Quality control in machining using order statistics

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A B S T R A C T

The quality of surface roughness for machined parts is essential in the manufacturing process. The cutting tool plays an important role in the roughness of the machined parts. The process of determining the number of tolerant faults is problematic; this is due to the fact that the behaviour of the cutting tool is random. In this paper, we use an approach based on order statistics to study the construction of functional and reliability characteristic for the faults tolerant machined parts in each five batch of ten machined parts. Our experiments show that the number of faulty machined parts will not exceed two and the distribution of the minimum gives the best interval of the surface roughness. We have shown that the distribution of extreme order statistics plays an important role in determining the lower and upper limits of the roughness measurements depending on the reliability of the cutting tool.

1. Introduction

In the manufacturing industry, the surface finish obtained on the final machined part is not always homogeneous. We will primarily focus on criteria of surface quality which are linked with surface roughness (see [1] for details). In machining processes, the surface roughness of machined parts depends on several factors. Some are systematic such as the geometrical conditions of the cutting tool associated with the cinematic conditions of the generation process. Other factors are associated with tool wear and random machining faults (see [7,15] for details). The evolution of the cutting tool behaviour is also random. A fault occurring in the process can lead to the lack of the machined parts conformity and can cause disastrous economic consequences. Such a situation must be identified as quickly as possible to repair the malfunction and thus allowing a normal operation of the production. We will consider, as is often the case in the manufacturing industry, a batch process where a given number of machined parts with a slight overshoot of the tolerance (faulty machined parts) are tolerated. The magnitude of the fault must be taken into account as well as the number of concerned machined parts. Due to this framework, an approach based on the use of the order statistics is particularly suitable to consider a contribution to a more efficient machining.

Order statistics describing random variables in order of magnitude are widely used in statistical methods and inference. They also play a key role in a wide variety of practical situations such as reliability, life testing, lifetime distributions, durability study, data compression, software reliability (see [2,11] for details), etc. These domains of statistics are concerned with the rank as well as the intensity of observations. It combines the techniques of conventional statistics which consider the intensity of the observation. The rank order statistics only consider the relative rank whatever the original observations were measured on an ordinary scale or not. Order statistics have many applications, such as modelling health data in [13], modelling the heights of sea waves (see [9] for details), in life testing of electrical components modelling (see [12] for details), or in the mechanic field (see [5] for details). In this paper, we study the construction of the functional process for the fault tolerant machined part detection based on order statistics.

Furthermore order statistics are very important in practice and especially the minimum and the maximum values because they are the critical values used in engineering, physics, medicine, etc. (see [2,5] for details). The minimum and maximum values in order statistics are called the extremes; they play an important part in the reliability and industry framework. For a more detailed presentation of extreme order statistics (see [3,6,11] for details). The Generalized Extreme Value (GEV) theory is a family of continuous probability distribution which deals with extreme events such as natural catastrophes, earthquakes, heart attacks, flying accidents. We distinguish three families of extreme value distributions namely Gumbel, Fréchet and Weibull; these distributions depend on the shape parameter (see [14] for more details). Moreover, the distributions can be obtained as limited distributions of properly normalized maxima of n independent and identically

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distributed (iid) random variables. Extreme values analysis finds wide applications in many areas including engineering for health engine managing. Commonly, the distribution which is widely used in a reliability model is viewed as a parameterization of Weibull distribution.

Applications of order statistics in surface roughness modelling is scarce in literature; we here present a case study in manufacturing areas. This study uses data and some outcomes of surface roughness modelling in [7]. The quality control of the machined parts in a manufacturing process is crucial. This control takes place in several process steps and especially at the end of this process. These several points could be checked like dimensional control, surface quality, resistance to mechanical stresses like tensile strength or fatigue behaviour (see [4] for details).

After the introduction, Section 2 exposes the extreme value theory, order statistics and some applications in literature. Section 3 draws the case study framework as well as all associated developments. Finally, Section 4 gives a conclusion.

2. Literature

This section provides information about basic concepts of the extreme value theory and order statistics which can be useful for the work and some examples of order statistics in literature are given.

