A comparative study between two control laws of an electropneumatic actuator

X.BRUN, S. SESMAT, D. THOMASSET, S. SCAVARDA

Laboratoire d'Automatique Industrielle

INSA Lyon Bât 303

20, Ave. A. Einstein, 69621 Villeurbanne Cedex, France

Tel: (33) 4 72 43 81 98 Fax: (33) 4 72 43 85 35

email : xbrun@laifluid2.insa-lyon.fr, sylvie@laifluid2.insa-lyon.fr, thom@laifluid2.insa-lyon.fr,
scavarda@laifluid2.insa-lyon.fr

Session : Sophisticated Control Hydraulic Drives (Invited paper)

Summary: This paper focuses on a comparison between two control laws of an half meter stroke electropneumatic dissymmetrical cylinder controlled by two three ways servo-distributors: a classical pole placement method and a with scheduling gains control law. Using physical laws, a nonlinear model of this process is described in the first part of this paper and the difficulty for the obtention of the servo-distributors mass flow rates is carried out. Nevertheless, a particular choice of the control input leads to a single input one and around an equilibrium point, a tangent linearized model is obtained. Two control laws are described and implemented using a well known dSPACE interface card. Experimental results obtained for point to point control are presented and discussed.

1 Introduction

During the last decade, many works have been related in the literature and especially in Fluidpower Workshops to the control of electropneumatic actuators. Both linear and nonlinear control have been presented [McCloy 80] [Shearer 56] [Moore 86] [Richard 96] [Scavarda 93] . Nevertheless, the main problem in the electropneumatic field remains the obtention of the servo-distributor mass flow rate because this term is a nonlinear function of the pressure in a cylinder chamber and the control input, and it is very difficult to model the mass flow. Then, different authors use several approximations of this mass flow rate and comparisons between the results presented and the related control laws are very difficult.

For a sake of clarity, our research team has built a kind of "industrial benchmark" and this article proposes the first results obtained on this experimental device. In the first part of this paper and after a presentation of the electropneumatic system, we recall the obtention of the actuator nonlinear model. Classical assumptions lead to a fourth order model : the two chambers pressures, the velocity and the position of the actuator are the state variables. After a study of the equilibrium set, a tangent linearized model is obtained. A model reduction leads to a third order one : the acceleration, the velocity and the position of the piston are the new state variables and the dynamic behavior of this linear model is parametrized by the equilibrium point.

With the performance of dSPACE environment, we can use a small sampling period (4 milliseconds) which is very smallest then the natural frequency of this electropneumatic system. So it is not necessary to discretize our model, we work in continuous time and we will do not talk about state affine control [Scavarda 92]. Two continuous control laws are then presented. The first uses a pole placement method with the linearized model around the central position. This method leads to a fixed gain state feedback. However, as previously mentioned, the dynamic behaviour of the cylinder varies from the central position to the end-stroke position and a tangent linearized model is known at each point of the equilibrium set. Then, multimodel control may be used in order to obtain a desired (constant or not) dynamic behavior independently of the equilibrium position. Because this kind of control leads to a switching from one control to another, a supplementary improvement is obtained using sceduling gains controller. The mathematical framework and the proposed control law are described in section 3. This technique leads to a unique nonlinear control valid for all operating points. Finally the last section presents the different results obtained with the two control laws and a comparative study is proposed.

2 The electropneumatic servodrive

The system under consideration (figure 1) is a linear electropneumatic servodrive controlled by two three-way servo-distributors.

fig. 1- The electropneumatic system

Notations

y, v, γ	position, speed, acceleration,	S_P, S_N	piston area,
p_X	pressure in the chamber X,	y^c	central position : $V_P(y^c) = V_N(y^c)$
p_S, p_E	supply and exhaust pressure,	T_X, T	chamber x and ambient temperatures,
V_X	volume of the chamber x,	u_P, u_N	servo-distributor voltage,
k	polytropic constant,	M	total load,
r	perfect gas constant,	f_v	viscous friction coefficient,
F_{ext}	external force,	F_f	dry friction force

$q_{mP}(u_P, p_S, p_E, p_P)$, $q_{mN}(u_N, p_S, p_E, p_N)$ are the mass flow rates provided from the servo-distributors to the cylinder chambers

2.1 Nonlinear model

The electropneumatic system model can be obtained using three physical laws: the mass flow rate through a restriction, the pressure behaviour in a chamber with variable volume and the mechanical equation.

