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Novel 4-D Algorithm for Functional MRI Image Regularization using Partial Differential Equations

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Synopsis

State-of-the-art techniques for denoising functional MRI (fMRI) images consider the problems of spatial and temporal regularization as decoupled tasks. In this work we propose a partial differential equations (PDEs) -based algorithm that acts directly on the 4-D fMRI image. Our approach is based on the idea that large image variations should be preserved as they occur during brain activation, but small variations should be smoothed to remove noise. Starting from this principle, by means of PDEs we were able to smooth the fMRI image with an anisotropic regularization, thus recovering the location of the brain activations in space and their timing and duration.

Introduction

Image denoising via different regularization algorithms have been addressed within the past years with many approaches [1, 2]. In particular, in the case of functional MRI (fMRI), deconvolution techniques are used to denoise the blood-oxygen-level-dependent (BOLD) response [3]. In this work we propose a novel approach to anisotropically regularize the 4-D fMRI image using partial differential equations (PDEs), in order to smooth the data by minimizing the image variations, while preserving the discontinuities, i.e. edges. The approach acts simultaneously in the 3-D space and the time dimension and does not require a priori information about the activation and their timing.

Methods

Let $\Omega$ be a 4-D (3-D space × time) domain and assume Neumann boundary conditions on $\partial \Omega$. We defined a regularization process, based on gradient descent
computed with PDEs, such that:

\[
\frac{\partial I}{\partial t} = (1 - \lambda) \bar{H} \ast (I_0 - H \ast I) + \lambda \frac{\text{div}(D\nabla I)}{B}
\]

(1)

where \( I_0 \) and \( I \) are the original and the denoised image respectively, \( A = \|I_0\| \) and \( B = \|\text{div}(D\nabla I_0)\| \) are normalization factors, \( H \) is the hemodynamic response function (HRF) [4], \( \bar{H} \) is the time-reversed HRF, \( \lambda \) is the regularization parameter and \( D = \nabla I \nabla I^T \|\nabla I\|_2 \ast G \) is the 4-D structure tensor of \( I \) smoothed by the gaussian kernel \( G \) \((\sigma_G = 1)\). In (1) the first term on the right is the data fitting term, which measures the correlation of the residual with \( H \), and the second term minimizes image variations. The convolution with \( H \) and \( \bar{H} \) were computed only along the time dimension. After computing the operator \( D \), we defined the directions of the image variations by an eigendecomposition of \( D \) such that:

\[
D = Q \Lambda Q^T
\]

where \( Q \) contains the orthogonal eigenvectors \((\theta_1, \theta_2, \theta_3, \theta_4)\) of \( D \) and \( \Lambda \) contains their associated eigenvalues \((\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4)\). Finally, we recomputed the matrix \( \bar{D} = \bar{Q} \Lambda \bar{Q}^T \) as follows: the highest eigenvalue, \( \lambda_1 \), was set according to a function of \( \|\nabla I\| \), i.e. \( f(\|\nabla I\|) \), such that if \( \|\nabla I\| >> 0 \) the current voxel may be located on a edge and the smoothing is performed just along the other three directions (anisotropic smoothing). Otherwise, if \( \|\nabla I\| \rightarrow 0 \) the smoothing will be isotropic in all the four directions. \( \lambda_2, \lambda_3, \lambda_4 \) were indeed set to 1 to perform an isotropic smoothing. After that, we replaced the operator \( D \) in (1) with \( \bar{D} \).

As a proof of concept, similarly to previous works [5, 6], we scaled a 3-D activation map computed with the FMRIB Software Library (FSL) in the range [0,3], with a 2-mm isotropic resolution (Figure 1.a). We multiplied it by a piece-wise constant signal of 100 s, with one onset of 40 s (Figure 1.b). We corrupted the image with model noise, we convolved it with the HRF [4] and we added gaussian noise thus simulating the fMRI time-courses \( y(t) \). We regularized the image as showed above, and we recovered \( u^*(t) \). Finally, to evaluate the results, we computed the root of the mean square errors (MSE) and standard deviation (STD) and the Pearson correlation and its STD between \( u(t) \) and \( u^*(t) \) averaged among the voxels belonging to the grey matter. We compared our results with those obtained using the Total Activation approach (TA) [6], implemented in the iCAPs toolbox (https://miplab.epfl.ch/index.php/software/total-activation).

Results

Figure 2 shows examples of regularized spatial maps and time series using our approach \((u^*_{\text{PDEs}})\) and the TA \((u^*_{\text{TA}})\). Figure 3 shows that the MSEs ± STDs change for different peak-SNRs (pSNRs) and that they are lower than the ones obtained using TA. Figure 4 shows that the activation recovered with our approach is more correlated with the ground truth, for different pSNRs, whereas the results obtained with TA are more sensitive to noise.

Discussion

Both techniques succeeded at finding the activated regions, but the results obtained using our approach were closer to the ground truth in terms of amplitude
and also of correlation between the simulated activations and the recovered ones, for different pSNRs. Our findings shows that our approach enabled us to solve a unique problem, coupling the space and the time dimension thus having to set just one regularization parameter, $\lambda$, rather than dividing it into two separate problems, i.e. time and space, as it was done by Farouj and colleagues [6].

**Conclusion**

We showed that a simultaneous 4-D approach for fMRI image regularization using PDEs diffusion allows to recover brain functional activations, without having to make a priori assumptions on the spatial features of the activations and their duration. Future works will validate the described method on real fMRI data.

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**References**


Figures

**Figure 1:** Ground truth for the functional MRI (fMRI) simulated data: (a) activation map. (b) Simulated activation $u(t)$, with a repetition time (TR) of 1 s.

**Figure 2:** (a) From left to right: spatial maps of the simulated functional MRI (fMRI) image $y$, ground truth activation $u$, recovered activation using the Total Activation (TA) approach ($u^{*}_{TA}$) and our approach ($u^{*}_{PDE}$). Each row corresponds to a different peak-SNR (pSNR): 6.54 dB, 5.99 dB, 5.9 dB, 3.93 dB from the top to the bottom. (b) Reconstructed time series $u^{*}$ obtained with our approach $u^{*}_{PDE}(t)$ (red) and the TA approach $u^{*}_{TA}(t)$ (blue) superimposed on the activation $u(t)$ (black) and fMRI signal $y(t)$ (green).
Figure 3: The graph shows, for different peak-SNRs, the roots of the mean square errors (MSE) and standard deviation (STD) between $u(t)$ and $u^*(t)$ averaged among the voxels belonging to the grey matter.

Figure 4: The graph shows, for different peak-SNRs (pSNRs), the Pearson correlation coefficient computed between $u(t)$ and $u^*(t)$ and averaged among the voxels belonging to the gray matter (GM) and their standard deviation. ($\mu_r$: mean correlation coefficient; $\sigma_r$: standard deviation of the correlation coefficients among the GM voxels.)