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Perspective-12-Quadric: An analytical solution to the camera pose estimation problem from conic-quadric correspondences.

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Goal: To estimate the pose of a perspective camera given a set of $n$ 3D quadrics in the world and their corresponding 2D projections (conics) in the image.

1 Problem statement.

- **2D**: Homogeneous quadratic form of a conic equation:
  
  \[ u^\top C_i u = 0, \]

  where $u \in \mathbb{R}^3$ is the homogeneous vector of a generic 2D point belonging to the conic defined by the symmetric matrix $C_i \in \mathbb{R}^{3 \times 3}$. The conic has 5 degrees of freedom given by the 6 elements of the lower triangular part of the symmetric matrix $C_i$, except one for the scale. The dual conic is defined by $C_i^* = \text{adj}(C_i)$.

- **3D**: Homogeneous quadratic form of a quadric equation:
  
  \[ x^\top Q_i x = 0, \]

  where $x \in \mathbb{R}^4$ is the homogeneous vector of a generic 3D point belonging to the quadric defined by the symmetric matrix $Q_i \in \mathbb{R}^{4 \times 4}$. The quadric has 9 degrees of freedom given by the 10 elements of the lower triangular part of the symmetric matrix $Q_i$, except one for the scale. The dual quadric is defined by $Q_i^* = \text{adj}(Q_i)$.

- **Projection equation**: Let’s consider the projection matrix

  \[ P = K[R|t] = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \]

  The projection equation [1] is:

  \[ \beta_i C_i^* = PQ_i^* P^\top \]  \hfill (1)
2 Solution.

In order to recover P in closed form from the set of conic-quadratic pairs, we have to re-arrange Eq. (1) into a linear system \([2, 3]\). Let us define the operator \(vec()\) that serialises all the elements of a generic matrix, and the operator \(vech()\) that serialises the elements of the lower triangular part of a symmetric matrix. Then let us define \(v^*_i = vech(Q^*_i)\) and \(c^*_i = vech(C^*_i)\). Then let us arrange the product of the elements of \(P\) and \(P^T\) in a single matrix \(G \in \mathbb{R}^{6 \times 10}\) as follows:

\[
G = D(P \otimes P)E
\]

where \(\otimes\) is the Kronecker product, and matrices \(D \in \mathbb{R}^{6 \times 9}\) and \(E \in \mathbb{R}^{16 \times 10}\), are two matrices such that \(vech(X) = Dvec(X)\) and \(vech(Y) = Evech(Y)\) respectively, where \(X \in \mathbb{R}^{9 \times 9}\) and \(Y \in \mathbb{R}^{16 \times 16}\) are two symmetric matrices.

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix},
E = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Given \(G\) we can rewrite Eq. (1) as a linear equation:

\[
\beta_i c^*_i = Gu^*_i \tag{2}
\]

Eq. (2) can thus be solved using DLT algorithm, e.g. can be written:

\[
0 = Bg.
\]
with:

\[ c_i^T = \begin{bmatrix} c_{i1} & c_{i2} & c_{i3} & c_{i4} & c_{i5} & c_{i6} \end{bmatrix}, \]

\[ v_i^T = \begin{bmatrix} v_{i1} & v_{i2} & v_{i3} & v_{i4} & v_{i5} & v_{i6} & v_{i7} & v_{i8} & v_{i9} & v_{i10} \end{bmatrix}, \]

\[ 0_{10}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

Each pair of conic-quadratic provides 15 homogeneous linear equations in the unknown elements of \( G \). However, only 5 are linearly independent. Thus 12 pairs of 3D-2D correspondences are required to solve the system. A total least squares solution can then be found by choosing \( g \) as a right singular vector corresponding to the smallest singular value of \( B \). Finally, \( P \) can be inferred from \( G \).

References