In our case, the bandwidth of the Servotronic Joucomatic servo-distributors and the actuator are respectively about 170Hz and 2,4Hz. Using the singular perturbation theory, Bouhal [Bouhal 94] has shown that the faster dynamic can be neglected. Then, the servo-distributor model can be reduced to a static one described by two relationships $q_{mP}(u_P, p_S, p_E, p_P)$ and $q_{mN}(u_N, p_S, p_E, p_N)$ between the mass flow rates q_{mP} and q_{mN} , the input voltages u_P and u_N , the output pressures p_P and p_N , the supply and exhaust pressures p_S and p_E .

The pressure evolution law in a chamber with variable volume is obtained assuming the following assumptions [Shearer 56, Andersen 67]:

- air is a perfect gas and its kinetic energy is negligible,
- the pressure and the temperature are homogeneous in each chamber,
- the process is polytropic (k coefficient).

For the actuator, the dynamical evolutions of the pressures in the two chambers; pressure p_X , volume V_X and temperature T_X with X equal 'P' or 'N' are given by equation 2.1. For linear cylinder we can neglect the leakage between the two chambers which leads to the consequence that q_{mout} is null.

$$\frac{dp_X}{dt} = \frac{krT_X}{V_X(y)} \left(\sum q_{min} - \sum q_{mout} - \frac{p_X}{rT_X} \frac{dV_X}{dt} \right) \quad (2.1)$$

$$\text{The last assumption (polytropic process) leads to an algebraic equation: } T_X^k P_X^{1-k} = cte \quad (2.2)$$

The application of the main principle of classic mechanic gives the following expression:

$$M \frac{d^2 y}{dt^2} = M\gamma = S_P p_P - S_N p_N - f_v v - F_f - F_{ext} \text{ with in our case } F_{ext} = (S_P - S_N) p_E \quad (2.3)$$

The electropneumatic system model is obtained by combining all the previous relations and assuming that the temperature variation is negligible with respect to the average one and equal to the supply temperature. Then: $T_P = T_N = T_S$

$$\left\{ \begin{array}{l} \frac{dp_P}{dt} = \frac{krT_s}{V_P(y)} \left[q_{mP}(u_P, p_S, p_E, p_P) - \frac{S_P}{rT_s} p_P v \right] \\ \frac{dp_N}{dt} = \frac{krT_s}{V_N(y)} \left[q_{mN}(u_N, p_S, p_E, p_N) + \frac{S_N}{rT_s} p_N v \right] \\ \frac{dv}{dt} = \frac{I}{M} [S_P p_P - S_N p_N - f_v v - F_f - F_{ext}] \\ \frac{dy}{dt} = v \end{array} \right. \quad (2.4)$$

where: $\left\{ \begin{array}{l} V_P(y) = V_P(\theta) + S_P y \\ V_N(y) = V_N(\theta) - S_N y \end{array} \right.$ with $\left\{ \begin{array}{l} V_P(0) = V_{DP} + S_P \frac{Stroke}{2} \\ V_N(0) = V_{DN} + S_N \frac{Stroke}{2} \end{array} \right.$ are the piping volumes of the

chambers for the zero position and V_{DX} are dead volumes present on each extremity of the cylinder.

Expression of the mass flow-rates

The main difficulty in the model (2.4) is the knowledge of the mass flow rates q_{mP} and q_{mN} . Servo-distributor manufacturers usually provide the mass flow rate gain characteristic $q_m(u)$, the mass flow rate law characteristic $q_m(p)$ for an output pressure equal to one bar and the pressure gain characteristic for a null flow rate. Indeed, this third curve is very important in the static phase near the equilibrium point but it is not sufficient for detailed analysis. In order to obtain a more accurate characterization, two methods are generally used : the first one is based on a local characterization of the servo-distributor orifice openings, the second proposes a global characterization.

