Heavy to Light Meson Semileptonic Decays Form Factors

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Abstract

Like the two-photon and two-gluon decays of the P-wave charmonium state for which the Born term produces a very simple decays amplitude, the Born term for the processes $c\bar{d} \to (\pi, K)\ell\nu$ and $b\bar{d} \to (\pi, K)\ell\nu$, could also produce a simple expression for D and B meson semileptonic decays with a light meson π, K in the final state. The pole term at $q^2 = m_B^2 + m_\pi^2$ for $B \to \pi$ and at $q^2 = m_D^2 + m_K^2$ for $D \to K$ form factor, are generated by the Born term and given as : $f_+(0)/(1 - q^2/(m_H^2 + m_\pi^2))$, with $H = D, B$ for $D, B \to \pi$ form factors, and $f_+(0)/(1 - q^2/(m_H^2 + m_K^2))$ for $B, D \to K$ form factor. These pole dominance terms describe rather well the q^2-behavior of the form factors observed in the BaBar, Belle and BESIII measurements and in lattice simulation. In particular, the $D \to K$ form factors are in good agreement with the measured values in the whole range of q^2 showing evidence for $SU(3)$ breaking with the presence of the m_K^2 term in the quark propagator, but some corrections to the Born term are needed at large q^2 for $D, B \to \pi$ form factors.

Keywords: Heavy to light meson semileptonic decays, Form factors, CKM quark mixing matrix

1. Introduction

The semileptonic heavy to light meson decay form factor as in the $D, B \to \pi\ell\nu$ decays, is given by the $V - A$ current matrix elements between heavy and light meson state. A precise knowledge of these form factors is required for an exclusive determination of the CKM matrix element V_{ub}. These form factors are known experimentally from BaBar[1], Belle[2] and BESIII[3] measurements. There have been previous calculations of the form factors in the light-cone sum rule approach(LCSR)[4] and in lattice simulation[5, 6]. The $B \to \pi$ form factor from lattice simulation and BaBar measurements could be fitted with a BK(Becirevic-Kaidalov) parametrization[7] with $\alpha = 0.52 \pm 0.05 \pm 0.03$ [8]:

$$f^+(q^2, \alpha) = \frac{f_0}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)} \quad (1)$$

For D meson semileptonic decays, the BaBar, Belle and BESIII measurements[3, 9] show that the $D \to \pi, K$ form factor could be fitted with an effective two-pole :

$$f^+(q^2, \alpha) = \frac{f_0}{(1 - q^2/m_{D^*}^2).(1 - \alpha q^2/m_{D^*}^2)} \quad (2)$$

or a single-pole parametrization :

$$f^+(q^2, \alpha) = \frac{f_0}{(1 - q^2/m_{\rm pole}^2)} \quad (3)$$

with $m_{\rm pole}^2 = (1.906 \pm 0.029 \pm 0.023)$GeV obtained by BaBar[9] and from BESIII $m_{\rm pole} = (1.911 \pm 0.012 \pm 0.004)$GeV for $D \to \pi$ form factor and $m_{\rm pole} = (1.921 \pm 0.010 \pm 0.007)$GeV for $D \to K$ form factor, quite different from the value of 2.010GeV for the D_s^{*+} mass, as noted by BESIII[3]. The lower value for the pole mass is also found by Belle[2] which gives a value of $1.82 \pm 0.04 \pm 0.03$GeV, but a higher value for pole mass of $(1.93 \pm 0.05 \pm 0.03)$GeV close to the BESIII value is obtained earlier by FOCUS[10]. Other data for $D \to K$ form factor are given in TABLE VI of Ref. [[11]], with the pole mass values for CLEO and FOCUS around 1.9GeV close to the BESIII values. .