2.1. Extreme value theory

Suppose $Y_1,\ldots,Y_n$ is a sequence of independent and identically distributed (iid) random variables and let.

$M_n = max(Y_1,\ldots,Y_n)$. If the distribution of Y_i is specified, then the exact distribution of M_n is known. On the other hand, in the absence of such specifications, the extreme value theory considers the existence of $\lim_{n\to+\infty} P[(M_n - b_n)a_n^{-1} \leqslant y] = F(y)$ for some sequence of real numbers $\{a_n > 0\}$, and $\{b_n \in \quad\}$. If the cumulative distribution $F(y)$ is a non-degenerate one distribution, then we obtain the following expression:

$$F(y) = G_{(\mu,\sigma,\xi)}(y) = \begin{cases} exp\left[-\left(1+\xi\frac{y-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right] & if\ \xi \neq 0 \\ exp\left[-exp\left(-\frac{y-\mu}{\sigma}\right)\right] & if\ \xi = 0 \end{cases}, \tag{1}$$

where $\mu \in \quad$ is the location parameter, $\sigma \in \quad$ is the scale parameter, $\xi \in \mathbb{R}$ is the shape parameter and $y_+ = \max(y,0)$. Gumbel, Frechet and Weibull families of distributions are obtained from (1) by considering $\xi = 0, \xi > 0, \xi < 0$ respectively (see [14] for more details).

2.2. Order statistics: basic distribution theory

Let Y be a random variable from a given population, $f(y)$ and $F(y)$ are the associated probability density function (*pdf*) and the cumulative distribution function (*cdf*), respectively. Assume that $(y_1,y_2,\ldots,y_m)$ is a random sample of size m from such a population and rearrange the observations in increasing order $(y_{1:m},y_{2:m},\ldots,y_{m:m})$, with $y_{1:m} \leqslant y_{2:m} \leqslant \ldots \leqslant y_{m:m}$. The r-th order statistic is defined as the member $y_{r:m}$ of the ordered collection of size m [2,6]. Depending on the application one or more order statistics can critically affect functionality, performances, or integrity of a complex system, and the statistical properties of $y_{r:m}$ drive design and maintenance activities [5,11].

Order statistics as a discipline can be used in this respect to determine the individual, joint, conditional probability of one or more order statistics from the parent functions of the population to which the sample belongs. Only the case of (iid) random samples is considered here.

Consider the observations in increasing order $(Y_{1:m},Y_{2:m},\ldots,Y_{m:m})$, such that $Y_{i:m} \leqslant y$ $(i = 1,\ldots,r)$, then the event $(Y_{r:m} \leqslant y)$ has *pdf* and *cdf* probability functions:

$$\begin{cases} f_{y_{r:m}}(y) = rC_m^r F^{r-1}(y).\ [1-F(y)]^{m-r} f(y) \\ F_{Y_{r:m}}(y) = \sum_{r=j}^{m} C_m^r F^r(y).\ [1-F(y)]^{m-r} \end{cases} \tag{2}$$

$F_{Y_{r:m}}(y) + R_{Y_{r:m}}(y) = 1,$

where R is the reliability function and

$$R_{Y_{r:m}}(y) = \sum_{r=1}^{j-1} C_m^r F^r(y).\ [1-F(y)]^{m-r}. \tag{3}$$

We can also write, for the sake of clarity:

$F_{Y_{r:m}}(y) = Pr\{at\ least\ r\ of\ the\ Y_j\ is\ less\ than\ or\ equal\ to\ y\}$ is also known as the unreliability function.

In particular cases of some interest in the applications;$Y_{1:m}$ is the minimum, with a density and cumulative distribution function respectively:

$$\begin{cases} f_{Y_{1:m}}(y) = m(1-F(y))^{m-1} f(y) \\ F_{Y_{1:m}}(y) = 1-(1-F(y))^m \end{cases}, \tag{4}$$

and the maximum, $Y_{m:m}$, with a density and cumulative distribution function respectively:

$$\begin{cases} f_{Y_{m:m}}(y) = mF^{m-1}(y)f(y) \\ F_{Y_{m:m}}(y) = F^m(y) \end{cases}. \tag{5}$$

The underlying idea of these results is that each y_i is a Bernoulli trial with only two possible outcomes: either $Y_i \leqslant y$ (success) or $Y_i > y$ (failure) [5,6].

