

The representation of the gradients of state variables in the numerical model (velocity, pressure, temperature) is expected to become progressively finer as the mesh resolution increases. Its influence on temperature predictions was evaluated by comparing results

obtained with three meshes, constituted of increasing number of elements in the whole computational domain (6.5×10^5 , 1.5×10^6 and 3.5×10^6 elements). These meshes were built as described above and then used for solving the problem under reference operating conditions ($\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3^\circ\text{C}$ and $f_{rot} = 750$ rpm).

Figure 10 compares Arellano et al. (2013a)'s measurements (indicated by circles) with the temperature profiles in the SSHE as predicted by the numerical model after using these three meshes. The three temperatures profiles behaved similarly in the central region of the heat exchanger ($0.05 < z < 0.35$ m), with differences below 0.1°C with respect to the results achieved with the intermediate mesh (1.5×10^6 elements). After increasing the mesh resolution (3.5×10^6 elements), the predicted temperature profile became slightly warmer near the exchanger inlet ($z < 0.05$ m); as a consequence, the initial freezing temperature is reached a few millimeters later than with the intermediate mesh. Effects of opposite sign and similar magnitude take place after decreasing the mesh resolution (6.5×10^5 elements). Weaker effects can be observed, in both cases, near the exchanger outlet ($z > 0.35$ m). The whole set of temperature predictions is coherent with Arellano et al. (2013a)'s measurements.

5.4. Effect of rotation frequency

The influence of the dasher rotation frequency was assessed by employing the numerical model and comparing its predictions with the observations reported by Arellano et al. (2013a) at 545, 750 and 1000 rpm. The results on the line through the temperature sensors are presented in Figure 11. The simulated values are in quite good agreement with the experimental data.

It is noteworthy that inside the exchanger the experimental data were already quite similar for all frequencies, and as well, the simulated values presented the same behavior. This could be due to two opposing effects: on one hand, an increase on the rotation frequency increases the heat transfer which tends to decrease fluid temperature; on the other hand, an increase in the rotation frequency increases the viscous dissipation in the fluid leading to higher temperatures.

5.5. Effect of refrigerant temperature

The influence of the refrigerant temperature on the profiles following the sensors can be appreciated in Figure 12. The profiles for the curves at $T_R \geq -15.3$ °C show a good agreement with the experimental values while the profile for $T_R = -20.1$ °C did not. However, the general trend of the profile is preserved; a sharp decrease in temperature is observed near the entrance of the SSHE until the initial freezing temperature of the sorbet mix, followed by a slow decrease for most of the exchanger and a small decrease near the exit of the SSHE (at about $z = 0.36$ m) due to the exit recirculation. Logically, the point at which the initial freezing temperature is reached is closer to the entrance for lower refrigerant temperature.

The lack of agreement at $T_R = -20.1$ °C could be explained by the fact that for low refrigerant temperatures, the wall temperature along the SSHE could reach values lower than -6 °C. As presented in Figure 4B, experimental values for sorbet consistency index are only available until this temperature. For temperatures below this point, the consistency index was extrapolated linearly in this work; however, lower temperatures in the mix would mean higher ice contents that strongly modify the apparent rheological properties of sorbet

(i.e. increasing the consistency index exponentially rather than linearly). This lack of extrapolation capacity could lead to miscalculations of the velocity profile in the exchanger and a strong under-prediction of the viscous dissipation effect in the simulations that, even if only located at the exit of the SSHE, could influence the results all over the exchanger due to the back-mixing observed.

5.6. Effect of mix flow rate

The influence of the mix flow rate was analyzed through simulations carried out at the conditions that were adopted by Arellano et al. (2013a): $\dot{m}_{mix} = 25, 50$ and 75 kg/h. Results over the line that follows the position of the temperature sensors are shown in Figure 13. The profiles calculated at $\dot{m}_{mix} \geq 50$ kg/h are in good agreement with the experimental data while the profile at $\dot{m}_{mix} = 25$ kg/h shows a good agreement until $z = 0.2$ m after which a strong decrease in temperature is observed. This could be explained since lower mix flow rates leads to an increase in the residence time within the exchanger, which in turn leads to fluid temperatures reaching values closer to the refrigerant temperature (the outlet temperature calculated value for this simulation was -11.1 °C). As explained before, there is an uncertainty on the rheological behavior of the sorbet under such conditions; that could affect the prediction of velocity profiles and lead to underestimate the viscous dissipation effect in the exchanger.