The local characterization uses a Wheatstone bridge representation of the servo-distributor openings and an experimental or normalized value of the mass flow rate through the different restrictions [Det 89, Richard 90, Zumbragel 90, ISO 6358, ...]. Nevertheless, these methods are valid if the servo-distributor leakage mass flow rate appears only on the variable restrictions. This is not generally true (and particularly on the Servotronic Joucomatic servo-distributor) because the spool sleeve building generally induces another orifices. In order to include the whole of this losses, Sesmat [Sesmat 96] performs a direct measurement of the mass flow rates $q_{mP}(u_P, p_S, p_E, p_P)$ or $q_{mN}(u_N, p_S, p_E, p_N)$. Figure 2 shows the evolutions of the output servo-distributor mass flow rate as a function of the input voltage and the output pressure.

fig. 2- Servo-distributor mass flow rate surface vs servo-distributor input voltage and output pressure

In order to obtain an analytical expression of the mass flow-rates and consequently an analytical nonlinear model, these above bidimensional characteristic may be approximated by one or several functions of p and u . This non trivial way is under investigation [Belgharbi 99]. However using simple geometric constructions and if only one control input $u = u_P = -u_N$ is considered, we can obtain a linearized tangent model with the following method.

2.2 Equilibrium set and tangent linearized model

The equilibrium set is defined by $\dot{x} = f(x^e, u^e) = 0$ so we obtain with equation (2.4)

$$y = y^e, v = v^e = 0, S_P p_P^e - S_N p_N^e - F_f - F_{ext} = 0, q_{mP}(u^e, p_S, p_E, p_P^e) = 0, q_{mN}(-u^e, p_S, p_E, p_N^e) = 0$$

The last two equations $q_{mP}(\cdot) = 0$ and $q_{mN}(\cdot) = 0$ define two relationships between p_P^e, p_N^e and u^e .

Figure 3 shows the experimental pressure gain characteristic and the pressure force gain characteristic $(S_P p_P - S_N p_N)(u)$ deduced from it, both for a null mass flow rate. These curves are obtained from the above bidimensional characteristics. They are monotonous and strictly increasing then for any value of u there exists only one equilibrium point defined by $\left\{ y = y^e, v = v^e = 0, p_P^e, p_N^e, u^e \right\}$. Consequently, the dimension of the equilibrium set is the same as the number of input of the system, which proves that the tangent linearized model is controllable.

fig. 3- Pressure force gain and Pressure gain at null mass flow rate

With variation near equilibrium set

$$\left\{ \begin{aligned} \delta p_P &= p_P - p_P^e, \delta p_N = p_N - p_N^e, \delta v = v - v^e, \delta y = y - y^e, \delta u = u - u^e \end{aligned} \right\}$$

The tangent linearized model is obtain of the form :

$$\frac{d}{dt} \begin{bmatrix} \delta p_P \\ \delta p_N \\ \delta v \\ \delta y \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_P^e} & 0 & -\frac{kp_P^e S_P}{V_P(y^e)} & 0 \\ 0 & -\frac{1}{\tau_N^e} & \frac{kp_N^e S_N}{V_N(y^e)} & 0 \\ \frac{S_P}{M} & -\frac{S_N}{M} & -\frac{f_v}{M} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta p_P \\ \delta p_N \\ \delta v \\ \delta y \end{bmatrix} + \begin{bmatrix} \frac{krT_S}{V_P(y^e)} G_{uP}^e \\ -\frac{krT_S}{V_N(y^e)} G_{uN}^e \\ 0 \\ 0 \end{bmatrix} \delta u \quad (2.5)$$