The problem is to obtain a theoretical expression with this pole-dominance q^2-behaviour for these form factors. In this talk, I would like to present a recent work[12] in which we show that the form factor for heavy meson semileptonic decay with pion or light hadron in the final state can be obtained from the Born terms for the $c\bar{d} \to \pi\ell\nu$ and $b\bar{d} \to \pi\ell\nu$ process, with the π, K meson treated as the Goldstone boson of the chiral $SU(3)\times SU(3)$ symmetry, rather than a two-body bound

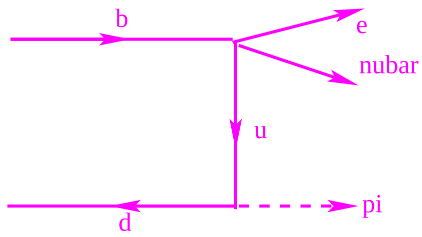

Figure 1: Diagram showing Born term for the semileptonic B decay form factor

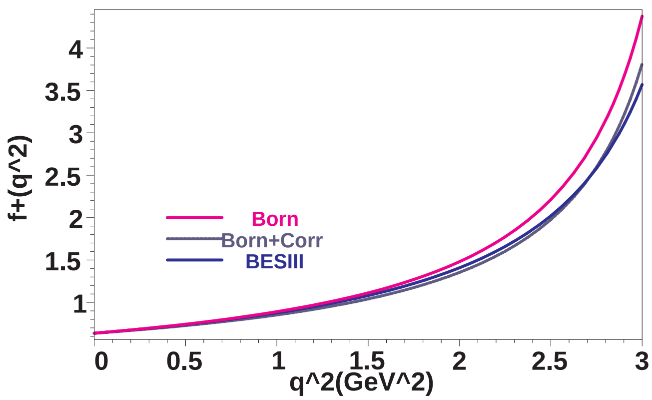

Figure 2: The Born term(upper curve), the Born term with a small polynomial term to fit the $D^0 \to \pi^-$ form factor BESIII data (lower curve) which are given in FIG.9 of Ref. [3]

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state which gives a rather small $\overline{B}^0 \to \pi^+\ell^-\nu$ branching ratio[13]. The use of the pion-quark coupling to obtain the heavy to light semileptonic decay amplitudes is similar to the study of strong and radiative decays of light vector mesons $\rho^- \to \pi^-\gamma$ with an essentially the same Born term shown in Figure 1 from which the extracted pion-quark coupling consistent with the theoretical value given by the bag model[14] to within 50%. Thus it is possible to obtain the semileptonic decays amplitude with pion in the final state treated as a Goldstone boson of chiral symmetry with the pion-quark coupling given by the Goldberger-Treiman relation. The Born term from this process then gives the semileptonic decay amplitudes and the $D \to \pi, K$ and $B \to \pi$ form factors with the pole dominance term, in agreement with the BaBar, Belle and BESIII measurements and lattice simulations.

2. Effective Lagrangian for $D \to \pi\ell\nu$ and $B \to \pi\ell\nu$

The $c\bar{c} \to \gamma\gamma$ annihilation in the two-photon decay of $P-$wave charmonium state proceeds through the Born term at the tree level approximation[15–17], the reac-

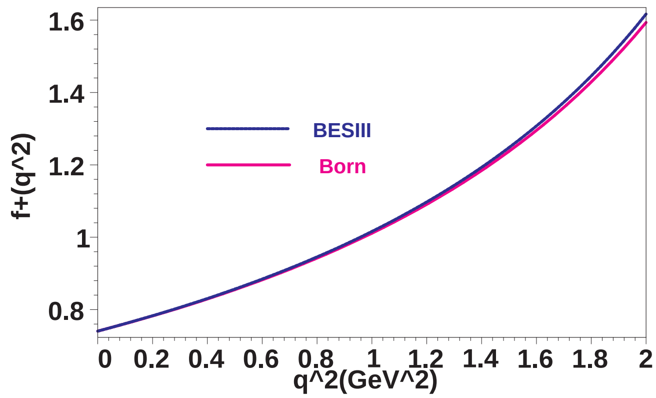

Figure 3: The Born term(upper curve), the Born term with a small polynomial term to fit the $D^+ \to \pi^0$ form factor BESIII data lower curve) which are given in FIG.6 of Ref. [19]

tions $c + \bar{d} \to \pi\ell\nu$ and $b + \bar{d} \to \pi\ell\nu$, can occur through a similar Born term, with the exchange of an u quark which combines with $\bar{d}$ quark to produce a pion in the final state.