6. Conclusion and perspectives

The production of non-aerated sorbet inside a scraped surface heat exchanger was studied with the help of a 3D numerical model able to solve the coupled problem of fluid flow, heat

transfer and product transformation. Fluid flow is very complex in the volume confined by the dasher, the blades and the wall of the SSHE inner cylinder. Fluid flow drives thermal convection which determines ice fraction and finally rheology evolution along the SSHE. Rheology, in turn, influences fluid flow. Model predictions compared favorably with available measurements when temperature remains in the range for which the sorbet rheology was available, indicating the global consistency of the modeling approach.

Sensitivity tests allowed us to assess the influence of key model parameters. On one hand, the influence of the heat transfer coefficient at the exchanger wall is relatively high; after increasing that coefficient by 20 %, the predicted temperatures near the SSHE outlet decreased by about 1.8 °C. Such an increase of the heat transfer coefficient could be achieved by intensifying the heat exchange between the evaporating refrigerant and the heat exchanger cylinder. In the scope of future experimental campaigns with the pilot unit here considered, we strongly recommend the inclusion of temperature monitoring at the inner wall of the SSHE as a necessary step for a finer estimation of that coefficient. On the other hand, the influence of conduction in solids (dasher, blades) is relatively weak: outlet temperatures decreased by 0.35 °C when conduction in solids was ignored. Reducing the conduction, by using a hollow dasher for example, is therefore expected to slightly improve the SSHE performance.

Post-processing of model predictions helped us to extend the understanding of phenomena under consideration. For instance, the occurrence of recirculation near the blades endings allows backflow from the outlet region. This backflow brings cold, high ice-content, fluid near to the inlet. This explains why temperatures between the blades and the dasher in the inlet region are much lower than the value prescribed at the SSHE entrance (5 °C, see for instance Figure 9). This aspect would not be predicted with a 1D model assuming plug-

696 flow and an overall uniform heat transfer coefficient. The temperature field summarizes the
697 strong coupling that occurs between the phenomena in the SSHE: on one hand, when the
698 temperature decreases below the initial freezing value $-2.72\text{ }^{\circ}\text{C}$, sharp changes affects the
699 product properties, including its rheology; on the other hand, in addition to rotation and
700 mean axial flow, the product rheology drives the fluid motion and hence the convective
701 flux of heat. We argue that only the fully-coupled solution of this challenging problem can
702 provide realistic predictions.

703 In further work, on the base on the modeling of velocity and temperature fields, CFD will
704 be used to predict ice crystals growth and bubble fragmentation. For ice crystal growth,
705 backflow will be also crucial because it seeds the inlet region with ice crystals, hence
706 avoiding super-cooling. Bubble fragmentation in aerated sorbet mainly depends on shear
707 rate, elongation rate and apparent viscosity of the surrounding fluid, which can be provided
708 by the present work. Nevertheless, it should be kept in mind that experimental rheology
709 data should be completed for high shear rate and ice content, because it could affect bubble
710 fragmentation prediction. This way, 3D modeling of the coupled relevant phenomena could
711 be really helpful for industrial applications.

712 Looking forward, we focus our final words on the necessity of a broader experimental
713 dataset than the one available to the present study. Firstly, pressure drop measurements
714 should be included, as a way to validate the model predictions of complex fluid flow
715 occurring under heat transfer and product transformation in the SSHE. Secondly, other
716 operating conditions and product properties should be considered, for instance the initial
717 freezing point. Beyond the opportunity for complementary assessment of model
718 predictions, the availability of a broad dataset is necessary to identify heat transfer

correlations in SSHE during crystallization. Recent efforts have been devoted on this
difficult subject (e.g. Saraceno et al., 2011; Rainieri et al., 2014). Lastly, the product
rheology must be fully characterized, as a way to improve the reliability of model
predictions under particular operating conditions (low refrigerant fluid temperature, low
mass flow rate). Such an enhanced characterization should include higher shear rates and
ice contents.

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Figure

FIGURE 1A)

FIGURE 1B)

Figure 1. Schematic view of the SSHE pilot unit used for freezing lemon sorbet (Arellano et al., 2013a, 2013b, 2013c):

A) 3D view of the SSHE as represented in the numerical model, and

B) longitudinal (left) and cross-sectional (right) views of the SSHE as employed for displaying model predictions.