In lieu of the iid hypothesis, the series of m trials follow a binomial distribution with a success rate, $p = F(y)$. The probability $F_{Y_{r:m}}(y)$, associated with $Y_{r:m} \leqslant y$ is readily computed as the exceedance of at least r trials being successful, whereas the $f_{y_{r:m}}(y)$ follows from derivation with respect to y. Distribution-independent results, such as Eq. (2) constitute the power of the theory of order statistics.

Let's note that $Y_{1:m} = min(y_1,y_2,\ldots,y_m)$ is the smallest order statistics and corresponds to the sample minimum, and $Y_{m:m} = max(y_1,y_2,\ldots,y_m)$ is the largest order statistics and corresponds to the sample maximum. These order statistics are specific cases of extreme value theory and are the most important in the order statistics approach.

For example [9], one uses the Rayleigh distribution and order statistic to model the waves heights, in a given location and the corresponding order statistic *pdf* and *cdf* to assess the survival probability of a rigid breakwater; in [12] one uses order statistic in the life test of bulbs, in [5] one uses extreme order statistics to control the strength in structural engineering; whereas in [16] one uses order statistics in the reliability of software.

3. Case study framework

The aim of this case study is to propose a decision support tool to define quality control criteria in manufacturing processes. The considered work aims at improving the production of machined parts by machining in turning process, in a production line under given cutting conditions. Making pre-series composed of 5 batches of 10 parts will allow to note the surface qualities obtained, these datasets are available in [7]. Both criteria must be highlighted: on the one hand, the best surface roughness measurement obtained and on the other hand, the number of faulty parts, which roughness measurements are greater than a maximum roughness allowed by the customer specification (upper limit). To achieve this goal, different stages have been carried out:

- Research of the data family distribution in a series of five batches;
- Parent distributions estimate of parameters of (method of moments);
- Expressions of order statistics distributions (*pdf* and *cdf*) in each of the five batches of 10 parts;

- Computation from cdf of order statistics to identify the faulty parts in each batch;
- Minimum and maximum of surface roughness measurement with extreme order statistics;
- Outcomes (number of faulty machined parts in each batch, best values of surface roughness).

The datasets have been provided by the work done in [7], it consists in the study of the surface roughness of machined parts. Surface roughness is given by $R(t)$ and depends on geometrical conditions of the cutting tool associated with the cinematic conditions of the machining process. The parameters of models are measured from 5 series of 10 parts each; on each part, 10 measurements have been achieved. We only consider the dynamic signal of the surface roughness.

This study shows that from 5 series of 10 parts each, the minimum of surface roughness is $1{,}79\ \mu\mathrm{m}$ and the upper limit of the surface roughness is around $4\ \mu\mathrm{m}$,

In the balance of this article, y denotes the value of surface roughness.

3.1. Maximum domain attraction of data distribution

From the generalized extreme value theory and maximum likelihood procedures in software R we obtain Table 1 showing that data follow a distribution in Weibull family with the shape parameter $\xi < 0$ for all five batches. Remember that each batch is composed of 10 machined parts.

For this modelling, we use the two parameters Weibull distribution. Density and the corresponding cumulative distribution are represented by:

$$\begin{cases} f_{k,\lambda}(y) = \frac{k}{\lambda}\left(\frac{y}{\lambda}\right)^{k-1} e^{-\left(\frac{y}{\lambda}\right)^k} \\ F_{k,\lambda}(y) = 1 - e^{-\left(\frac{y}{\lambda}\right)^k} \end{cases} \tag{6}$$

where k is the shape parameter and λ the scale parameter, which will be estimated in the next subsection.