In the weak binding approximation with $b, \bar{d}$ quark taken at rest in the B^0 meson, the effective Lagrangian for $B, D \to \pi\ell\nu$ is given by :

$$O_\mu = \bar{v}(p_d)V_\mu u(p_b) \tag{4}$$

with

$$V_\mu = \frac{1}{i}\Big[(-ig_{\pi qq}\gamma_5)i\frac{(\not{p} - \not{p}_d + m_u)}{(p - p_d)^2 - m_u^2}(-ig\gamma_\mu)\Big] \tag{5}$$

In terms of the local 2-quark operator , the vector current matrix element in $B \to \pi\ell\nu$ decays is given by :

$$O_P = \frac{2m_B(\bar{d}\gamma_5 b)p_\mu}{(m_B^2 + m_\pi^2 - q^2)} \tag{6}$$

showing the appearance of the pole at $q^2 = (m_B^2 + m_\pi^2)$ generated by the Born terms. For $D \to K$ form factor, the pole is at $q^2 = (m_D^2 + m_K^2)$. This result explains the success of the single-pole or two-pole fits of the BaBar and BESIII data as shown below. By putting the B meson at rest with $m_b + m_d = m_B$, $< 0|\bar{d}\gamma_5 b|B) = m_B f_B$, and the pion-quark coupling for a constituent quark from the Goldberger-Treiman relation with $g_A = 3/4$ [18] we obtain immediately the form factor for $B \to \pi\ell\nu$ decay, we have:

$$f_+(0) = \Big(\frac{f_B}{f_\pi}\Big)g_A\frac{1}{(1 + m_\pi^2/m_B^2)} \tag{7}$$

and similar expression for $D \to K$ form factor.

With $f^+(q^2)$ at $q^2 = 0$ known from experiment, we have:

$$f_+(q^2)_{D\pi} = \frac{f_+(0)_{D\pi}}{(1 - q^2/(m_D^2 + m_\pi^2))}$$
$$f_+(q^2)_{DK} = \frac{f_+(0)_{DK}}{(1 - q^2/(m_D^2 + m_K^2))}$$
$$f_+(q^2)_{B\pi} = \frac{f_+(0)_{B\pi}}{(1 - q^2/(m_B^2 + m_\pi^2))} \quad (8)$$

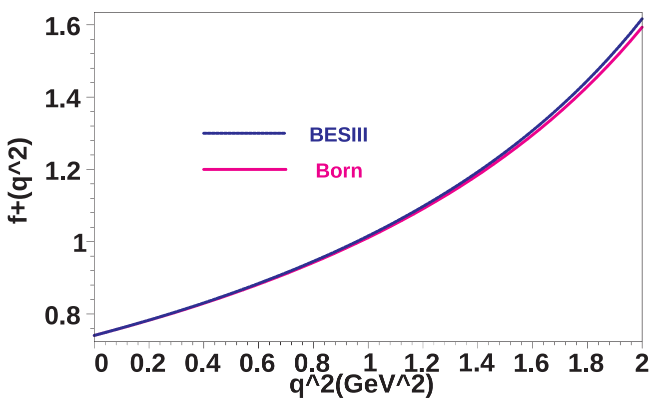

Figure 4: The Born term(lower curve) and the fit to the $D^0 \to K^-$ BES measured form factor (upper curve) given in FIG.8 of Ref. [3]