FIGURE 2)

Figure 2. Cross-section view of the SSHE. The computation domain taken into account by the numerical model is pictured in gray (see the whole cross-section view at top-left). This domain is delimited by the thickest line, which corresponds to the internal wall of the SSHE inner cylinder. Other black lines delimitate the SSHE outer and inner cylinders. The wall of the inner cylinder is characterized by thickness e_w , thermal conductivity k_w , and the outside and inside temperatures ($T_{outside}$ and T_{inside} respectively). The dashed zone corresponds to the refrigerant fluid, characterized by evaporating temperature T_R .

FIGURE 3)

Figure 3. Freezing depression curve of concentrated lemon sorbet mix and sucrose solution:

- (○) obtained experimentally for lemon sorbet mix by Gonzalez Ramirez (2012),
- (—) estimated for lemon sorbet mix using equation (11),
- (△) obtained experimentally for sucrose solutions by Reiser et al. (2012), and
- (—) estimated for sucrose using equation (11).

FIGURE 4A)

FIGURE 4B)

FIGURE 4C)

FIGURE 4D)

Figure 4. Sorbet physical properties as taken into account in this study:

A) ice mass fraction in sorbet according to temperature using equations (9-12);

B) consistency index:

(□) obtained experimentally by Arellano et al. (2013b) and

(—) estimated using equation (13);

C) heat capacity:

(—) estimated as proposed by Cogné et al. (2003) and (—) estimated using equations (15);

D) thermal conductivity:

(—) estimated as proposed by Cogné et al. (2003) and (—) estimated using equations (16).

FIGURE 5A)

FIGURE 5B)

Figure 5. A) Actual cross-section view of the SSHE inlet bowl (left), and its representation in the numerical model (right). B) The same for the SSHE outlet bowl.

FIGURE 6)

Figure 6. Mesh built for the simulations carried out in this work. Right: mesh around the solid elements (dasher and blades); left: cut view in the center of the SSHE inner cylinder.

FIGURE 7)

Figure 7. Velocity field in the SSHE, as predicted by the numerical model under reference operating conditions: $\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3$ °C and $f_{rot} = 750$ rpm. Velocity vectors presented in the plane YZ containing the axial direction are not in the same scale than those in the three planes XY crossing the axial direction.

FIG. 8A) Velocity magnitude [m/s]

...and Shear Rate [1/s]

FIGURE 8B) Pressure [Pa]

FIGURE 8C) Temperature [°C]

FIG. 8D) Specific enthalpy [J/kg] and ice mass fraction

FIGURE 8E) Viscous dissipation rate [W/m³]

Figure 8. Views of the profiles in the SSHE as predicted by the numerical model under reference operating conditions: $\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3$ °C and $f_{rot} = 750$ rpm:

- A)** velocity magnitude (top) and shear rate (bottom),
- B)** pressure,
- C)** temperature,
- D)** enthalpy and ice fraction, and
- E)** viscous dissipation rate.

FIGURE 9A)

FIGURE 9B)

Figure 9. A) Measurements of temperature by Arellano et al. (2013a) at selected positions of the SSHE (circles) and temperature axial profiles predicted by the numerical model under reference operating conditions ($\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3$ °C and $f_{rot} = 750$ rpm), after setting the thermal conductivity of solids to $15 \text{ W.m}^{-1}.\text{K}^{-1}$ (black line) and to zero (grey line). Squares indicate mean surface values of temperature as predicted by the numerical model at the exit of the outlet pipe. The dashed line indicates the initial freezing temperature. **B)** The same, but setting the heat transfer coefficient at the SSHE inner wall to $2500 \text{ W.m}^{-2}.\text{K}^{-1}$ (black line), to a value 20 % larger ($3000 \text{ W.m}^{-2}.\text{K}^{-1}$) (dark grey line) and to a value 20 % smaller ($2000 \text{ W.m}^{-2}.\text{K}^{-1}$) (light grey line).