3.2. Estimation of Weibull parameters with the method of moments

The method of moments is a technique commonly used in the field of parameters estimation. If the numbers $y_1, y_2, \ldots, y_m$ represent a set of data, then an unbiased estimator for the $m-th$ origin moment is given by:

$$\widehat{M_m} = \frac{1}{m}\sum_{i=1}^{m} y_i^m,$$

where $\widehat{M_m}$ stands for the estimation of M_m. In a Weibull distribution, the m-moment is readily obtained from Eq. (5). With an expression of the Gamma function $\Gamma(y)$, the average surface roughness can be expressed as a function of k and λ. The found integral cannot be resolved. However it can be reduced to a standard integral, the Gamma function, as follows:

$$\Gamma(s) = \int_0^{\nu} q^{s-1} e^{-q} dq, \tag{7}$$

where $q = \left(\frac{y}{\lambda}\right)^k, \frac{y}{\lambda} = q^{s-1}$ and $s = 1 + \frac{1}{k}$.

The expression of the $m-th$ moment is:

$$\mu_m = \left(\frac{1}{\lambda}\right)^{\frac{m}{k}} \Gamma\left(1 + \frac{m}{k}\right).$$

As we have two parameters to estimate, we can find the first and the second moments as follows:

$$\widehat{\mu_1} = \left(\frac{1}{\lambda}\right)^{\frac{1}{k}} \Gamma\left(1 + \frac{1}{k}\right) = m_1,$$

$$\widehat{\mu_2} = \left(\frac{1}{\lambda}\right)^{\frac{2}{k}} \left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right] = m_2.$$

When we divide m_2 by the square of m_1, we get the following expression:

$$\frac{m_2}{m_1^2} = \frac{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)}{\Gamma^2\left(1 + \frac{1}{k}\right)}. \tag{8}$$

On taking the square roots of Eq. (7), we have the coefficient of variation C_v:

$$C_v = \frac{\sqrt{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)}}{\Gamma\left(1 + \frac{1}{k}\right)} = \frac{\sigma}{\bar{y}},$$

where σ is the standard deviation and $\bar{y}$ the means of surface roughness (see [8,10] for more details).

In this case, the method of moments can be used as an alternative to maximum likelihood. The values of k and λ can be determined by the following equations:

$$\sigma = \lambda\left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right]^{\frac{1}{2}},$$

$$C_v = \sqrt{\frac{\Gamma\left(1 + \frac{2}{k}\right)}{\Gamma^2\left(1 + \frac{1}{k}\right)} - 1}.$$

After some calculations, we have:

$$k \simeq \left(\frac{0.9874}{C_v}\right)^{1.0983}. \tag{9}$$

The Weibull scale parameters can be calculated by:

$$\lambda = \frac{\bar{y}}{\Gamma\left(1 + \frac{1}{k}\right)}.$$

Table 2 presents the summary of computations for the five batches.

For the scale parameter calculation, we must find $\Gamma\left(1 + \frac{1}{k}\right)$; This integral has no closed form; thus, we use software Geogebra to compute $\Gamma\left(1 + \frac{1}{k}\right)$.

For example, for the first batch, using Eq. (6) and Table 2, we get:

$$\begin{cases} f_{3.769,3.526}(y) = \frac{3.769}{3.526}\left(\frac{y}{3.526}\right)^{2.769} e^{-\left(\frac{y}{3.526}\right)^{3.769}} \\ F_{3.769,3.526}(y) = 1 - e^{-\left(\frac{y}{3.526}\right)^{3.769}} \end{cases}. \tag{10}$$

Similarly, we obtain the *pdfs* and *cdfs* of the four remaining batches.

Table 1
GEV shape parameter ξ.

Batches	1	2	3	4	5
Shape parameter ξ	−0.6258	−0.707	−0.6819	−0.7014	−0.675

Table 2
Weibull parameters for five batches.