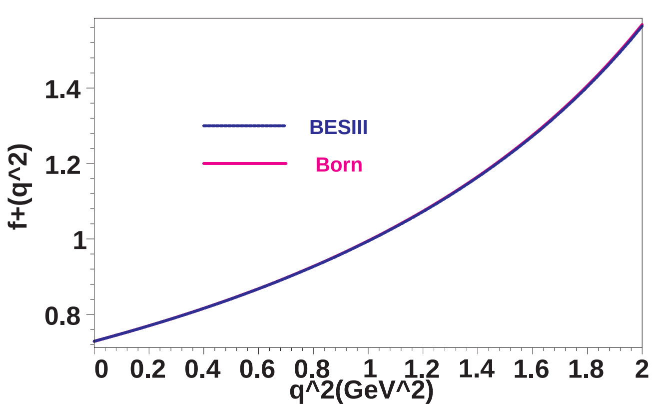

Figure 5: The Born term(lower curve) and the fit to the $D^+ \to \bar{K}^0$ BES measured form factor (upper curve) given in FIG.6 of Ref. [19]

for the Born term contribution to the form factor.

The above expressions are essentially the same to those used in the parametrization of the form factors measured at BaBar, Belle and BESIII. As the Born term is of pure kinematic origin, there is no D^*, B^* pole term in the above expressions.This explains the fact that the single-pole fits for $D \to K, \pi$ form factors do not have a D^* pole, consistent with the quark propagator pole term. For the $B \to \pi$ form factor, the mass difference between B^* and B is negligible, to be consistent, $m_{B^*}^2$ should be replaced by m_B^2 without affecting the BaBar BK fit, making it consistent with the Born term. What is new here is that the Born term could generate this pole dominance term which seems impossible to obtain otherwise.

There is also possible suppression of the quark-pion coupling due to the off-shell effects of the quark propagator, as the momentum of the u–quark in the Born term gets large for small q^2, the value of $f^+(q^2)$ would be suppressed for small q^2. The QCD sum rule calculation[22] gives $f_+(0) = 0.23 \pm 0.02$, while in the calculation of Ref. [23], $f_+(0) = 0.24 \pm 0.025$ and $f_+(0) = 0.26 \pm 0.02$. These calculations show that the $B \to \pi$ form factor is strongly suppressed at $q^2 = 0$. The QCDSF Collaboration lattice calculation also shows suppression with $f_+(0) = 0.27 \pm 0.07 \pm 0.07$. For $D^0 \to \pi^-$ form factor, the BESIII data give $f_+(0) = 0.6372 \pm 0.0.0080 \pm 0.0044$ and for $D^0 \to K^-$ form factor $f_+(0) = 0.7368 \pm 0.0080 \pm 0.0044$, and similarly for $D^+ \to \pi^0$, $f_+(0) = 0.622 \pm 0.012 \pm 0.003$ and $f_+(0) = 0.725 \pm 0.004 \pm 0.012$ for $D^+ \to \bar{K}^0$ showing no large suppression compared with the $B \to \pi$ form factor. As shown in Figure 2 and Figure 3 for $D^0 \to \pi^-$ and $D^+ \to \pi^0$ form factor, the Born term plotted in the upper curve is slightly above the lower curve obtained with the BESIII fit with $f_+^\pi(0) = 0.6372 \pm 0.0080 \pm 0.0044$, For the $D^0 \to K^-$ form factor, in Figure 4, the lower curve(Born term) is in excelllent agreement with the fit to the BESIII data(upper curve) with $f_+^K(0) = 0.7768 \pm 0.0026 \pm 0.0036$, $M_{pole} = 1.921 \pm 0.010 \pm 0.007$GeV $V_{cs} = 0.97343 \pm 0.00015$ and $V_{cs} = 0.97343 \pm 0.00015$. This good agreement between the two curves in Figure 4 could be explained by the m_K^2 term in the u–quark propagator. If we replace the factor $q^2/(m_D^2 + m_K^2)$ by $q^2/m_{\rm eff}^2$, with $m_{\rm eff} = \sqrt{m_D^2 + m_K^2}$ as the effective mass in the pole term, then $m_{\rm eff} = 1.931$GeV, very close to the pole mass of the BESIII fit, $M_{pole} = 1.921 \pm 0.010 \pm 0.007$GeV.