FIGURE 10)

Figure 10. Measurements of temperature by Arellano et al. (2013a) at selected positions of the SSHE (circles) and temperature axial profiles predicted by the numerical model under reference operating conditions ($\dot{m}_{mix} = 50 \text{ kg/h}$, $T_R = -15.3 \text{ °C}$ and $f_{rot} = 750 \text{ rpm}$), after using three meshes constituted by an increasing number of elements: (--) 6.5×10^5 , (—) 1.5×10^6 and (···) 3.5×10^6 elements. Corresponding values of the surface-averaged value of predicted temperatures at the exit of the outlet pipe are also indicated, for (▲) 6.5×10^5 , (●) 1.5×10^6 and (■) 3.5×10^6 elements.

FIGURE 11)

Figure 11. Effect of the dasher rotation frequency on the temperature profile along the exchanger:

(▼) prescribed value at the inlet, for all the operating conditions (5 °C);

measured values for (Δ) $f_{rot} = 545 \text{ rpm}$, ($\square$) $f_{rot} = 750 \text{ rpm}$, and ($\circ$) $f_{rot} = 1000 \text{ rpm}$;

predicted profiles for (▣) $f_{rot} = 545 \text{ rpm}$, (—) $f_{rot} = 750 \text{ rpm}$, and (—) $f_{rot} = 1000 \text{ rpm}$; and

surface-averaged values at the exit for ($\blacktriangle$) $f_{rot} = 545 \text{ rpm}$, ($\blacksquare$) $f_{rot} = 750 \text{ rpm}$, and ($\bullet$) $f_{rot} = 1000 \text{ rpm}$.

FIGURE 12)

FIGURE 13)

Figure 13. Effect of the mix mass flow rate on the temperature profile along the exchanger: (▼) prescribed value at the inlet, for all the operating conditions (5 °C); measured values for (Δ) $\dot{m}_{mix} = 25$ kg/h, ($\square$) $\dot{m}_{mix} = 50$ kg/h, and ($\circ$) $\dot{m}_{mix} = 75$ kg/h; predicted profiles for (···) $\dot{m}_{mix} = 25$ kg/h, (---) $\dot{m}_{mix} = 50$ kg/h, and (—) $\dot{m}_{mix} = 75$ kg/h; and surface-averaged values at the exit for ($\blacksquare$) $\dot{m}_{mix} = 50$ kg/h and ($\bullet$) $\dot{m}_{mix} = 75$ kg/h.

Figure Captions

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40 solid elements (dasher and blades); left: cut view in the center of the SSHE inner cylinder.

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43 operating conditions: $\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3$ °C and $f_{rot} = 750$ rpm. Velocity vectors
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49 reference operating conditions: $\dot{m}_{mix} = 50$ kg/h, $T_R = -15.3$ °C and $f_{rot} = 750$ rpm:

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65

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71 using three meshes constituted by an increasing number of elements: (--) 6.5×10^5 , (—) 1.5×10^6
72 and (■■■) 3.5×10^6 elements. Corresponding values of the surface-averaged value of
73 predicted temperatures at the exit of the outlet pipe are also indicated, for
74 (▲) 6.5×10^5 , (●) 1.5×10^6 and (■) 3.5×10^6 elements.

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78 (▼) prescribed value at the inlet, for all the operating conditions (5 °C);
79 measured values for (Δ) $f_{rot} = 545$ rpm, (□) $f_{rot} = 750$ rpm, and (○) $f_{rot} = 1000$ rpm;
80 predicted profiles for (■■■) $f_{rot} = 545$ rpm, (—) $f_{rot} = 750$ rpm, and (—) $f_{rot} = 1000$ rpm; and
81 surface-averaged values at the exit for (▲) $f_{rot} = 545$ rpm, (■) $f_{rot} = 750$ rpm, and (●) $f_{rot} =$
82 1000 rpm.

83

84

85

86 **Figure 12.** Effect of the refrigerant evaporation temperature on the temperature profile
87 along the exchanger:

88 (▼) prescribed value at the inlet, for all the operating conditions (5 °C);

89 measured values for (Δ) $T_R = -10.3$ °C, (□) $T_R = -15.3$ °C, and (○) $T_R = -20.1$ °C;

90 predicted profiles for (▬▬▬) $T_R = -10.3$ °C, (— —) $T_R = -15.3$ °C, and (—) $T_R = -20.1$ °C;

91 and

92 surface-averaged values at the exit for (▲) $T_R = -10.3$ °C, (■) $T_R = -15.3$ °C, and (●) T_R

93 $= -20.1$ °C.