With time constants τ_P^e and τ_N^e : $\tau_P^e = \frac{V_P(y^e)}{krT_S C_{pPP}^e}$ et $\tau_N^e = \frac{V_N(y^e)}{krT_S C_{pNN}^e}$ and

$$\left\{ \begin{aligned} C_{pPP}^e &= -\frac{\partial q_{mP}(u^e, p_P^e)}{\partial p_P} \Big|_e, G_{uP}^e = \frac{\partial q_{mP}(u^e, p_P^e)}{\partial u} \Big|_e, C_{pNN}^e = -\frac{\partial q_{mN}(-u^e, p_N^e)}{\partial p_N} \Big|_e, G_{uN}^e = \frac{\partial q_{mN}(-u^e, p_N^e)}{\partial u} \Big|_e \end{aligned} \right\}$$

2.3 Analysis of the reduced model

In the pneumatic field, the conventional position control law is composed of position, velocity and acceleration feedbacks. Using acceleration feedback instead of pressure or differential pressure can be justify by the fact that acceleration is quickly influenced by an external perturbation force. Nevertheless this choice need to obtain a good information on the acceleration without noise. To obtain a

model of third order, with position, velocity and acceleration state variables, [Kellal 86] has proposed to replace each time constant of each chamber by an average time constant τ_m^e (geometric mean). So we obtain :

$$\frac{d}{dt} \begin{bmatrix} y \\ v \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -w_{ol}^2 & -2z_{ol}w_{ol} \end{bmatrix} \begin{bmatrix} y \\ v \\ \gamma \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} u \quad (2.6)$$

$$\text{with : } b = \frac{krTS}{M} \left[\frac{S_P G_{uP}^e}{V_P(y^e)} + \frac{S_N G_{uN}^e}{V_N(y^e)} \right] \text{ and } z_{ol} = \frac{I}{2w_{ol}} \left(\frac{1}{\tau_m^e} + \frac{f_v}{M} \right) \quad (2.7)$$

$$w_{ol} = \sqrt{w_{cyl}^2 + \frac{f_v}{\tau_m^e M}} \text{ where } w_{cyl} = \sqrt{\frac{k}{M} \left(\frac{S_P^2 p_P^e}{V_P(y^e)} + \frac{S_N^2 p_N^e}{V_N(y^e)} \right)}$$

All coefficients of this model are dependent of the piston position and we can see on figure 4 that the pulsation is minimum for the central position

fig. 4- Evolution of the electropneumatic system model parameters with the piston equilibrium position.

3. Control

3.1 Presentation of the partial state feedback

The state feedback coefficients are obtained by choosing a desired behaviour for the closed loop system. The most common choice [Shearer 56, Burrows 72] consists of fixing a third order characteristic polynomial in closed loop, which is composed of dominant second order and a first order :

$$\left(\lambda + \frac{I}{\tau_{cl}} \right) \left(\lambda^2 + 2w_{cl}z_{cl}\lambda + w_{cl}^2 \right)$$

τ_{cl} : the pulsation corresponding to the first order ($1/\tau_{cl}$) is equal to six times the closed loop pulsation.

z_{cl} : the damping coefficient is equal to one

ω_{cl} : the closed loop pulsation is proportional to the open loop pulsation

Then we obtain the control law $u = u^e + K_y(y_d - y) - K_v v - K_\gamma \gamma$, y_d is the desired position. Simple

calculus lead to $K_y = \frac{6w_{cl}^3}{b}$, $K_v = \frac{(13w_{cl}^2 - w_{ol}^2)}{b}$, $K_\gamma = \frac{(8w_{cl} - 2z_{ol}w_{ol})}{b}$

3.2 Fixed gains

So the gains of control law depend on the piston position, the tangent linearized model and consequently the control law are calculated in the central position [Shearer 56], which corresponds to the smallest value of the cylinder natural pulsation, with Routh-Hurwitz criterion it is easy to show that if the system is stable on the center position it will be also stable on the extremity. Consequently the behaviour of the system will be different in extremity. However, for electropneumatic system this is not really true as shown on figure 5 with a pole representation.

fig. 5- Pole evolution in closed loop for different equilibrium position

It shows the evolution of the poles for a control law calculated in the middle and applied on different position (in this case we impose $w_{cl} = w_{ol}$). In all the stroke the imaginary part of the pole is negligible in comparison with real part that leads to a step reponse without oscillation.