Batches	1	2	3	4	5
Mean $\bar{y}$	3.138	3.079	3.151	3.11	3.11
Standard deviation σ	0.925	0.923	0.938	0.905	0.898
Coefficient of variation C_v	0.295	0.3	0.298	0.291	0.289
Shape k	3.769	3.7	3.728	3.826	3.855
Scale parameter λ	3.526	3.499	3.54	3.494	3.494

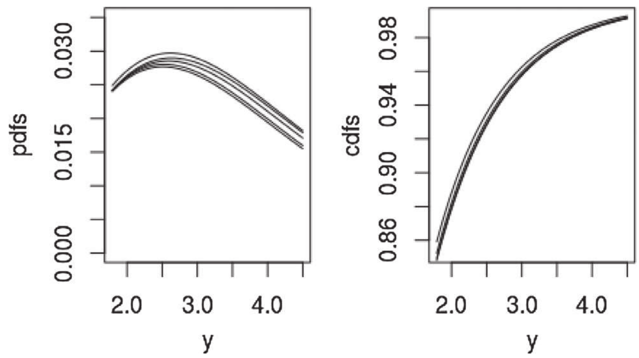

Fig. 1. Pdfs and cdfs of Weibull distribution of the surface roughness.

Results are reported in Fig. 1.

3.3. *Distribution of ordered observations for the five batches*

The 10 observations have been rearranged in size order for each batch.

The *pdfs* and the corresponding *cdfs* of the *rth* order statistic in a sample of size $m = 10$ for the first batch, values from Table 2 are:

$$\begin{cases} f_{Y_{r:10}}(y) = rC_{10}^{r}\left(1-e^{-\left(\frac{y}{3.526}\right)^{3.769}}\right)^{r-1}.\left(e^{-\left(\frac{y}{3.526}\right)^{3.769}}\right)^{10-r} \\ \qquad\qquad \times\left(\frac{3.769}{3.526}\right)\left(\frac{y}{3.526}\right)^{2.769}e^{-\left(\frac{y}{3.526}\right)^{3.769}} \\ F_{Y_{r:10}}(y) = \sum_{r=j}^{10} C_{10}^{r}\left(1-e^{-\left(\frac{y}{3.526}\right)^{3.769}}\right)^{r}.\left(e^{-\left(\frac{y}{3.526}\right)^{3.769}}\right)^{10-r} \end{cases} \tag{11}$$

Similarly, we obtain the *pdfs* and *cdfs* of the *rth* order statistic for all other batches.

From results in [7], we assume that the part is faulty when $y > 4\,\mu\text{m}$. It is shown that the fault origin is generally in a small breakage of the cutting tool. This fault can occur at any stage of the machining process. If it happens at the beginning of the operation, the effect of this breakage can diminish due to a self polishing phenomenon. This situation must not be confused with the classicalwear of the cutting tool where the behaviour evolution of the cutting tool can be modelized. However, we modelized the reliability of the cutting tool which is linked with the roughness measurement. The probability to have one measurement above $4\mu m$ is $F_{Y_{10:10}}(4)$ and the corresponding reliability of the cutting tool is $R_{Y_{10:10}}(4)$; Table 3 presents the value of each batch.

All considered batches show that $F_{Y_{10:10}}(4)$ is large enough to consider the presence of one faulty part. Due to all values of $F_{Y_{10:10}}(4)$; it is also clear that it will be more than one. The reliability of the cutting tool is weak in all batches.

Let us consider $F_{Y_{9:10}}(4)$, which indicates the probability of having 2 parts above $4\,\mu\text{m}$.

Table 4 gives results and according to all values close to 1, the probability of a certain event. We conclude that the number of faulty parts will not exceed 2. Moreover, $F_{Y_{8:10}}(4) \simeq 0.99$, this means that the probability of three measurements above $4\,\mu\text{m}$ is approximately the same for two measurements and confirms that there are no more than two faulty parts. The reliability values of cutting tools are associated to measurements; these values show that in each batch the magnitude of the measurement $4\,\mu\text{m}$ is very important in the behaviour of the cutting tool.

Table 3
Probability of one measurement above $4\,\mu\text{m}$ and reliability of the cutting tool.

Batches	1	2	3	4	5
$F_{Y_{10:10}}(4)$	0.87	0.86	0.86	0.88	0.89
$R_{Y_{10:10}}(4)$	0.13	0.14	0.14	0.12	0.11

Table 4
Probability of two measurements above $4\,\mu\text{m}$ and reliability of the cutting tool.