94

95 **Figure 13.** Effect of the mix mass flow rate on the temperature profile along the exchanger:

96 (▼) prescribed value at the inlet, for all the operating conditions (5 °C);

97 measured values for (Δ) $\dot{m}_{mix} = 25$ kg/h, (□) $\dot{m}_{mix} = 50$ kg/h, and (○) $\dot{m}_{mix} = 75$ kg/h;

98 predicted profiles for (▬▬▬) $\dot{m}_{mix} = 25$ kg/h, (— —) $\dot{m}_{mix} = 50$ kg/h, and (—) $\dot{m}_{mix} = 75$

99 kg/h; and

100 surface-averaged values at the exit for (■) $\dot{m}_{mix} = 50$ kg/h and (●) $\dot{m}_{mix} = 75$ kg/h.

2
34

Nomenclature

- 1 **Nomenclature**
- 2
- 3 A_{wall} : heat exchange area (m^2)
- 4 c_{pi} : ice heat capacity ($J.mol^{-1}.K^{-1}$)
- 5 c_{pw} : water heat capacity ($J.mol^{-1}.K^{-1}$)
- 6 C, C_1, C_2 : dimensionless parameters
- 7 C_p : apparent specific heat capacity ($J.kg^{-1}.K^{-1}$)
- 8 C_{pi} : specific heat capacity of ice ($J.kg^{-1}.K^{-1}$)
- 9 C_{ps} : specific heat capacity of solute ($J.kg^{-1}.K^{-1}$)
- 10 C_{pw} : specific heat capacity of water ($J.kg^{-1}.K^{-1}$)
- 11 e_w : wall thickness of the SSHE inner cylinder (m)
- 12 f_{rot} : frequency of rotation (rev/s)
- 13 $h_{outside}$: heat transfer coefficient, external side of SSHE inner wall ($W.m^{-2}.K^{-1}$)
- 14 $\hat{H}_{inlet}, \hat{H}_{outlet}$: specific enthalpies at SSHE inlet and outlet ($J.kg^{-1}$)
- 15 k : thermal conductivity of product ($W.m^{-1}.K^{-1}$)
- 16 k_w : thermal conductivity of stainless steel wall ($W.m^{-1}.K^{-1}$)
- 17 K : consistency index ($Pa.s^n$)
- 18 $\dot{m}$: mass flow rate ($kg.s^{-1}$)
- 19 M_s : molar mass of solute ($kg.mol^{-1}$)
- 20 M_w : molar mass of water ($kg.mol^{-1}$)
- 21 n : flow behavior exponent (–)

- 22 p : pressure (Pa)
- 23 $\vec{q}$: conductive heat transfer vector (W.m^{-2})
- 24 $\vec{r}$: position vector
- 25 r : radial component of the position vector (m)
- 26 r_{pipe} : pipe radius (heat exchanger cylinder) (m)
- 27 R : ideal gas constant ($8.314 \text{ J.mol}^{-1}.\text{K}^{-1}$)
- 28 T : temperature (K)
- 29 T_{if} : initial freezing temperature (K)
- 30 T_{mp} : pure-water melting-point temperature (K)
- 31 T_R : refrigerant evaporating temperature (K)
- 32 U : global heat transfer coefficient ($\text{W.m}^{-2}.\text{K}^{-1}$)
- 33 $\vec{u}$: velocity vector in the Galilean frame (m.s^{-1})
- 34 $\vec{v}$: velocity vector in the rotating frame (m.s^{-1})
- 35 w_i : ice mass fraction (—)
- 36 w_s : solute mass fraction (—)
- 37 w_{s0} : initial solute mass fraction, at the exchanger inlet (—)
- 38 w_s^* : solute mass fraction at saturation (—), see eq. (9)
- 39 x_w : water molar fraction at saturation (—)
- 40 z : axial component of the position vector (m)
- 41
- 42

43 Greek symbols

44

45 $\Delta h_{f,mp}$: ice-water heat of fusion (J.mol^{-1}), see eq. (9)

46 ΔH_f : specific heat of fusion of ice (J.kg^{-1}), see eq. (14)

47 $\vec{\Omega}_{rotor}$: angular velocity vector (rad.s^{-1})

48 ρ : density (kg.m^{-3})

49 $\bar{\tau}$: viscous stress tensor (Pa)