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Perspective-12-Quadric: An analytical solution to the camera pose estimation problem from conic-quadric correspondences.

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**Goal:** To estimate the pose of a perspective camera given a set of \( n \) 3D quadrics in the world and their corresponding 2D projections (conics) in the image.

1 Problem statement.

- **2D:** Homogeneous quadratic form of a conic equation:
  \[ u^\top C_i u = 0, \]
  where \( u \in \mathbb{R}^3 \) is the homogeneous vector of a generic 2D point belonging to the conic defined by the symmetric matrix \( C_i \in \mathbb{R}^{3 \times 3} \). The conic has 5 degrees of freedom given by the 6 elements of the lower triangular part of the symmetric matrix \( C_i \), except one for the scale. The dual conic is defined by \( C_i^* = \text{adj}(C_i) \).

- **3D:** Homogeneous quadratic form of a quadric equation:
  \[ x^\top Q_i x = 0, \]
  where \( x \in \mathbb{R}^4 \) is the homogeneous vector of a generic 3D point belonging to the quadric defined by the symmetric matrix \( Q_i \in \mathbb{R}^{4 \times 4} \). The quadric has 9 degrees of freedom given by the 10 elements of the lower triangular part of the symmetric matrix \( Q_i \), except one for the scale. The dual quadric is defined by \( Q_i^* = \text{adj}(Q_i) \).

- **Projection equation:** Let's consider the projection matrix

\[
P = K[R|t] = \begin{bmatrix}
p_{11} & p_{12} & p_{13} & p_{14} 
p_{21} & p_{22} & p_{23} & p_{24} 
p_{31} & p_{32} & p_{33} & p_{34}
\end{bmatrix}
\]

The projection equation [1] is:

\[
\beta_i C_i^* = PQ_i^* P^\top
\] (1)
2 Solution.

In order to recover \( P \) in closed form from the set of conic-quadric pairs, we have to re-arrange Eq. (1) into a linear system \([2, 3]\). Let us define the operator \( \text{vec}() \) that serialises all the elements of a generic matrix, and the operator \( \text{vech}() \) that serialises the elements of the lower triangular part of a symmetric matrix. Then let us define \( v^* \) = \( \text{vech}(Q^*) \) and \( c^* = \text{vech}(C^*) \). Then let us arrange the product of the elements of \( P \) and \( P^\top \) in a single matrix \( G \in \mathbb{R}^{6 \times 10} \) as follows:

\[
G = D(P \otimes P)E
\]

where \( \otimes \) is the Kronecker product, and matrices \( D \in \mathbb{R}^{6 \times 9} \) and \( E \in \mathbb{R}^{16 \times 10} \), are two matrices such that \( \text{vech}(X) = D \text{vec}(X) \) and \( \text{vec}(Y) = E \text{vech}(Y) \) respectively, where \( X \in \mathbb{R}^{9 \times 9} \) and \( Y \in \mathbb{R}^{16 \times 16} \) are two symmetric matrices.

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}, \quad E = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Given \( G \) we can rewrite Eq. (1) as a linear equation:

\[
\beta_i c^*_i = Gv^*_i \quad (2)
\]

Eq. (2) can thus be solved using DLT algorithm, e.g. can be written:

\[
0 = Bg
\]
where $g = \text{vec}(G) = [g_{11} \cdots g_{6,10}]^\top \in \mathbb{R}^{60}$, and $B \in \mathbb{R}^{15n \times 60}$ is the concatenation of matrices $b_i \in \mathbb{R}^{15 \times 60}$:

$$b_i = \begin{bmatrix}
c_{i1}v_{1i}^\top & -c_{i1}v_{1i}^\top & 0_{10}^\top & 0_{10}^\top & 0_{10}^\top & 0_{10}^\top \\
c_{i2}v_{1i}^\top & c_{i2}v_{1i}^\top & -c_{i2}v_{1i}^\top & 0_{10}^\top & 0_{10}^\top & 0_{10}^\top \\
c_{i3}v_{1i}^\top & 0_{10}^\top & c_{i3}v_{1i}^\top & -c_{i3}v_{1i}^\top & 0_{10}^\top & 0_{10}^\top \\
c_{i4}v_{1i}^\top & 0_{10}^\top & 0_{10}^\top & c_{i4}v_{1i}^\top & -c_{i4}v_{1i}^\top & 0_{10}^\top \\
c_{i5}v_{1i}^\top & 0_{10}^\top & 0_{10}^\top & 0_{10}^\top & c_{i5}v_{1i}^\top & -c_{i5}v_{1i}^\top \\
c_{i6}v_{1i}^\top & 0_{10}^\top & 0_{10}^\top & 0_{10}^\top & 0_{10}^\top & c_{i6}v_{1i}^\top \\
\end{bmatrix}.$$ 

with:

$$c_i^\top = \begin{bmatrix} c_{i1} & c_{i2} & c_{i3} & c_{i4} & c_{i5} & c_{i6} \end{bmatrix},$$

$$v_{1i}^\top = \begin{bmatrix} v_{1i} & v_{2i} & v_{3i} & v_{4i} & v_{5i} & v_{6i} & v_{7i} & v_{8i} & v_{9i} & v_{10i} \end{bmatrix},$$

$$0_{10}^\top = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Each pair of conic-quadric provides 15 homogeneous linear equations in the unknown elements of $G$. However, only 5 are linearly independent. Thus 12 pairs of 3D-2D correspondences are required to solve the system. A total least squares solution can then be found by choosing $g$ as a right singular vector corresponding to the smallest singular value of $B$. Finally, $P$ can be inferred from $G$.

**References**