We can explain that the dynamic behaviour is nearly the same by the fact that for all position of the piston the two complex poles are like one double real pole and impose the dynamic because the third pole is in ratio of six or more.

3.3 Scheduling gains

K_y, K_v and K_γ are dependent of desired position, the appropriate control law is :

$u(y_d) = u^e + K_y(y_d)(y_d - y) - K_v(y_d)v - K_\gamma(y_d)\gamma$. For some position on the stroke we can calculate the gains, and deduce the expression of the gains function of the desired position, with interpolation and

approximation in sense of least square. We obtain for each gain a fourth order polynomial. With this control law we take advantage of the characteristic of electropneumatic systems and we reach best performances in closed loop, at the cylinder ends since the system dynamic is naturally higher.

4 Experimental results

With the intention of doing comparison between experimental results of different control laws we have created a kind of "industrial benchmark". Table 1 summarizes the results obtained for fixed and variable gains in the central position and on the two extreme position. In each case the magnitude of the movement is 10% of the total stroke. So we can consider that the tangent linearized model is true in all desired positions and it will of course be interesting to compare results for different magnitude of the piston. To limit the influence of stochastic perturbation like dry friction variation, supply pressure evolution, noise measurement... all tests presented in this table have been done twenty times in same conditions. It should be noted that we work in point to point desired position, which means more important static error and greater standard deviation (Std) than for tracking trajectory, because we have only final desired position for control and not desired position, velocity and acceleration during all the movement. In term of repetability, we notice that the standard deviation is independant of the position, we obtain a value around 0.1 mm for fixed gains control law and about 30 % less for variable gains.

Control law :	Fixed Gains						Variable Gains					
Desired Position [mm]	-200 to -150		-25 to +25		150 to 200		-200 to -150		-25 to +25		150 to 200	
Direction of piston	-	+	-	+	-	+	-	+	-	+	-	+
Std [mm]	0.114	0.121	0.120	0.110	0.006	0.116	0.027	0.021	0.089	0.105	0.035	0.119
Mean ε * [mm]	0.41	0.21	0.27	0.01	0.40	0.20	0.35	0.13	0.61	0.20	0.42	0.29
Max error [mm]	0.61	0.59	0.32	0.30	0.50	0.23	0.38	0.17	0.74	0.56	0.46	0.39
Mean pos [mm]	-199.59	-149.79	-24.73	24.99	150.40	199.80	-199.65	-149.87	-24.39	25.20	150.42	199.71
τ_r ** [ms]	253	263	251	259	259	272	141	220	262	248	243	165
v_{max} [mm/s]	252	247	244	264	222	253	427	313	244	264	251	393
γ_{max} [m/s ²]	4.56	4.70	4.35	4.59	3.48	4.59	6.39	4.59	4.05	4.59	4.75	4.59

* ε = static error $|y_d - y|$ in millimeter ** τ_r = response time between 10 and 90 % of the movement in second

table 1 : Experimental results with fixed and variable gains control laws.

As explained in section three, the results obtained with variable gains control law are more satisfactory than with fixed gains, particularly in term of velocity of displacement, which have for consequence a satisfactory improvement. Table 2 and figure 6 bring to the fore which we present in section three, we

note 40 % improvement on the extremity in term of response time calculated between 10 and 90 % of the movement.

Desired Position [mm]	-200 to -150		150 to 200	
Direction	-	+	-	+
Std	- 43 %			
Static error [mm]	- 0.08	- 0.36	- 0.11	+ 0.22
τ_r	-43 %	- 17 %	- 16 %	- 39%
Max velocity	+ 69 %	+ 27 %	+ 13%	+ 55%

table 2 : Improvement of the variables gains control

law compared to fixed gains

fig. 6- Experimental results

5. Conclusion

This paper shows and explains why the fixed gains control laws calculated in central position leads approximatively to the same behaviour in all the stroke (without overshoot or oscillation). It also recall that the conventional control law used by a lot of manufacturers in pneumatic positioning system can be improved taking advantage of the natural characteristics of this system. If we use variable gains control law for which the cost in term of sensor is the same as for fixed gains and the implementation phase contains just some additional multipliers. It also note and explain the very interesting improvement with variable gains control law in term of dynamic and static behaviours.