Batches	1	2	3	4	5
$F_{Y_{9:10}}(4)$	0.99	0.98	0.99	0.99	0.99
$R_{Y_{9:10}}(4)$	0.01	0.02	0.01	0.01	0.01

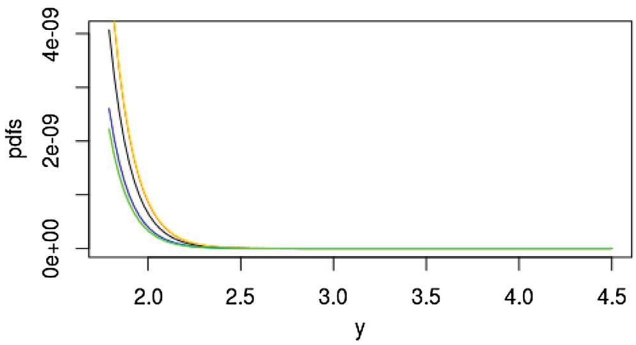

Fig. 2. Distribution of minimum of five experiments.

3.4. *Distributions of minima and maxima of Weibull iid samples of size 10*

The distribution of the first-order statistics, which is the minimum, is also the best value of surface roughness.

The *pdf* of random minima is obtained from Eq. (4) by setting $r = 1$, which leads to the following *pdf*:

$f_{Y_{1:10}}(y) = 10.\, f_{3.769,3.526}(y).\, [1-F_{3.769,3.526}(y)]^{9}.$

$f_{Y_{1:10}}(y)$ is shown in Fig. 2 for all 5 batches. As expected, a high value of $f_{Y_{1:10}}(y)$ is obtained for the small values of the surface roughness. The best values of surface roughness are between 1.79 and 2.25. This interval is now clearly described and the expected quality in line production could be imposed.

Based on the same logic the *pdf* of random maxima can be studied from Eq. (5) by setting $r = m$, the *pdf* for each batch can be written as:

$f_{y_{10:10}}(y) = 10.\, f_{3.769,3.526}(y).\, [F(y)]^{9}$

Fig. 3 shows $f_{y_{10:10}}(y)$ and logically high values of $f_{y_{10:10}}(y)$ correspond to high values (around $4\mu m$) of the surface roughness.

3.5. *Flowchart of the different steps of proceeding*

In this subsection, we propose a flowchart (Fig. 4) which illustrates the practical stages of our proceeding. This proposed methodology includes the following steps:

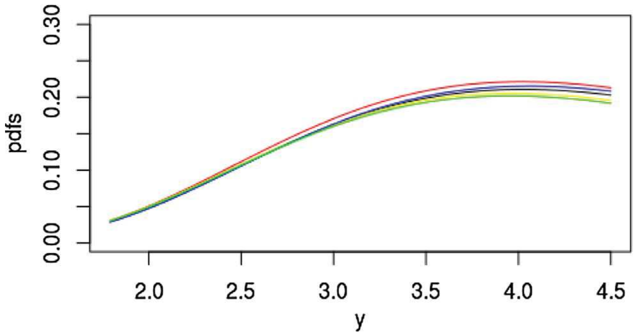

Fig. 3. Pdfs distribution of maximum for five batches.