Agreement is also found between the Born term and the BESIII fit for the $D^+ \to \bar{K}^0$ form factor obtained with $f_+^\pi(0) = 0.7094 \pm 0.0035 \pm 0.0111$, $M_{pole} = 1.935 \pm 0.017 \pm 0.006$GeV shown in Figure 5 , very close to the effective mass $m_{\rm eff} = 1.931$GeV in the Born term. This dependence on m_K^2 in both $D^0 \to K^-$ and $D^+ \to \bar{K}^0$ form factor shows evidence for the dominance of the Born term for the $D \to K$ semileptonic decay form factors.

There is also a correction to the $B \to \pi$ form factor Born term to compensate for a suppression at large q^2 induced by $f_+(0)_{B\pi}$ mentioned above. Thus with these corrections included, the middle curve of Figure 6 is

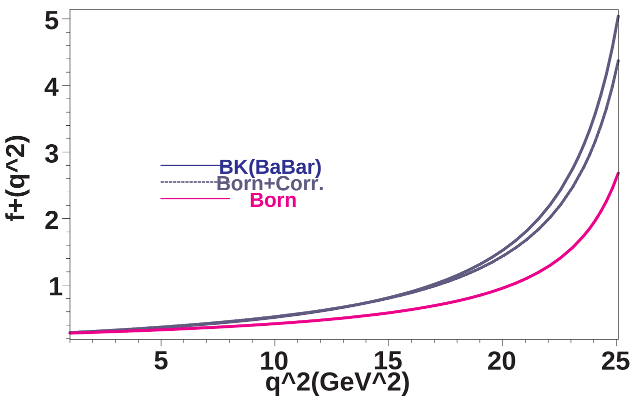

Figure 6: The Born term(lower curve), the Born+Corr plot has a small polynomial term added (middle curve) and the BaBar data (upper curve) represented by the BK fit to the BaBar data of Ref. [8]

now in agreement with data and almost coincides with the lower curve obtained with a BK fitted to the BaBar data. The $D \to K, \pi$ and $B \to \pi$ form factors with the Born term as the main contribution, and assuming the same correction term for $D^0 \to \pi^-$ and $D^+ \to \pi^0$ form factor, are now in agreement with data and are given by :

$$f_+(q^2)_{D\pi} = \frac{f_+(0)_{D\pi}(1 - 0.15q^2/(m_D^2 + m_\pi^2))}{(1 - q^2/(m_D^2 + m_\pi^2))}$$
$$f_+(q^2)_{DK} = \frac{f_+(0)_{DK}}{(1 - q^2/(m_D^2 + m_K^2))}$$
$$f_+(q^2)_{B\pi} = \frac{f_+(0)_{B\pi}(1 + 0.70q^2/(m_B^2 + m_\pi^2))}{(1 - q^2/(m_B^2 + m_\pi^2))} \quad (9)$$

3. Conclusion

In conclusion, the tree-level Born term for the process $c + \bar{d} \to \pi\ell\nu$ and $b + \bar{d} \to \pi\ell\nu$ in semileptonic decays of a heavy meson to a light meson in the final state is found to describe rather well the q^2-dependence of the $D \to \pi, D \to K$ and $B \to \pi$ form factors. The $D^0 \to K^-$ and $D^+ \to K^0$ form factors show possible evidence for the K mass term in the q^2-dependence generated by this Born term.

Note added: While preparing this talk, I found a previous paper[26] in which a B-meson pole q^2-dependence term $1/(1 - q^2/m_B^2)$ for the $B \to \pi$ form factor is also obtained.

4. Acknowledgments

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