References

- [McCloy 80] : Mc Cloy, D., Martin, H.R. *Control of fluid power : analysis and design*. Chichester (England), Ellis Horwood, 1980. 510 p.
- [Shearer 56] : Shearer, J.L. *Study of pneumatic processes in the continuous control of motion with compressed air*. Parts I and II. Trans. Am. Soc. Mech. Eng. 1956, Vol. 78, p.233-249.
- [Richard 96] : Richard, E., Scavarda, S., *Comparison between linear and nonlinear control of an electropneumatic servodrive*. Journal of dynamic system, Measurement, and Control, June 1996, Vol 118, p 245-252.
- [Moore 86] : Moore, P.R., Ssenkungu, F.W., Weston, R.H., Thatcher, T.W. and Harrison, R. *Control Strategies for pneumatic servo-drives*. Int.J. Prod. Res., 1986, Vol 24, N°6, p 1363-1382.
- [Scavarda 93] : Scavarda, S. *Movement control of electropneumatic positioning systems*. Congresso Internazionale della Trasmissione di Potenza, Meccanica, Oleodinamica, Pneumatica, Assago, Italia, 20-21 Juin 1995, p 117-131.
- [Scavarda 92] : Scavarda, S., Thomasset, D., Richard, E., Bourdat, S. *State-affine position control of an electropneumatic cylinder*. Winter Annual Meeting of American Society of Mechanical Engineers, Anaheim, Californie (USA), November 8-13, 1992, n° 92-WA/FPST-12, 6p.
- [Bouhal 94] : Bouhal, A., *Contribution à la commande linéaire et non linéaire adaptative des systèmes électropneumatiques*. Thèse: Sci.: Institut National des Sciences Appliquées de Lyon, 1994. 203p.

- [Andersen 67] : Andersen, B.W., *The analysis and design of pneumatic systems*. New-York : John Wiley and Sons, 1967. 302p.
- [Det 89] : Det, F., Scavarda, S., Richard, E., *Simulated and experimental study of charging and discharging of a cylinder using an electropneumatic servovalve*. In : Prepr. First Japan Hydraulics and Pneumatics Society Int. Symp. on Fluid Power, Tokyo, March 13-16, 1989, p.199-206.
- [Zumbrägel 90] : Zumbrägel, J., *Flexible servopneumatische Greifersysteme für die Handhabungstechnik*. Dissertation RWTH, Aachen, 1990. 143 p.
- [ISO 6358] : Norme ISO 6358 *Transmissionspneumatiques - Eléments traversés par un fluide compressible - Détermination des caractéristiques de débit*. 1989, 15 p.
- [Richard 90] : Richard, E., Scavarda, S., *Non linear control of a pneumatic servodrive*. Proc. Second Bath Int. Fluid Power Workshop - 21-22 Septembre 1989 Bath (U.K.) - and in : Fluid Power Components and Systems Second Bath International Fluid Power Workshop Ed. by C.R. BURROWS, K.A. EDGE. 1990 - RESEARCH STUDIES PRESS LTD Tounton, Somerset (England). p.59-75.
- [Sesmat 96] : Sesmat, S., Scavarda, S. *Static characteristics of a three way servovalve*. In: 12th Aachen Conference on Fluid Power Technology, Aachen, Allemagne, March 12-13, 1996
- [Belgharbi 99] : to appear in the sixth Scandinavian International Conference on Fluid Power, Tampere, Finland, May 26-28, 1999.
- [Kellal 86] : Kellal, A., Scavarda, S., Fontaine, J.G. *Electropneumatic servodrive for a robot*. Proceedings of the 16th ISIR - International Symposium on Industrial Robots, Brussels, Belgium, 29 Sept. - 2 Oct, 1986, p. 117-128
- [Burrows 72] : Burrows, C.R. *Fluid Power Servomechanisms*. London: Van Nostrand Reinhold Company, 1972. 237 p.