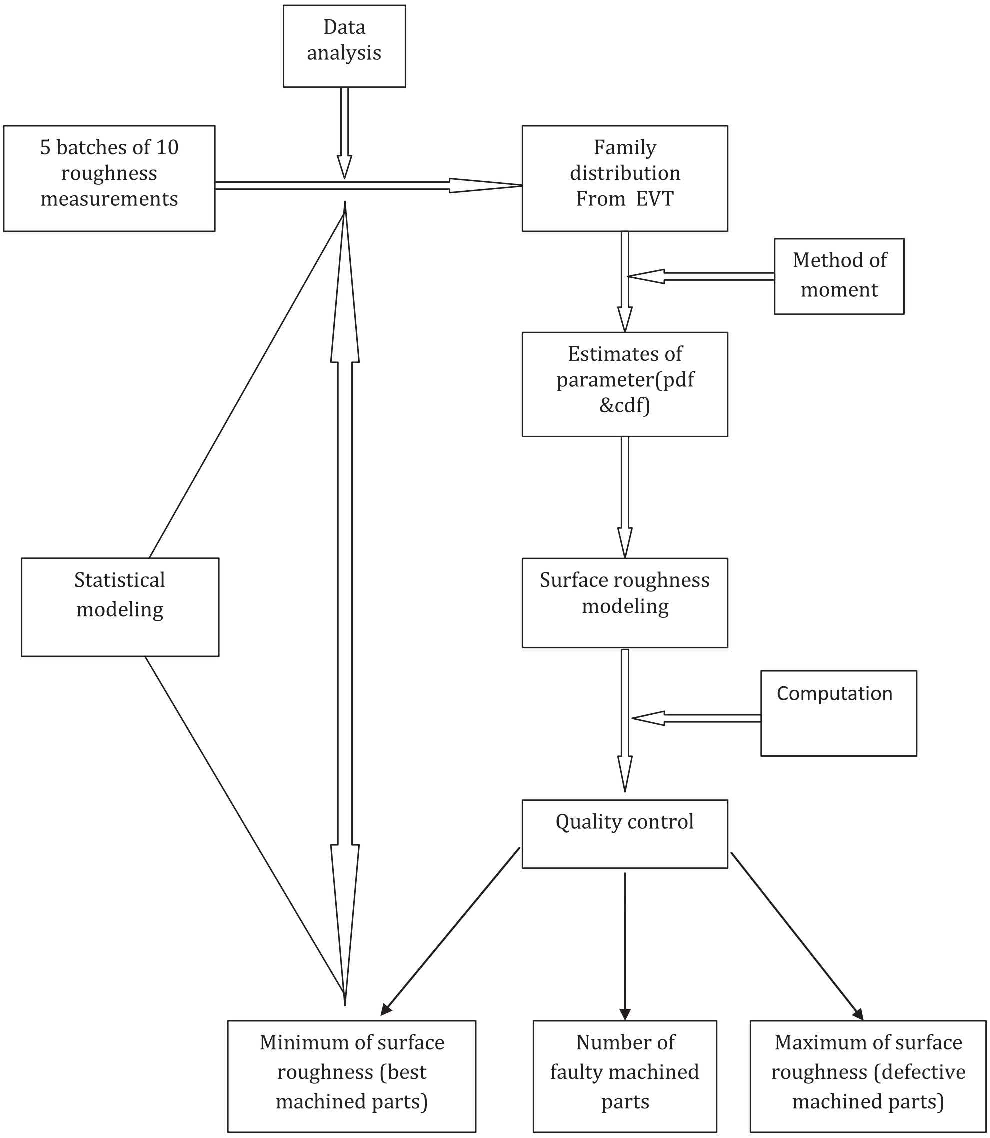

Fig. 4. Flowchart of the different stages.

- Data analysis: it concerns the analysis of raw data from industrial components;
- Method of moment: the method of moments is a technique commonly used in the field of parameters estimation;
- Computation: it concerns the use of cdf and order statistics to identify the faulty machined parts in each batch;

Finally, we have the best values of surface roughness, the number of faulty machined parts and the corresponding roughness values in each batch.

4. Conclusion

In manufacturing, preliminary steps consist of defining the quality criteria expected when the process will be operated in line production. These steps are achieved by making pre-series.

For this purpose, we have considered a case study which is a turning process with some specific cutting conditions. We have proceeded the machining of 5 batches of 10 parts. The quality criteria in our case study are twofold: the minimum surface roughness for each part and the maximum number of faulty parts in a batch. A part is considered as faulty when the surface roughness is greater than $4\mu m$. It is shown that the dataset follows a Weibull distribution with shape parameters ($k > 1$).

Extreme order statistics in surface roughness were applied to highlight the 2 considered criteria.

The computation of the *cdf* of order statistic has revealed a number of faulty parts whereas the minimum distributions give the best interval, [1.79,…, 2.25], of the surface roughness. The reliability of the cutting tool around 4 μm is not satisfactory; this result confirms the best interval of roughness measurement for machined parts.

As a classical limit of this study one can argue that 5 batches could be too small to be sure that the good criteria were found. Nevertheless, by keeping the same procedure, the dataset can surely be enhanced with measurements of the line at the beginning of the parts production.

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